Design and Simulation of a Coaxial Exponential Transmission Line for a Half Impulse Radiating Antenna

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Abstract
This paper describes the simulation and design of an exponential matching transmission line for the connection of a 50-Ω generator to a 100-Ω Half Impulse Radiating Antenna (HIRA).

1. Introduction
The aim of the present study is the matching of a 50-Ω impulse generator to a Half Impulse Radiating Antenna (HIRA) having an input impedance \(Z_A = 100\) Ω. The impulse voltage contains energy in a band of frequencies ranging from DC to about 1 GHz. Achieving good matching at low frequencies requires the use of a large adaptor (in size), which is impractical for this application. We decided therefore to choose a minimum frequency of 50 MHz, which corresponds to the minimum working frequency of the HIRA.

The HIRA consists of a half parabolic reflector placed over a ground plane. The reflector is illuminated by two TEM Horn feeders connected to an impulse voltage source. The feeders are connected to the reflector by a set of matching resistors. A description of the ensemble can be seen in Figure 1. The HIRA is a modified monopolar version of the Full IRA, presented by Baum in [1].

2. Tapered transmission lines
Tapered transmission lines have been used as high pass matching adaptors for both harmonic and transient signals. For frequency domain applications, the work presented by Collin [2] constitutes the reference for exponentially varying tapered transmission lines. In [3], Klopfenstein proposed an optimum tapered transmission line minimizing the input reflection coefficient for a determined length. In [4], Baum and Lehr analyzed the use of tapered transmission line transformers for high voltage pulses, concluding that the an exponential tapered transmission line minimizes the pulse droop after the initial rise time. In [5], the construction of a linearly varying coaxial adaptor for HIRAs is presented.

A circuit diagram of a tapered transmission line (TL) for matching a source to a load is shown in Figure 2. The characteristic impedance of the matching line \(Z(x)\) should vary smoothly as a function of the distance \(x\), from the impedance of the source \(Z_0\) at Port 1, \(x=0\), to the impedance of the load \(Z_L\) at Port 2, \(x=L\).

The quality of the matching in the frequency domain can be expressed in terms of the reflection coefficient at Port 1 \(\Gamma(f)\) which can be evaluated [2] by solving the differential equation:
\[
\frac{d\Gamma(f)}{dx} = \Gamma(f)2j\beta - \frac{1}{2}(1-\Gamma(f)^2)\frac{d}{dx}(Ln(Z(x)))
\]  
(1)

where \( \beta = \frac{2\pi f}{v_r} \) and \( v_r \) is the wave propagation speed.

If \( Z(x) \) is assumed to vary exponentially, an analytical solution for the equation (1) can be derived [2]

\[
Z(x) = Z_0 e^{ax} \quad 0 \leq x \leq L
\]  
(2)

\[
\Gamma(f) = \frac{\sin \left( \frac{RL\sqrt{\varepsilon_r}}{2} \right)}{R \cos \left( \frac{RL\sqrt{\varepsilon_r}}{2} \right) + 2j\beta \sin \left( \frac{RL\sqrt{\varepsilon_r}}{2} \right)}
\]  
(3)

The exponential factor \( a \) can be determined as a function of the source and load impedances and the factors \( C \) and \( R \) in (3) are given by:

\[
a = \frac{1}{L} \ln \left( \frac{Z_1}{Z_0} \right)
\]  
(4)

\[
C = \ln \left( \frac{Z_L}{Z_0L} \right)
\]  
(5)

\[
R = \sqrt{4\beta^2 - C^2}
\]  
(6)

From equation (3) it can be deduced that \( J(f) \) depends on the ratio \( L(\sqrt{\varepsilon_r})/\lambda \). The bigger this factor, the faster the decrease of \( \Gamma \) as a function of frequency. However, due to space restrictions, the line should not be larger than 0.7 m. Therefore in order to increase the electric length of the line, a dielectric filling material (mineral oil) with \( \varepsilon_r = 2.2 \) was chosen.

Replacing these values in equation (3) and taking into account that the required minimum matching frequency is 50 MHz, the calculated physical and electrical lengths of the line to ensure a reflection coefficient \( \Gamma(f) < -10 \text{ dB} \) are given by:

\[
L = 0.6(m) \quad L = 0.8(m)
\]  
(7)

Figure 3 shows \( \Gamma(f) \) as a function of the frequency. It can be seen that matching criterion \( \Gamma(f) < -10 \text{ dB} \) is fulfilled for frequencies above 50 MHz.

The transfer function \( T \) between \( V_L(f) \) and \( V_i(f) \) can be evaluated as [6]:

\[
T(f) = \frac{V_L}{V_S} = \frac{Z_L}{Z_o} \frac{k_1 e^{-\beta_1 L}}{(1+k_1) + e^{-2\beta_1 L} (-1+k_1)}
\]  
(8)

where: \( k_1 = \sqrt{1 - \frac{a^2}{4\beta^2}} \) \( (9) \), \( k_2 = \sqrt{\beta^2 - \frac{a^2}{4}} \) \( (10) \)

The tapered transmission line performs a physical transition between the exit connector of the generator (Coaxial N, chassis-mount connector) and the antenna’s input connector.

At the input port, the sizes of the inner \( r_1 \) and outer \( r_2 \) conductors of the taper should correspond to the N-type chassis-mount connectors, namely: \( r_1(0) = 1.5(\text{mm}) \), \( r_2(0) = 5.2(\text{mm}) \). At the output port, the taper is connected to the
coaxial entry port of the antenna, with radiuses: \( r_1(L) = 2.5 \text{ mm} \), \( r_2(L) = 30 \text{ mm} \).

It can be seen that the radius of the inner conductor must vary from 1.5 mm to 2.5 mm over a length of 600 mm, following some progressive profile. The simplest assumption is a linear variation between these two values. Manufacturing such a geometry in a CNC machine or in a lathe is quite a challenge. In order to avoid that, the whole taper was divided into three sections, the profile of which is illustrated in Figure 5.

![Figure 5 Profile of the taper as a function of the longitudinal distance (x and r axes are not in the same scale).](image)

![Figure 6 Impedance of the tapered transmission line along the longitudinal distance.](image)

In Region 1, we have a 30-mm long, 50 \( \Omega \) coaxial conical transmission line. In this section, both the inner and the outer conductors increase linearly with the longitudinal distance. In Region 2, we have a coaxial exponential transition, with an impedance varying from 50 \( \Omega \) to 100 \( \Omega \), over a total length of 600 mm. Finally, in Region 3, we have a 30-mm long, 100 \( \Omega \) coaxial cylindrical transmission line.

### 3. Numerical Simulations

#### 3.1 Frequency Domain Simulations

The designed taper was simulated in the frequency domain using the 2D-axial symmetry, frequency-domain TM-wave module in Comsol®. The geometry used in the simulation is presented in Figure 9. It consists of a 2-D cut of the taper line. In order to compare the results with the analytical solution (equation (3)), we have considered a purely exponential profile along the taper. The inner and outer conductors are modeled as perfect electric conductors (PEC). The central axis of the coaxial is the rotational symmetry axis of the geometry. A signal, 1 W in power was applied to Port 1 (50 \( \Omega \)). At the output port (Port 2) a 100 \( \Omega \) coaxial port was defined. The frequency range of the signal is 50 MHz to 500 MHz.

The magnitude of the simulated and analytical transfer functions are shown in Figure 8. The simulated \( \tilde{T}(f) \) was calculated using the scattering parameters produced by the simulation and is shown in Figure 9: 

![Figure 7 2-D geometry simulation setup. The exponential profile was imported in Comsol ®.](image)

![Figure 8 Simulated and analytical \( \Gamma(f) \).](image)

![Figure 9 Transfer function \( \tilde{T}(f) \) obtained by numerical simulations and using Equation (8) . Both graphics coincide.](image)
3.1 Time Domain Simulations

In order to analyze the distortion and attenuation of the applied signal while traveling through the taper, the geometry was simulated using the time-domain, 2D-axial symmetry, TM-wave, module in Comsol®. The geometry is the same one illustrated in Figure 7.

A voltage source producing a double exponential pulse \( V_s(t) \) was connected to Port 1. The output impedance of the source is 50 \( \Omega \). Port 2 is terminated on a coaxial impedance of 100 \( \Omega \).

The signal of the source is defined as:

\[
V_s(t) = V_p \left(e^{-\alpha t} - e^{-\beta t}\right)
\]  

(11)

where: \( \alpha = 3 \times 10^8 \) s\(^{-1}\), \( \beta = 1 \times 10^9 \) s\(^{-1}\), \( V_p = 2.42 \) V

The risetime and decay time of \( V_s(t) \) correspond to the waveform of the pulser which will be applied to the HIRA, namely a risetime of 800 ps, and a FWHM of 4.7 ns.

The signal at the source \( V_s(t) \), at the input port \( V_i(t) \), and at the load \( V_L(t) \) are shown in Figure 10. Note that the risetime of the output pulse is preserved.

![Figure 10 Simulation results. Voltage at the source, input and load. Note that the risetime of the signal at the load is preserved.](image)

The amplitude of \( V_L(t) \) is 70\% of that of \( V_s(t) \), therefore an “amplification” factor \( G = 1.4 \) can be inferred, as predicted in Figure 4.

A 3-D view of the exponential taper can be seen in Figure 11.

![Figure 11 3-D exploded view of the exponential taper](image)

5. Conclusion

We presented the theoretical analysis and the design of a coaxial exponentially tapered transmission line to adapt a 50-\( \Omega \) impulse generator to a 100-\( \Omega \) Half Impulse Radiating Antenna (HIRA). The performance of the designed taper was evaluated using time-domain and frequency-domain simulations. The obtained results suggest that this device can be effectively used for connecting a 50-\( \Omega \) source (either a pulser or a continuous wave generator) to a 100-\( \Omega \) HIRA.

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7. References