On analytic expressions for fast estimation of coupling between electrically short thin-wire antennas within cavities

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Abstract

Method of moment solutions for thin-wire antenna currents can turn to analytic solutions if the antennas are electrically small. This observation is useful for the analysis of interior problems of Electromagnetic Compatibility when transmitting and receiving thin-wire antennas serve to model electromagnetic sources and victims within a resonating environment. The analytic solutions allow fast evaluation of corresponding couplings but also are of limited accuracy. This is exemplified by the example of two coupled antennas within a rectangular cavity where coupling results between an analytical solution and an actual method of moment solution are compared.

1 Introduction

In Electromagnetic Compatibility (EMC) it is often of interest to model electromagnetic interference (EMI) sources and victims by means of transmitting and receiving antennas [1]. This also applies to the interior EMC problem, where EMI sources and victims are enclosed by a resonating environment, such as given by the interior of a cavity. To calculate antenna coupling within a cavity it is necessary to merge concepts of conventional antenna theory with those of microwave theory. In antenna theory the main task is to calculate antenna currents and the usual approach to do this is to solve for a given electromagnetic excitation an integral equation for the unknown current [2]. On the other hand, in microwave theory the main task is to determine the electromagnetic field within a waveguide or cavity and the usual approach is to construct an appropriate Green's function [3]. Once the Green's function is constructed from the solution of a boundary value problem, which reflects the geometry of the waveguide or cavity, the principal structure of the electromagnetic field is known. It is given in terms of the eigenmodes of the cavity and which eigenmodes actually are excited depends on the electromagnetic excitation It follows that the main task of antenna theory in resonating environments is to solve integral equations with Green's functions as kernels which incorporate the properties of eigenmodes. The main difficulties that are encountered in the solution of this class of integral equations are of a numerical nature and given by the fact that the Green's function of a resonating system exhibits the two complementary singularities of the electromagnetic field, namely the Coulomb singularity and electromagnetic resonances [4]. In a cavity the presence of resonances adds singular field effects and these additional effects will also be reflected by antenna characteristics. It is of primary interest to calculate these additional effects. Single resonances can have a dominating influence and drastically change the behavior of antenna configurations inside resonating systems if compared to free space [5].

2 Electrically short antennas

As pointed out in the introduction, antenna currents can be determined from integral equations that can be solved by the method of moments. Usually, the method of moments is taken as a numerical solution procedure, but it can be turned into an approximative, analytical solution procedure if the unknown functions, that is, the antenna currents, are approximated by only a few basis functions. For linear antennas, sinusoidal basis are often chosen, which, for z-directed antennas, are of the form [9]

$$S_k(\boldsymbol{r}^{(j)}) = \begin{cases} \frac{\sin k(z^{(j)} - z_k^{(j)})}{\sin kh^{(j)}}, & \text{if} \quad z_{k-1}^{(j)} \leq z^{(j)} \leq z_k^{(j)} \\ \frac{\sin k(z_{k+1}^{(j)} - z^{(j)})}{\sin kh^{(j)}}, & \text{if} \quad z_k^{(j)} \leq z^{(j)} \leq z_{k+1}^{(j)} \\ 0, & \text{else} \end{cases} \qquad k = 1, \dots 2M-1\,. \tag{1}$$

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where it is assumed that the antenna is indexed by j, divided into $2M$ intervals of length $h^{(j)} = L^{(j)}/(2M)$, and that the $2M-1$ points between adjacent intervals are chosen as matching points $\boldsymbol{r}_k^{(i)}$. Then the resulting linear system of equations can analytically be solved, as shown by example below. Generally, the approximation of an antenna current by only a few terms is physically meaningful if the antenna considered is electrically small, $kL \ll 1$.

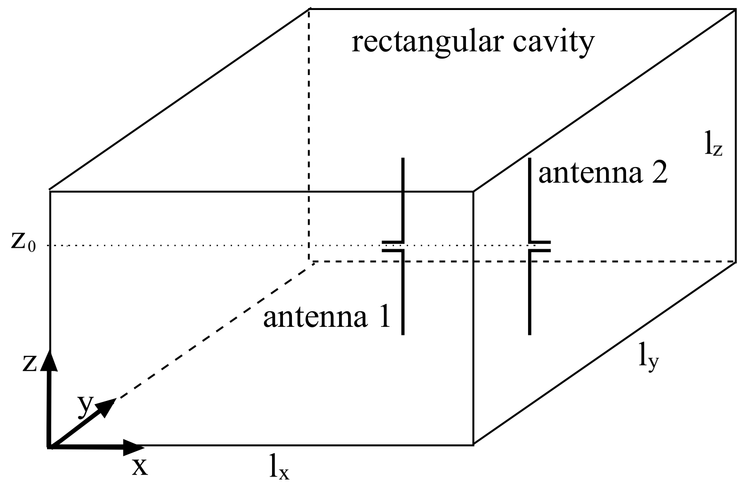

Figure 1: Two straight dipole antenna which are aligned to the z-axis and placed within a rectangular cavity of dimensions l_x, l_y, and l_z.

An antenna-cavity configuration as given by Fig. 1 is considered. The rectangular cavity is only chosen as an example, the following formulas apply to any closed cavity. Also the relative orientation of the antennas could be more general. To calculate the antenna coupling between both antennas one may concentrate on the expression for their mutual impedance [6],

$$Z_{12} = -\frac{\langle \boldsymbol{E}^b, \boldsymbol{J}^a \rangle_p}{I_2^a I_1^b} . \tag{2}$$

With reference to the method of moments [7, 8] the antenna currents are expanded by means of one sinusoidal basis function

$$I_1^b(\boldsymbol{r}^{(1)}) = \alpha^{(1)} S_1(\boldsymbol{r}^{(1)}) , \tag{3}$$

$$I_2^a(\boldsymbol{r}^{(2)}) = \alpha^{(2)} S_1(\boldsymbol{r}^{(2)}) . \tag{4}$$

The input currents of the antennas are simply given by the (unknown) expansion coefficients,

$$I_1^b = \alpha^{(1)} , \tag{5}$$

$$I_2^a = \alpha^{(2)} . \tag{6}$$

To evaluate the inner product of (2) the electric field is first evaluated by an elementary calculation, compare [9],

$$E_z^b(\boldsymbol{r}^{(2)}) = -j\omega\mu \int_{\text{antenna 1}} G_{zz}^E(\boldsymbol{r}^{(2)}, \boldsymbol{r}^{(1)}) \alpha^{(1)} S_1(\boldsymbol{r}^{(1)})\, d\boldsymbol{r}^{(1)} \tag{7}$$

$$= -\frac{j\omega\mu\alpha^{(1)}}{k\sin(kh)} \left[G_{zz}^A(\boldsymbol{r}_2, \boldsymbol{r}_1 + L/2\boldsymbol{e}_z) - G_{zz}^A(\boldsymbol{r}_2, \boldsymbol{r}_1 - L/2\boldsymbol{e}_z) \right.$$
$$\left. -2\cos(kL/2) G_{zz}^A(\boldsymbol{r}_2, \boldsymbol{r}_1) \right] . \tag{8}$$

Here, the position vectors $\boldsymbol{r}_1, \boldsymbol{r}_2$ point to the centers of the antennas. The symbols G_{zz}^E and G_{zz}^A denote the zz-component of the dyadic Green's function for the electric field and the vector potential in Lorenz gauge, respectively.

With (5), (6), and (8) the expression for the mutual impedance (2) becomes

$$Z_{12} = \frac{j\omega\mu}{k\sin(kL/2)} \left[G^A_{zz}(\boldsymbol{r}_2, \boldsymbol{r}_1 + L/2\boldsymbol{e}_z) - G^A_{zz}(\boldsymbol{r}_2, \boldsymbol{r}_1 - L/2\boldsymbol{e}_z) \right.$$
$$\left. -2\cos(kL/2) G^A_{zz}(\boldsymbol{r}_2, \boldsymbol{r}_1)\right] \int_{\text{antenna2}} S_1(\boldsymbol{r}^{(2)})\, d\boldsymbol{r}^{(2)} \,. \tag{9}$$

The last integral can be approximated according to

$$\int_{\text{antenna2}} S_1(\boldsymbol{r}^{(2)})\, d\boldsymbol{r}^{(2)} = \frac{2}{\sin(kL/2)} \left(1 - \cos(kL/2)\right) \tag{10}$$
$$\approx \frac{L}{2} \tag{11}$$

and one obtains as final result the analytic expression

$$Z_{12} = \frac{j\omega\mu L}{2k\sin(kL/2)} \left[G^A_{zz}(\boldsymbol{r}_2, \boldsymbol{r}_1 + L/2\boldsymbol{e}_z) - G^A_{zz}(\boldsymbol{r}_2, \boldsymbol{r}_1 - L/2\boldsymbol{e}_z) \right.$$
$$\left. -2\cos(kL/2) G^A_{zz}(\boldsymbol{r}_2, \boldsymbol{r}_1)\right] \,. \tag{12}$$

In Fig. 2 the mutual impedance as calculated by the approximate, analytic result (12) is compared to an ordinary method of moment solution of Hallén’s equation [10], utilizing several basis functions. The lengths of the antennas are taken as L=0.2m, the cavity dimensions are given by $l_x = 6$m, $l_y = 7$m, and $l_z = 3$m. It is recognized that the approximate solution reproduces well the major resonance peaks that occur whenever both antennas couple to a common mode. Differences are observed inbetween these resonances, but the qualitative behavior of the curves agree fairly well. Of course, the advantage of the approximate solution is that it is calculated much faster if compared to the method of moment solution and also provides a final answer in terms of formula (12).

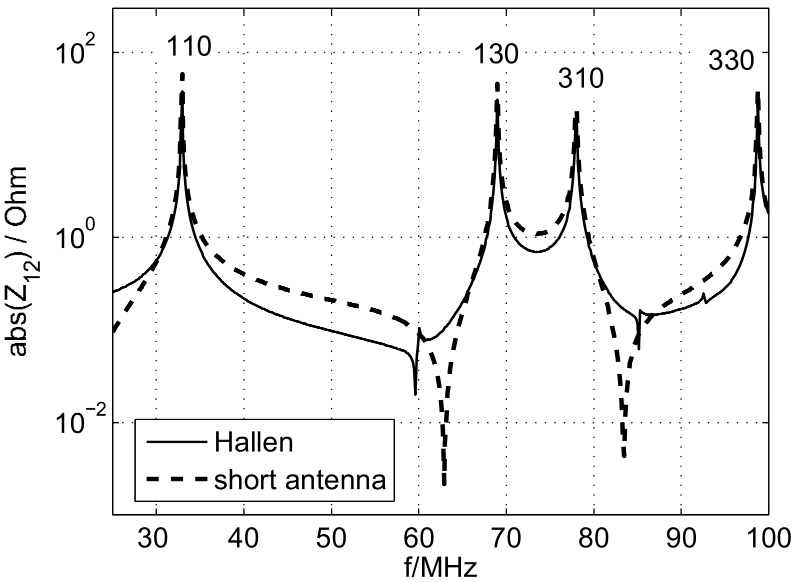

Figure 2: Mutual impedance between electrically short antennas, calculated by a method of moments solution of Hallén’s equation and the approximate, analytic result (12) which is based on the assumption of electrically short antennas. The numbers in the figure indicate resonance peaks that correspond to eigenmodes of the rectangular cavity.

3 Concluding remarks

Clearly, as one turns to the simplified models of electrically short antennas, one generally will no longer be able to take into account detailed features of the antenna current. For example, the approximation by a simple sinusoidal function will fail to yield any reliable value for the input impedance of an antenna. Therefore, turning to electrically short antennas means turning away from a number of details and subtleties of antenna theory. If antennas are drastically reduced in size to represent localized current elements that do not exhibit their own degrees of freedom one is left to study the behavior of the electromagnetic field. Then an antenna theory in resonating environments reduces to conventional microwave theory. Models of electrically short antennas nevertheless are useful to get an overall understanding of antenna coupling in resonating environments and also allow to estimate the order of coupling effects that are of practical interest. This has been exemplified in particular in the works of Tkachenko, see [11, 12], for example.

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