The Analysis of Stochastic EMC Problems Using the Concept of Diffuse Field Reciprocity

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Abstract

There has been much progress in recent years in the analysis of complex random vibro-acoustic systems, and general analysis methods have been developed which are based on the properties of diffuse wave fields. It is shown in the present paper that such methods can also be applied to high frequency EMC problems, avoiding the need for costly full wave solutions to Maxwell's equations in complex cavities. The theory behind the approach is outlined and then applied to the relatively simple case of a wiring system which is subject to reverberant electromagnetic wave excitation.

1. Introduction

There are many practical situations of interest in which wiring systems are subjected to potentially harmful electromagnetic excitation; examples include the response of automotive electric systems to external mobile phone transmitters [1], or to internal bluetooth devices [2], and the threat of electromagnetic pulse weapons to aircraft [3]. This type of problem is characterised by the fact that some form of excitation produces an electromagnetic field inside the vehicle, which can then interfere with the vehicle electronic systems. Often the frequency of the electromagnetic excitation is relatively high, so that the electromagnetic wavelength is short in comparison to the dimensions of the vehicle interior, meaning that the electromagnetic field produced within a typical automotive cabin will have a spatially complex distribution. In principle, such fields can be predicted numerically by solving Maxwell's equations within the vehicle, although very many grid points will be required by either the finite element method or the finite difference method to capture the detailed spatial distribution of the field [2]. A further consequence of the short wavelength excitation is that the response of a system can be very sensitive to detail; for example, moving a wiring harness by several centimetres can significantly change the resulting currents, as can small random changes in the surrounding vehicle structure. Holland and St John [4], Ladbury et al. [5] and Hill [6] have put forward strong arguments for a statistical view of high frequency electromagnetics, and this can be approached by either randomising existing detailed deterministic models (which may be computationally unfeasible), or by directly exploiting the statistics of short wavelength reverberant fields. With regard to the latter approach, there is strong evidence to suggest that a short wavelength electromagnetic field inside a vehicle cabin can be well approximated as an ideal diffuse wavefield [1,6,7]. Exploiting this fact can lead to an analysis method which is much more efficient than a direct deterministic calculation, while also being more realistic from a statistical point of view (for example [8,9]). Such statistical methods are remarkably similar in philosophy to techniques that have been developed over recent decades for the analysis of high frequency vibro-acoustic problems, where the concern is with the interaction of structural components (beams, plates, and shells) and acoustic volumes (see for example the text by Lyon and DeJong [10] for an overview of this subject). The aim of the present paper is to adapt methods which have been developed recently in the field of vibro-acoustics, in order to produce a computationally efficient technique for predicting the response of wiring systems to short wavelength reverberant electromagnetic fields.

The key result from the vibro-acoustics literature which will be employed in the present work is known as the "diffuse field reciprocity relation" [11-13]. This relation expresses the excitation arising from a random diffuse wave field (for example, an acoustic field in a room or an electromagnetic field in a cavity) in terms of the radiation properties of the excited system. Thus the relatively difficult problem of computing the random excitation arising

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from a complex reverberant field is replaced by the relatively easy problem of computing the radiation characteristics of the system into free space. The general basis of the approach is outlined in Section 2, and the method is then applied to the electromagnetic excitation of a two-wire system in Section 3. A full description of the approach is given in reference [14].

2. Outline of the Method

Consider a system (for example a wiring system) which is to be subjected to harmonic reverberant electromagnetic excitation of frequency ω. Let the surface of the system be represented by S, and let $\mathbf{e}$ and $\mathbf{h}$ respectively represent generalised coordinates that are used to describe the surface (tangential) components of the electric and magnetic fields, i.e. given knowledge of $\mathbf{e}$ and $\mathbf{h}$, the tangential components of the electromagnetic field at every point on the surface are fully described. Consider initially the case in which the system is radiating into free space in the absence of any reverberant excitation: the fact that Maxwell's equations must be satisfied in free space imposes a relation between $\mathbf{e}$ and $\mathbf{h}$ in the form

$$\mathbf{Z}_{\mathrm{D}}\mathbf{h} = \mathbf{e}\,, \qquad \mathbf{Z}_{\mathrm{D}} = (\mathbf{Z}_{\mathrm{D}} + \mathbf{Z}_{\mathrm{D}}^{*\mathrm{T}})/2 + (\mathbf{Z}_{\mathrm{D}} - \mathbf{Z}_{\mathrm{D}}^{*\mathrm{T}})/2 = \mathbf{Z}_{\mathrm{DH}} + \mathbf{Z}_{\mathrm{DA}}\,, \tag{1,2}$$

where $\mathbf{Z}_{\mathrm{D}}$ is an impedance matrix which can be represented as the sum of a Hermitian part $\mathbf{Z}_{\mathrm{DH}}$ and an anti-Hermitian part $\mathbf{Z}_{\mathrm{DA}}$ as shown in equation (2). The physical significance of the Hermitian part of the impedance matrix is that the time averaged power P radiated by the system has the form

$$P = (1/2)\mathbf{h}^{*\mathrm{T}}\mathbf{Z}_{\mathrm{DH}}\mathbf{h}\,. \tag{3}$$

The impedance matrix can be calculated by any relevant approach including, for example, the method of moments [15,16]. When the system is placed inside a cavity and subjected to a reverberant electromagnetic field then equation (1) must be modified; also, an additional equation must be introduced to allow for the fact that Maxwell's equations must be satisfied *inside* the system. These considerations lead to the equations

$$\mathbf{Z}_{\mathrm{D}}(\mathbf{h} - \mathbf{h}_{\mathrm{inc}}) = \mathbf{e} - \mathbf{e}_{\mathrm{inc}}\,, \qquad \mathbf{Z}_{\mathrm{C}}\mathbf{h} = -\mathbf{e}\,, \tag{4,5}$$

where $\mathbf{h}_{\mathrm{inc}}$ and $\mathbf{e}_{\mathrm{inc}}$ are respectively the magnetic and electric field components arising from the incident reverberant field (in the absence of any scattering or diffraction), and $\mathbf{Z}_{\mathrm{C}}$ is the impedance matrix associated with the material that lies inside the surface S. Equations (4) and (5) then yield

$$(\mathbf{Z}_{\mathrm{D}} + \mathbf{Z}_{\mathrm{C}})\mathbf{h} = \mathbf{Z}_{\mathrm{D}}\mathbf{h}_{\mathrm{inc}} - \mathbf{e}_{\mathrm{inc}} = -\mathbf{e}_{\mathrm{b}}\,, \tag{6}$$

where, to borrow vibro-acoustics terminology, $\mathbf{e}_{\mathrm{b}}$ is referred to here as the *blocked* electric field, representing the reverberant excitation acting on the system. Equation (6) can be solved to yield the response of the system, although the prior calculation of $\mathbf{e}_{\mathrm{b}}$ requires in principle a full wave solution of Maxwell's equations within the cavity, together with a detailed consideration of the statistical uncertainties in the computed field. Such computational difficulties are avoided here by assuming that the excitation arises from a diffuse wave field, in which case the *diffuse field reciprocity relation* [11-14] can be employed, which states

$$\mathrm{E}[\mathbf{e}_{\mathrm{b}}\mathbf{e}_{\mathrm{b}}^{*\mathrm{T}}] = \left(\frac{4U}{\pi\nu}\right)\mathbf{Z}_{\mathrm{DH}}\,, \qquad U = (1/2)\mu V\mathrm{E}\left[|\mathbf{H}|^2\right], \qquad \nu = V\omega^2/(\pi^2 c^3)\,, \tag{7-9}$$

where U is the electromagnetic energy of the field in the cavity, $\mathbf{H}$ is the magnetic field vector, and ν is the cavity modal density, i.e. the average number of electromagnetic resonances in a unit frequency band. The symbol E[] in equations (7) and (8) represents an ensemble average, i.e. an average taken over a collection of random cavities having slightly different properties. If the strength of the excitation is specified (represented by U), then equations (6) and (7) yield the response of the system in the form

$$\mathrm{E}[\mathbf{h}\mathbf{h}^{*\mathrm{T}}] = \left(\frac{4U}{\pi\nu}\right)(\mathbf{Z}_{\mathrm{D}} + \mathbf{Z}_{\mathrm{C}})^{-1}\mathbf{Z}_{\mathrm{DH}}(\mathbf{Z}_{\mathrm{D}} + \mathbf{Z}_{\mathrm{C}})^{-\mathrm{T}*} . \tag{10}$$

Equation (10) states that the response of the system to reverberant excitation can be expressed simply in terms of the free-space and internal impedance matrices of the system, without any need for a full wave solution of the field in the cavity. The result for the surface magnetic field (given above in terms of the mean squared field) can readily be converted into a current, thus allowing the integrity of the system to be assessed.

3. Application to a Two-Wire System

The foregoing theory has been applied to a two-wire transmission line in a cavity. A set of *N* evenly spaced reference points is placed along each wire, and the vector **e** consists of the axial component of the electric field at each point; likewise, the vector **h** represents the circumferential component of the magnetic field at each point, which is proportional to the current. The surface fields are taken to be axisymmetric around each wire, and the impedance matrices $\mathbf{Z}_{\mathrm{D}}$ and $\mathbf{Z}_{\mathrm{C}}$ have been calculated by using analytical solutions to Maxwell's equations in cylindrical coordinates [14] (the method of moments [15] or some other numerical technique could alternatively have been employed).

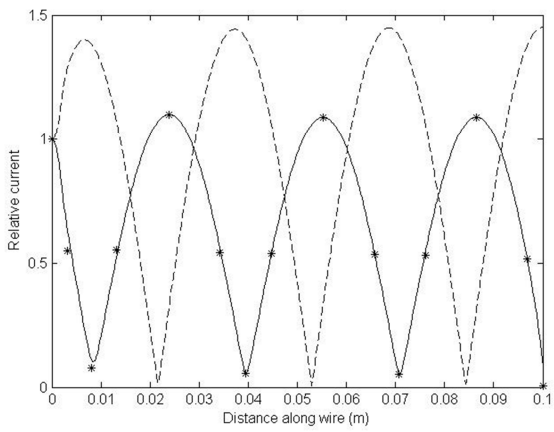

Figure 1: Response to a point voltage. Solid curve, present results; * benchmark results [16]; dashed curve, short circuit.

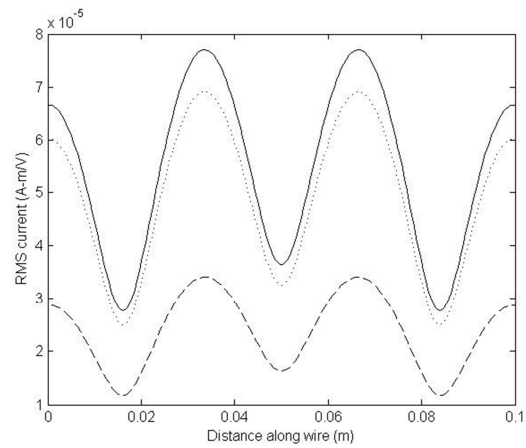

Figure 2: Response to a reverberant field. Upper curve, total current; middle curve, antenna mode; lower, transmission mode.

The wires are spaced 1 cm apart and each wire is of length 0.1m and radius 1 mm. As a validation check on the computed impedance matrices the response of the system to a voltage applied at one end was computed, and a comparison was made with previous results obtained by Maffucci [16], as shown in Figure 1. The results of reference [16] concern a wire with a zero-current boundary condition at the non-excited end of the wire; results yielded by the present method are shown in Figure 1 for both this case and the case in which the non-excited end is short circuited. Having validated the impedance matrices, the response of the system to a diffuse electromagnetic field of unit mean squared electrical field strength was computed using equation (10). The results obtained for a short circuit applied to each end of the system are shown in Figure 2. The Figure shows separately the total current and the contributions from the antenna mode current (equal current in each wire) and the transmission mode current (equal and opposite current in each wire). Clearly the current in the wire is dominated by the antenna mode, although any current transmitted to connected equipment would be governed solely by the transmission mode.

4. Conclusions

It has been shown that recent developments in the analysis of vibro-acoustic systems can also be applied to problems in electromagnetics. The present work has focussed on the use of the diffuse field reciprocity relation to avoid the need to model in detail the electromagnetic field in a cavity. The key point is that knowledge of the way in which a

system radiates in free space is sufficient to compute the response of the system to a diffuse field of electromagnetic waves. Other developments in vibro-acoustics [10-14] could be employed to allow the efficient analysis of complex multi-cavity systems, and work in this area is ongoing.

5. References

1. T. Konefal, A.C. Marvin, J.F. Dawson and M.P. Robinson, "A statistical model to estimate an upper bound on the probability of failure of a system installed in an irradiated vehicle," *IEEE Trans. Electromagn. Compat.,* **49**, Nov. 2007, pp. 840-848.

2. L. Low, H. Zhang, J. Rigelsford and R.J. Langley, "Computed field distributions within a passenger vehicle at 2.4 GHz," in *Proc. LAPC2009,* Loughborough, 2009.

3. P. Marks, "Just one pulse and the plane drops from the sky," *New Scientist,* 4th April Issue, 2009, pp. 16-17.

4. R. Holland and R.H. St. John, *Statistical Electomagnetics.* New York: Taylor Francis, 1999.

5. J.M. Ladbury, T.H. Lehman and G.H. Koepke, "Coupling to devices in electrically large cavities, or why classical EMC evaluation techniques are becoming obsolete," in *Proc. EMC 2002, IEEE International Symposium on Electromagnetic Compatibility*, Minneapolis, 19-23 August, 2002, pp. 648-655.

6. D.A. Hill, *Electromagnetic Fields in Cavities: Deterministic and Statistical Theories.* Piscataway, NJ: Wiley-IEEE Press, 2009.

7. D.A. Hill and J.M. Ladbury, "Spatial-correlation functions of fields and energy density in a reverberation chamber," *IEEE Trans. Electromagn. Compat.,* **44**, Feb. 2002, pp95-101.

8. I. Junqua, J.-P. Parmantier and P. Degauque, "Coupling on cables in an electrically large cavity:a theoretical approach," in *Proc. IEEE Int. Symp. Electomagnetic Compatibility,* Zurich, 2009, pp. 89-92.

9. I. Junqua, J.-P. Parmantier and F. Issac, "A network formulation of the power balance method for high frequency coupling," *Electromagnetics*, **25**, 2005, pp 603-622.

10. R.H. Lyon and R.G. DeJong, *Theory and Application of Statistical Energy Analysis*. Boston: Butterworth-Heinemann, 1995.

11. P.J. Shorter and R.S. Langley, "On the reciprocity relationship between direct field radiation and diffuse reverberant loading," *Journal of the Acoustical Society of America,* **117**, 2005, pp 85-95.

12. R.S. Langley, "On the diffuse field reciprocity relation and vibrational energy variance in a random subsystem at high frequencies," *Journal of the Acoustical Society of America,* **121**, 2007, pp 913-921.

13. P.J. Shorter and R.S. Langley, "Vibro-acoustic analysis of complex systems," *Journal of Sound and Vibration,* **288**, 2005, pp 669-700.

14. R.S. Langley, "A reciprocity approach for computing the response of wiring systems to diffuse electromagnetic fields," *IEEE Transactions on electromagnetic Compatibility,* **52**, 2010, pp 1041-1055.

15. D.B. Davidson, *Computational Electromagnetics for RF and Microwave Engineering.* Cambridge: Cambridge University Press, 2005.

16. A. Maffucci, G. Miano and F. Villone, "An enhanced transmission line model for conducting wires," *IEEE Trans. Electromagn. Compat.,* **46**, Nov. 2004, pp 512-528.