Electromagnetic Detection in Natural and Man-Made Disasters

I. Kohlberg[1], S.A. von Laven[2], and R.W. McMillan[3]

[1]Kohlberg Associates, Inc., Reston, Virginia 20190-4440, ira.kohlberg@gmail.com

[2]Scientic, Inc., 555 Sparkman Drive, Suite 214, Huntsville, Alabama 35816-3440, scott.vonlaven@scientic.us

[3]U.S. Army Space and Missile Defense Command, Huntsville, Alabama 35807-3801, bob.mcmillan@us.army.mil

1. Introduction

In a previous investigation **[1-3]** a theory for detecting complex objects embedded in complex dielectrics using the mathematical structure of the dyadic Green's function **[4]** was developed. The purpose of that study was to improve upon existing approximations for predicting the electromagnetic fields generated by and onto canonical structures such as loops and dipoles in conducting media. In this study, we develop a mathematical theory for detecting irregularly shaped structures in two region geometries that could be created by natural causes and man-made actions.

2. Detection in Region Two

This section addresses two related calculations: (1) the computation of the electromagnetic field generated by a target transmitter/receiver and striking a target, and (2) the computation of the electromagnetic field generated by the target that strikes the transmitter/receiver. For brevity the plane of separation shown in **Figure 1** is called the ground plane; for example Region 1 could be air or a moist region and Region 2 could be snow (an avalanche condition). The target could, in fact, be a transponder, designed to respond in such an avalanche condition.

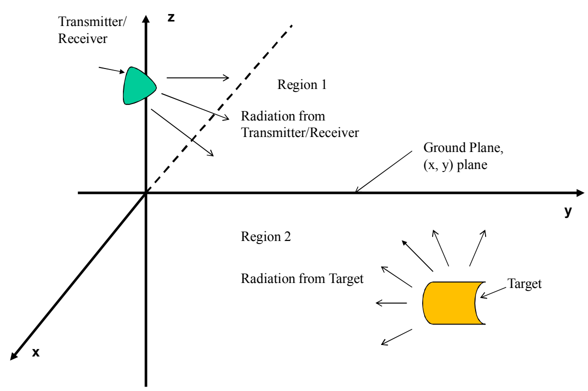

Figure 1. Detecting and identifying embedded object.

If $\vec{\vec{G}}(\vec{R}\,|\,\vec{R}')$ is the general expression for the dyadic Green's function and $\vec{J}(\vec{R}')$ is the general current source, the electric field is given by $\vec{E}_n(\vec{R}) = -j\omega\mu_0 \int_{V'} \vec{\vec{G}}_n(\vec{R}\,|\,\vec{R}') \bullet \vec{J}(\vec{R}')dV'$. We show that $\vec{\vec{G}}(\vec{R}\,|\,\vec{R}')$ can be written in dyadic form as $\vec{\vec{G}}(\vec{R}\,|\,\vec{R}') = \sum_n \vec{\vec{G}}_n(\vec{R}\,|\,\vec{R}')$, where n is a mode index to be defined in the following equations. We then have

$$\vec{\vec{G}}_n(\vec{R}\,|\,\vec{R}') = \sum_{q=1}^{q=4} \frac{1}{4\pi j} \int_0^\infty \frac{(2-\delta_0)}{\lambda h_1(\lambda)} d\lambda \vec{\vec{g}}_{nq}(\vec{R}\,|\,\vec{R}',\lambda), \tag{1}$$

where

$$\vec{\vec{g}}_{nq}(\vec{R}\,|\,\vec{R}',\lambda) = \Omega_q(\lambda)\vec{g}^{\,f}_{nq}(\vec{R},\lambda)\vec{g}^{\,s}_{nq}(\vec{R}',\lambda). \tag{2}$$

978-1-4244-6051-9/11/$26.00 ©2011 IEEE

In the foregoing expressions n is the mode index, $\Omega_q(\lambda)$ is a derived algebraic function of λ, $\vec{g}^{f}_{nq}$ is the far field component of the n^{th} mode, and $\vec{g}^{s}_{nq}$ is the n^{th} source region component of the n^{th} mode. Substituting Equations (1) and (2) into the expression for the electric field yields and using some of the notation from Tai's book **[4]** gives

$$\vec{E}_n(\vec{R}) = -j\omega\mu_0 \sum_{q=1}^{q=4} \frac{1}{4\pi j} \int_0^{\infty} \frac{(2-\delta_0)\Omega_q(\lambda)}{\lambda h_1(\lambda)} d\lambda \vec{g}^{f}_{nq}(\vec{R},\lambda)\Gamma_{nq}(\lambda) \tag{3}$$

$$\Gamma_{nq}(\lambda) = \int_{V'} \vec{g}^{s}_{nq}(\vec{R}',\lambda) \bullet \vec{J}(\vec{R}')dV' \tag{4}$$

$$\vec{\vec{g}}_{n1}(\vec{R}\,|\,\vec{R}',\lambda) = c(\lambda)\vec{M}_{en\lambda}(-h_2,\lambda)\vec{M}'_{en\lambda}(-h_1,\lambda) \tag{5}$$

$$\vec{\vec{g}}_{n2}(\vec{R}\,|\,\vec{R}',\lambda) = c(\lambda)\vec{M}_{on\lambda}(-h_2,\lambda)\vec{M}'_{on\lambda}(-h_1,\lambda) \tag{6}$$

$$\vec{\vec{g}}_{n3}(\vec{R}\,|\,\vec{R}',\lambda) = d(\lambda)\vec{N}_{en\lambda}(-h_2,\lambda)\vec{N}'_{en\lambda}(-h_1,\lambda) \tag{7}$$

$$\vec{\vec{g}}_{n4}(\vec{R}\,|\,\vec{R}',\lambda) = d(\lambda)\vec{N}_{on\lambda}(-h_2,\lambda)\vec{N}'_{on\lambda}(-h_1,\lambda) \tag{8}$$

The general expressions for the $\vec{M}$ and $\vec{N}$ terms are given by Equations (9-12). Similar equations are used for $\vec{M}'$ and $\vec{N}'$, and are obtained with the replacement of R,ϕ,z by R',ϕ',z'. In the following equations the n^{th} order Bessel function is denoted as J_n.

$$\vec{M}_{en\lambda}(R,\phi,z) = \left[-\frac{nJ_n(\lambda R)}{R}\sin n\phi\,\vec{a}_r - \frac{\partial J_n(\lambda R)}{\partial R}\cos n\phi\,\vec{a}_\varphi \right]\exp(-jhz) \tag{9}$$

$$\vec{M}_{on\lambda}(R,\phi,z) = \left[+\frac{nJ_n(\lambda R)}{R}\cos n\phi\,\vec{a}_r - \frac{\partial J_n(\lambda R)}{\partial R}\sin n\phi\,\vec{a}_\phi \right]\exp(-jhz) \tag{10}$$

$$\vec{N}_{en\lambda}(R,\varphi,z) = \frac{1}{k_\lambda}\begin{bmatrix} -jh\dfrac{\partial J_n(\lambda R)}{\partial R}\cos n\phi\,\vec{a}_r + \dfrac{jhn}{R}J_n(\lambda R)\sin n\phi\,\vec{a}_\phi \\ +\lambda^2 J_n(\lambda R)\cos n\phi\,\vec{a}_z \end{bmatrix}\exp(-jhz) \tag{11}$$

$$\vec{N}_{on\lambda}(R,\phi,z) = \frac{1}{k_\lambda}\begin{bmatrix} -jh\dfrac{\partial J_n(\lambda R)}{\partial R}\sin n\phi\,\vec{a}_r + \dfrac{-jhn}{R}J_n(\lambda R)\cos n\phi\,\vec{a}_\phi \\ +\lambda^2 J_n(\lambda R)\sin n\phi\,\vec{a}_z \end{bmatrix}\exp(-jhz) \tag{12}$$

$$k_\lambda^2 = \lambda^2 + h^2\ , \qquad \delta_0 = 1\ \ for\ m\ or\ n = 0, \qquad \delta_0 = 0\ \ for\ m\ or\ n \neq 0$$

$$k_1 = \omega\sqrt{\mu_0\varepsilon_0}\ , \qquad k_2 = \omega\sqrt{\mu_0\varepsilon(1 - j\sigma/\omega\varepsilon)}\ , \qquad h_1 = \sqrt{k_1^2 - \lambda^2}$$

$$h_2 = \sqrt{k_2^2 - \lambda^2}\ , \qquad c = \frac{2h_1}{h_1 + h_2}\ , \qquad d = \frac{2k_1k_2h_1}{k_2^2h_1 + k_1^2h_2}$$

From Equation (2) we have $\vec{\vec{g}}_{nq}(\vec{R}\,|\,\vec{R}',\lambda) = \Omega_q(\lambda)\vec{g}^{f}_{nq}(\vec{R},\lambda)\vec{g}^{s}_{nq}(\vec{R}',\lambda)$, and

$$\Omega_1(\lambda) = c(\lambda), \qquad \vec{g}^{f}_{n1}(\vec{R},\lambda) = \vec{M}_{en\lambda}(-h_2,\lambda), \qquad \vec{g}^{s}_{n1}(\vec{R}',\lambda) = \vec{M}'_{en\lambda}(-h_1,\lambda)$$

$$\Omega_2(\lambda) = c(\lambda), \quad \vec{g}^{\,f}_{n2}(\vec{R},\lambda) = \vec{M}_{on\lambda}(-h_2,\lambda), \quad \vec{g}^{\,s}_{n2}(\vec{R}',\lambda) = \vec{M}'_{on\lambda}(-h_1,\lambda)$$
$$\Omega_3(\lambda) = d(\lambda), \quad \vec{g}^{\,f}_{n3}(\vec{R},\lambda) = \vec{N}_{en\lambda}(-h_2,\lambda), \quad \vec{g}^{\,s}_{n3}(\vec{R}',\lambda) = \vec{N}'_{en\lambda}(-h_1,\lambda)$$
$$\Omega_4(\lambda) = d(\lambda), \quad \vec{g}^{\,f}_{n4}(\vec{R},\lambda) = \vec{N}_{on\lambda}(-h_2,\lambda), \quad \vec{g}^{\,s}_{n4}(\vec{R}',\lambda) = \vec{N}'_{on\lambda}(-h_1,\lambda)$$

The significant part of the foregoing methodology is that all subsequent calculations are carried out in the limits where $\lambda R \to \infty$ (the far field) and in the source region, where $\lambda R' << 1$. We first consider the far field: $\lambda R \to \infty$ **[2-3]**

$$J_n(\lambda R) \to = \sqrt{\frac{2}{\pi\lambda R}}\alpha(\lambda R, n), \qquad \frac{\partial J_n(\lambda R)}{\partial R} = \lambda\sqrt{\frac{2}{\pi\lambda R}}\beta(\lambda R, n) \tag{13}$$

$$\alpha(\lambda R, n) = \cos(\lambda R - \frac{n\pi}{2} - \frac{\pi}{4}), \qquad \beta(\lambda R, n) = -(f(n,\lambda R)\sin\chi + g(n,\lambda R)\cos\chi)$$

$$f(n,\lambda R) \sim 1 - \frac{(u-1)(u+15)}{2!(8\lambda R)^2}, \qquad g(n,\lambda R) \sim \frac{u+3}{8\lambda R},$$

$$\chi = \lambda R - (\frac{n}{2} + \frac{1}{4})\pi, \qquad 4n^2 = u$$

The small $\lambda R'$ approximation **[2-3]** is

$$J_n(\lambda R') \cong \frac{1}{n!}(\frac{1}{2}\zeta)^n = K_n \lambda^n (R')^n \tag{14}$$

$$K_n = \frac{1}{n!}(\frac{1}{2})^n$$

In the far field the leading terms for $\vec{M}$ and $\vec{N}$ are

$$\vec{M}_{en\lambda}(R,\phi,z) = -\sqrt{\frac{2}{\pi\lambda R}}\left[\lambda\beta\cos n\phi\,\vec{a}_\varphi\right]\exp(jh_2 z) \tag{15}$$

$$\vec{M}_{on\lambda}(R,\phi,z) = -\sqrt{\frac{2}{\pi\lambda R}}\left[\lambda\beta\sin n\phi\,\vec{a}_\phi\right]\exp(jh_2 z) \tag{16}$$

$$\vec{N}_{en\lambda}(R,\varphi,z) = \frac{1}{k_\lambda}\sqrt{\frac{2}{\pi\lambda R}}\left[-jh_2\lambda\beta\cos n\phi\,\vec{a}_r + \lambda^2\alpha\cos n\phi\,\vec{a}_z\right]\exp(jh_2 z) \tag{17}$$

$$\vec{N}_{on\lambda}(R,\phi,z) = \frac{1}{k_\lambda}\sqrt{\frac{2}{\pi\lambda R}}\left[-jh_2\lambda\beta\sin n\phi\,\vec{a}_r + \lambda^2\alpha\sin n\phi\,\vec{a}_z\right]\exp(jh_2 z) \tag{18}$$

In the source region we get

$$\vec{M}'_{en\lambda}(R',\phi',z') = -nK_n(R')^{n-1}\left[\sin n\phi'\,\vec{a}_r + \cos n\phi'\,\vec{a}_\varphi\right]\exp(jh_1 z') \tag{19}$$

$$\vec{M}'_{on\lambda}(R',\phi',z') = nK_n(R')^{n-1}\left[\cos n\phi'\,\vec{a}_r - \sin n\phi'\,\vec{a}_\phi\right]\exp(jh_1 z') \tag{20}$$

$$\vec{N}_{en\lambda}(R',\varphi',z') = \frac{K_n(R')^{n-1}}{k_\lambda}\begin{bmatrix} -jh_1 n\cos n\phi'\,\vec{a}_r + jh_1 n\sin n\phi'\,\vec{a}_\phi \\ +\lambda^2 R'\cos n\phi'\,\vec{a}_z \end{bmatrix}\exp(jh_1 z') \tag{21}$$

$$\vec{N}'_{on\lambda}(R',\varphi',z') = \frac{K_n(R')^{n-1}}{k_\lambda}\begin{bmatrix} -jh_1 n\sin n\phi'\,\vec{a}_r - jh_1 n\cos n\phi'\,\vec{a}_\phi \\ +\lambda^2 R'\sin n\phi'\,\vec{a}_z \end{bmatrix}\exp(jh_1 z') \tag{22}$$

To demonstrate the nature of the calculation for illustrative purposes, consider Equation (4), $\Gamma_{nq}(\lambda) = \int_{V'} \vec{g}^{\,s}_{nq}(\vec{R}',\lambda)\bullet\vec{J}(\vec{R}')dV'$, for $q=1$. From the previous equations we get

$$\vec{g}^{\,s}_{n1}(\vec{R}',\lambda) = \vec{M}'_{en\lambda}(-h_1,\lambda), \qquad \Gamma_{n1}(\lambda) = \int_{V'} \vec{M}'_{en\lambda}(-h_1,\lambda)\bullet\vec{J}(\vec{R}')dV' \tag{23}$$

$$\vec{J}(\vec{R}') = J_r(R',\phi',z')\vec{a}_r + J_\phi(R',\phi',z')\vec{a}_\varphi + J_z(R',\phi',z')\vec{a}_z$$

$$dV' = R'dR'd\phi'dz'$$

$$\vec{M}'_{en\lambda}(\vec{R}')\bullet\vec{J}(\vec{R}')dV' = -nK_n(R')^n\begin{bmatrix} \sin n\phi' J_r(\vec{R}') + \\ \cos n\phi' J_\phi(\vec{R}') \end{bmatrix}\exp(jh_1 z')dR'd\phi'dz' \tag{24}$$

Similar expressions result for the cases $q = 2,3,4.$ The major calculation is

$$\vec{E}_n(\vec{R}) = -j\omega\mu_0\sum_{q=1}^{q=4}\frac{1}{4\pi j}\int_0^\infty \frac{(2-\delta_0)\Omega_q(\lambda)}{\lambda h_1(\lambda)}d\lambda\vec{g}^{\,f}_{nq}(\vec{R},\lambda)\Gamma_{nq}(\lambda), \tag{25}$$

which is then summed over significant values of *n*: $\vec{E}(\vec{R}) = \sum_n \vec{E}_n(\vec{R}).$ The integration in Equation (25) has been accomplished without any singularity problems. In general, the divide-and-conquer advantage of the dyadic formulation will mitigate any potential computational difficulties, even in the near field.

3. Conclusion

Electromagnetic fields for a two-region case are derived for detecting complex and irregularly shaped objects much less than a wavelength is developed using the mathematical structure of the dyadic Green's function in conjunction with components of a target's surface and/or internal current density. The mathematical structure allows one to examine the salient features of the far field generated by the source current density in a straightforward manner.

4. Acknowledgment

We gratefully acknowledge the support of the U.S. Army Space and Missile Defense Command.

5. References

1. I. Kohlberg, S.A. von Laven, N.G. Albritton, T.A. Baginski, and R.W. McMillan, "Electromagnetic Detection and Identification of Complex Structures," *26th Army Science Conference*, Orlando, Florida, December 1-4, 2008.
2. I. Kohlberg, "Application of Dyadic Green's Function and Reciprocity to Detect Obscure Novel Structures," *Final Report for Army Space and Missile Defense Command*, Prime Contract W911NF-07-D-0001, TCN 07-302, Battelle, November, 2008.
3. I. Kohlberg, R.W. McMillan, and S.A. von Laven, "Computation of IEMI Fields through Layered using the Radiation Principle," *Proceedings of the International Conference on Electromagnetics in Advanced Applications (ICEAA)*, Torino, Italy, September 18-21, 2009.
4. C.T. Tai, *Dyadic Green's Functions in Electromagnetic Theory*, 2nd ed. IEEE Press. New York, 1994.