Statistical description of shielding problems using multipole analysis

***K. Körber*[1] *and* *L. Klinkenbusch*[2]**

Technische Fakultät der CAU Kiel, Kaiserstr. 2, 24143 Kiel, Germany,

[1] email: kko@tf.uni-kiel.de, phone: +49 431 880 6257, fax: +49 431 880 6253
[2] email: lbk@tf.uni-kiel.de, phone: +49 431 880 6252, fax: +49 431 880 6253

Abstract

The paper describes the influence of small statistical variations in the physical parameters on the shielding behavior of an enclosure. The varying parameters are given in terms of the first few statistical moments of their distributions. From these the amplitudes of a spherical-multipole expansion are derived analytically. These multipole amplitudes contain the statistical properties of the electromagnetic field valid not only in a single point but in a spherical region around the center of the expansion inside of a shield. Two first examples demonstrate the validity of the approach.

1 Introduction

Recent research in the field of statistical electromagnetics in general and statistical EMC in particular seems to focus on extremely non-deterministic fields as observed in a reverberation chamber. Much progress has been achieved in this area over the last decades, leading to very general statistical power distributions largely independent of the chamber geometry [1]. Comparatively less work has been done on the topic of small statistical variations of a known electromagnetic field impinging on a shield [2].

In this paper we discuss the case of small statistical variations in the physical parameters of a shielding problem. The varying parameters are given by the first few statistical moments of their distributions. From these the statistical moments of the amplitudes of a spherical-multipole expansion are derived analytically. These multipole amplitudes also contain the statistical properties of the electromagnetic field in a spherical region around the center of expansion which typically is located inside of a shield. For non-canonical problems the calculation of the multipole amplitudes cannot be achieved analytically and measurement or numerically obtained data are needed to estimate their statistical multipole-moments.

2 Theory

2.1 Spherical multipole analysis

Inside of a homogeneous spherical region (e.g. vacuum) any time-harmonic electromagnetic field can be described by means of a spherical-multipole expansion with complex-valued coefficients (multipole amplitudes) $A_{n,m}$ and $B_{n,m}$:

$$\begin{aligned} \mathbf{E} &= \sum_{n,m} A_{n,m}\mathbf{N}_{n,m} + \frac{Z}{j}\sum_{n,m} B_{n,m}\mathbf{M}_{n,m} \\ \mathbf{H} &= \frac{j}{Z}\sum_{n,m} A_{n,m}\mathbf{M}_{n,m} + \sum_{n,m} B_{n,m}\mathbf{N}_{n,m} \end{aligned} \tag{1}$$

where n, m are integers with $n > 0$, $-n \leq m \leq n$, $\mathbf{N}_{n,m}$ and $\mathbf{M}_{n,m}$ are the vector spherical-multipole functions, and Z is the wave impedance [3].

2.2 Statistical moments of random variables

A random vector [4] $\mathbf{X} = (X_1, \ldots, X_m)$ is described by its probability density function $f_x(x_1, \ldots, x_m)$ or alternatively by its moments. We are especially interested in the first order moments (expected values)

$$\eta_{X_i} = \int x_i f_x(x_1, \ldots, x_m) dx_1 \ldots dx_m \tag{2}$$

978-1-4244-6051-9/11/$26.00

the central second order moments (variances and covariances)

$$C_{X_i,X_j} = \int (x_i - \eta_{X_i})(x_j - \eta_{X_j}) f_x(x_1, \ldots, x_m) dx_1 \ldots dx_m \quad (3)$$

as well as the central moments of third and fourth order. A second random vector $\mathbf{Y} = (Y_1, \ldots, Y_n)$ depending on the random vector $\mathbf{X}$ according to the functions

$$Y_\nu = g_i(X_1, \ldots, X_m) \quad (4)$$

can be calculated in terms of its moments, if the moments of $\mathbf{X}$ and the functions g_i are known. We use this fact to calculate the moments first of the multipole amplitudes and then of the electromagnetic field inside of a shield, with respect to some variations in the impinging field.

3 Statistical formulation of the spherical-multipole analysis

We consider the multipole amplitudes of the field to be random variables. If the multipole expansion is cut off at $n = n_{max}$ there are $(n_{max}^2 + 2n_{max})$ $A_{n,m}$'s and $B_{n,m}$'s, which each consist of two real components. This leads to a random vector $\mathbf{X}$ consisting of $4(n_{max}^2 + 2n_{max})$ real random variables:

$$\mathbf{X} = (\mathrm{Re}\{A_{1,-1}\}, \mathrm{Im}\{A_{1,-1}\}, \mathrm{Re}\{B_{1,-1}\}, \mathrm{Im}\{B_{1,-1}\}, \mathrm{Re}\{A_{1,0}\}, \ldots, \mathrm{Im}\{B_{n_{max},n_{max}}\}) \quad (5)$$

The electromagnetic field (at a given point) is a random vector of length twelve:

$$\mathbf{Y} = (\mathrm{Re}\{E_x\}, \mathrm{Im}\{E_x\}, \mathrm{Re}\{E_y\}, \mathrm{Im}\{E_y\}, \mathrm{Re}\{E_z\}, \ldots, \mathrm{Im}\{H_z\}) \quad (6)$$

Since the relation (1) between $\mathbf{X}$ and $\mathbf{Y}$ is linear, the moments of $\mathbf{Y}$ can be derived relatively simply from those of $\mathbf{X}$:

$$\eta_{Y_\nu} = g_\nu(\eta_X) \quad (7)$$

$$C_{Y_\nu Y_\mu} = \sum_{i=1}^{m} \sum_{j=1}^{m} C_{X_i X_j} \left.\frac{\partial g_\nu}{\partial x_i}\right|_{\eta_X} \left.\frac{\partial g_\mu}{\partial x_j}\right|_{\eta_X} \quad (8)$$

Higher orders of statistical moments are calculated similarly. So the statistical moments of the field components can easily be calculated from the statistical moments of the multipole amplitudes.

4 First Results

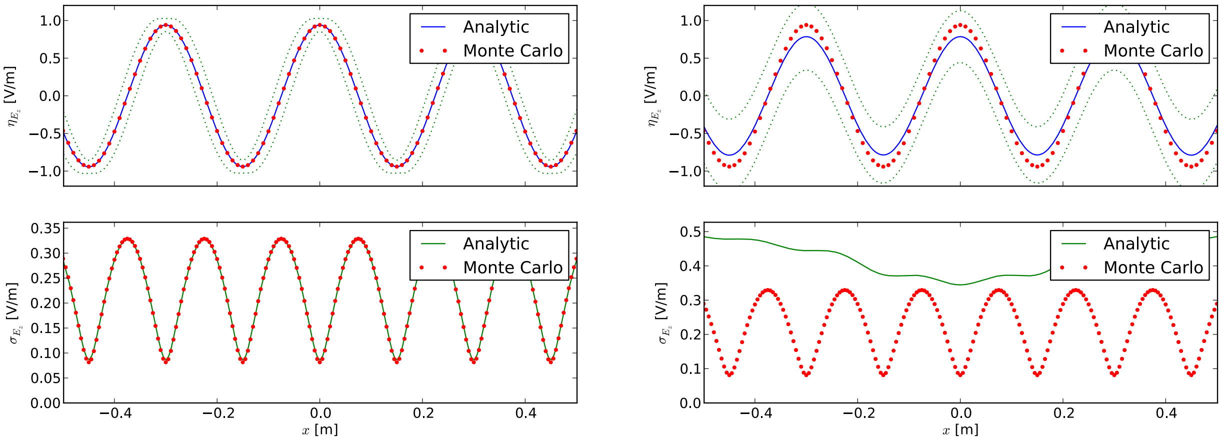
Two canonical examples will be presented. To introduce a statistical variation we consider one parameter X to be normally distributed with estimated value η_X and standard deviation σ_X. It follows that only central moments of even order are non-vanishing and that the fourth central moment equals $3\sigma_X^4$ [5]. Higher orders are not considered in the following examples. If we furthermore neglect those terms with sixth and higher orders of σ_X, the estimated values and (co-)variances of the multipole amplitudes $\mathbf{Y}$ (these are equal to $\mathbf{X}$ in equation (5)) can be calculated as follows:

$$\eta_{Y_\nu} = g_\nu(\eta_X) + \frac{\sigma_X^2}{2} g_\nu''(\eta_X) + \frac{3\sigma_X^4}{24} g_\nu^{(4)}(\eta_X) \quad (9)$$

$$C_{Y_\nu Y_\mu} = \sigma_X^2 \left[g_\nu'(\eta_X) g_\mu'(\eta_X)\right] + \frac{\sigma_X^4}{2} \left[g_\nu'''(\eta_X) g_\mu'(\eta_X) + g_\nu''(\eta_X) g_\mu''(\eta_X) + g_\nu'(\eta_X) g_\mu'''(\eta_X)\right] \quad (10)$$

4.1 Arbitrary field with varying phase

Beginning with any known electromagnetic field given in terms of its multipole amplitudes $A'_{n,m}$ and $B'_{n,m}$ we introduce a statistical variation by adding a Gauss-distributed shift of the phase ψ with expected value $\eta_\psi = 0$ and standard deviation $\sigma_\psi > 0$.



a) Correlation accounted for

b) Correlation not accounted for

Figure 1: Plane wave with varying phase $\sigma_\psi = \pi/9$

Figure 1a illustrates the results for a plane wave travelling in x-direction and polarized in z-direction with a varying phase of $\sigma_\psi = \pi/9$. It shows the estimated value (top blue line) and standard deviation (bottom green line) of E_z as calculated analytically in contrast to a Monte Carlo approach (red dots) of 10^5 plane waves with the same variation in phase. The upper figure also shows the estimated value plus/minus one standard deviation (small green dots). As expected the varying phase leads to a maximum field variation at the inflection points of the sine curve and is minimal at the extrema. Analytical and numerical results are in good agreement for the chosen variance of $\sigma_\psi = \pi/9$. Including the multipole amplitudes' covariances—i.e. the off-diagonal elements of the covariance matrix $C_{Y_\nu Y_\mu}$—is essential. Figure 1b shows the result for the same calculation as in 1a but without taking into account correlation. The influence of higher order correlation remains to be analyzed.

4.2 Plane wave with varying angle of incidence

The multipole amplitudes $A^{pw}_{n,m}$ and $B^{pw}_{n,m}$ of a plane wave incident from (ϑ_0, φ_0) and polarized in $\hat{\xi}$-direction can be given using the transverse vector functions [3] $\mathbf{n}_{n,m}(\vartheta, \varphi)$ and $\mathbf{m}_{n,m}(\vartheta, \varphi)$

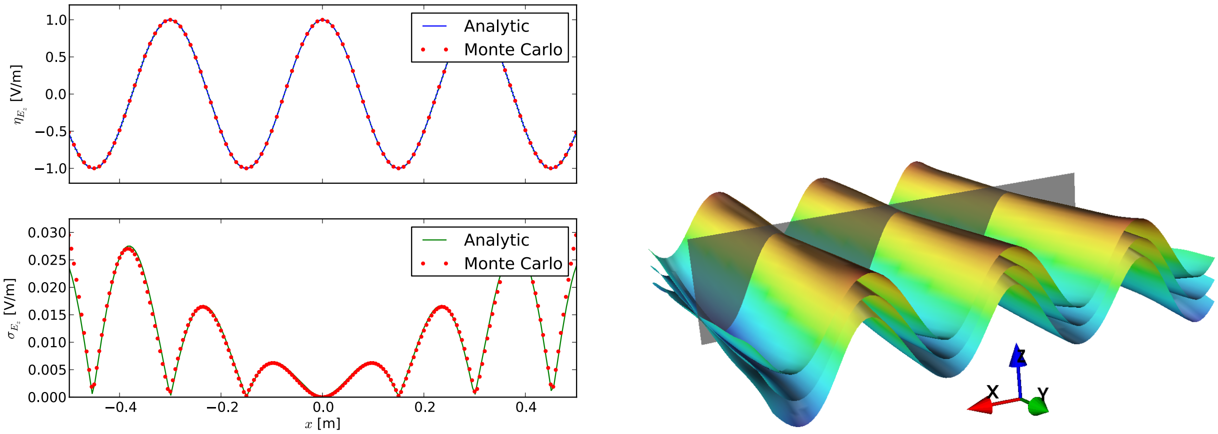
$$\begin{aligned} A^{pw}_{n,m} &= E_0 4\pi (j)^{n+1} \tfrac{(-1)^m}{n(n+1)} \left[\mathbf{n}_{n,-m}(\vartheta, \varphi) \cdot \hat{\xi}\right]_{\vartheta=\vartheta_0, \varphi=\varphi_0} \\ B^{pw}_{n,m} &= \tfrac{E_0}{Z} 4\pi (j)^{n+1} \tfrac{(-1)^m}{n(n+1)} \left[\mathbf{m}_{n,-m}(\vartheta, \varphi) \cdot \hat{\xi}\right]_{\vartheta=\vartheta_0, \varphi=\varphi_0} \end{aligned} \tag{11}$$

Assuming the angle of incidence φ_0 to be normally distributed with $\sigma_{\varphi_0} = 4^\circ$ leads to the fields displayed in Figure 2. 2a again shows the estimated value and standard deviation along the x-axis for the analytical (lines) and the numerical (red dots) method. Figure 2b depicts η_{E_z} as well as $\eta_{E_z} \pm \sigma_{E_z}$ in the (x, y)-plane with the plane cut indicating the position of the plot in Figure 2a. These results show good agreement between analytical and numerical (Monte-Carlo) data.

4.3 Outlook: Slitted cylinder in two dimensions

The slitted cylinder is a two-dimensional shielding structure where an approach similar to the spherical multipole expansion can be used. It serves as a more interesting example for the possibilities our method offers, while still being relatively easy to calculate.

To investigate the shielding properties we use as a first figure of merit the absolute value of the electric field, while the shield is illuminated by a plane wave with varying parameters. Since the phase of the incoming wave has no influence on the shielding, we determine the impact of a varying angle of incidence.



a) $y = 0, z = 0$ b) $z = 0$

Figure 2: Plane wave with varying angle of incidence $\sigma_{\varphi_0} = 4°$

5 Summary

We presented a technique to calculate the electromagnetic field (e.g. inside a shield) in terms of multipole amplitudes, while small variations in the parameters of the impinging field are present. First results were given for two canonical cases involving a plane electromagnetic wave with either varying phase or varying angle of incidence.

Current work includes the application of this method to practically more interesting canonical structures like the slitted circular cylinder. The determination of the multipole amplitudes for slightly varying conditions could then be used to estimate their statistical moments and thus extend our method to real shields.

6 References

[1] R. Holland and R. St. John. *Statistical Electromagnetics*. Taylor and Francis, 1999.

[2] A. Ajayi, P. Ingrey, P. Sewell, and C. Christopoulos. Direct Computation of Statistical Variations in Electromagnetic Problems. *Electromagnetic Compatibility, IEEE Transactions on*, 50(2):325–332, may 2008.

[3] L. Klinkenbusch. On the shielding effectiveness of enclosures. *Electromagnetic Compatibility, IEEE Transactions on*, 47(3):589–601, aug. 2005.

[4] Athanasios Papoulis and S. Unnikrishna Pillai. *Probability, random variables, and stochastic processes*. McGraw-Hill series in electrical and computer engineering. McGraw-Hill, Boston, 4. ed., internat. edition, 2008.

[5] Milton Abramowitz and Irene A. Stegun, editors. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Applied mathematics series 55. US Government Printing Office, Washington, DC, Tenth Printing, with corrections edition, 1972.