Combination of the Failure Probability with a Random Angle of Incidence of the Radiated Interference

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Abstract

Electronic devices exposed to high level electromagnetic interference with certain amplitude will fail with a certain probability. Knowing this failure probability is essential when a system is being designed to withstand intentional electromagnetic interference (IEMI). Based on this knowledge additional redundancy can be included in order to reduce the risk. In previous investigations [1] failure probability was analyzed for the case where a device was illuminated from one direction only. If the device is illuminated from other (random) directions, then the failure probability will change. In this contribution it is discussed how the failure probability determined for one direction can be extended in order to include a random angle of incidence of the interference. The main focus of this contribution is on failure probability caused by pulsed wideband signals. However, the approach presented here can also be extended to narrowband signals.

1. Introduction

In previous research Camp et al. [1] have shown that the susceptibility of electronic devices can be described by the so-called breakdown failure rate (*BFR*), also known as the upset rate or the latch-up rate. The *BFR* is defined as the ratio of the number of breakdowns (upsets) to the number of exposures of the component to electromagnetic pulses:

$$BFR = \frac{N_{breakdown}}{N_{puls}}. \tag{1}$$

If the number of pulses is increased, the BFR will converge to breakdown failure probability (*BFP*):

$$BFP = \lim_{N_{puls}\to\infty} BFR \,. \tag{2}$$

The *BFP* depends on the pulse characteristics and is typically presented as a function of the pulse amplitude, e. g. see Fig. 1. For the determination of the *BFP* the equipment under test is mounted in a certain position (expected to be the one with the worst case coupling) and is exposed to electromagnetic pulses with a step-by-step increasing amplitude of the incident field E_{inc}. Hence, the resulting *BFP* is only valid for the interferences coming from the same direction.

In case of IEMI it is not known from which direction the attacker will expose the equipment to electromagnetic interference. Most likely, this will not be the worst case situation, because the attacker does not know where this direction is. It is even possible that for some angles of incidence and some polarizations the attack will not have any

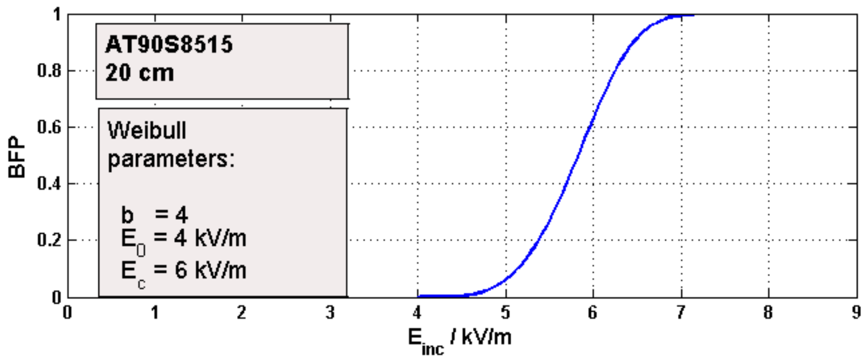

Fig. 1. Breakdown Failure Probability approximated as Weibull distribution [1]

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effect no matter how high the amplitude of the pulse is. That is why the classical definition of the *BFP* for the radiated exposure has to be extended in order to include the random angle of incidence and polarization.

The classical definition of BFP describes the failure probability as a function of an incident field E_{inc}. Here, it is assumed that the voltage which is induced at the critical component is a function of the incident electromagnetic field. For a constant amplitude of the incident field E_{inc} the coupled voltage V_c remains constant. Since for the same coupled voltage V_c there are failures and non-failures, it follows that there is a random threshold voltage level V_T. Whenever the threshold voltage V_T is lower than the coupled voltage V_c, a failure occurs. Thus, the BFP can be interpreted as the probability that the random threshold voltage V_T is lower than the coupled voltage V_c:

$$BFP(V_c) = P(V_T < V_c). \tag{3}$$

Mathematically, the coupled voltage V_c can be described as a product of the incident electric field E_{inc} and a coupling factor h: $V_c = E_{inc} \cdot h$, which is similar to the definition of effective antenna height. In previous research [1] the coupling factor was held constant. However, by selecting random angles of incidence and polarization the coupling factor becomes also random variable and hence V_c is also random. In the following, it is investigated how the BFP in eq. (3) can be calculated for a random value of V_c. For that purpose, the first step is to describe the distribution of the threshold voltage V_T (see section 2). The second step is to determine the distribution of the coupled voltage V_c (see section 3). The third step is to combine these two distributions in order to get the resulting failure probability (see section 4).

2. Distribution of the threshold voltage

Camp et al. [1] have shown that the *BFP* can be described by the Weibull distribution:

$$BFP(E_{inc}) = 1 - \mathrm{e}^{-\left[\frac{E_{inc} - E_0}{E_c - E_0}\right]^b}, \tag{4}$$

where E_{inc} is the peak incident field, E_0 is the breakdown-free field strength, E_c is the characteristic field strength (such that $BFP(E_{inc} = E_c) = 1 - \mathrm{e}^{-1}$)* and b is the shape-factor. For example, the Weibull parameters shown in Fig. 1 are extracted from [1] for the AT90S8515 microcontroller connected over a 20 cm ribbon cable.

In order to determine the distribution of the threshold voltage a few assumptions and definitions are made:

- The angle at which the *BFP* has been measured is assumed to be the worst case angle.
- The polarization at which the *BFP* has been measured is assumed to be the worst case polarization.
- If the coupling factor in the worst-case is denoted as h_{WC}, then the cumulative distribution function (cdf) of the threshold voltage can be written as:

$$cdf_{V_T}(V_T) = 1 - \mathrm{e}^{-\left[\frac{V_T/h_{WC} - E_0}{E_c - E_0}\right]^b}. \tag{5}$$

The parameter h_{WC} is an unknown in eq. (5). As can be seen later in section 4, its knowledge is not needed for determining the failure probability for a random angle of incidence.

3. Coupling for a random angle of incidence

In order to keep it simple, an electronic device with an attached cable is described by a two conductor transmission line. The parameters of this transmission line can be modified in order to represent the electronic system. The coupling to a two conductor transmission line is a well known problem and is described in [2, p. 85]. The transmission line configuration is depicted in Fig. 2. The peak voltage coupled onto the transmission line depends on the angle of incidence (ψ, ϕ) and the polarization γ. To describe this dependency the angle and polarization dependent coupling factor $h(\psi, \phi, \gamma)$ is introduced which span is between zero and the maximum value h_{WC}: $h \in [0, h_{WC}]$. A relative coupling factor is defined as $h_r = h/h_{WC}$ with $h_r \in [0, 1]$. The distribution function of the relative coupling

* Please note the difference to the definition of the characteristic field strength in [1], where E_c is defined as $BFP(E_{inc} = E_c - E_0) = 1 - \mathrm{e}^{-1}$

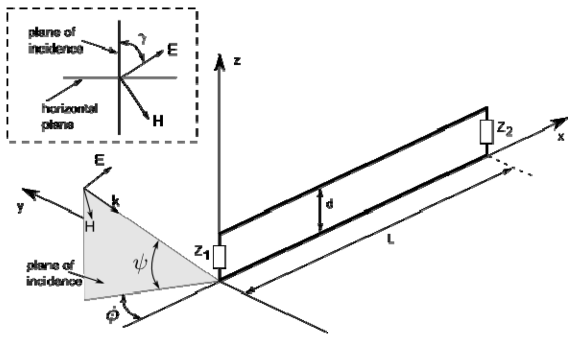

Fig. 2. Two-wire line excited by an incident plane wave [4, p. 85]

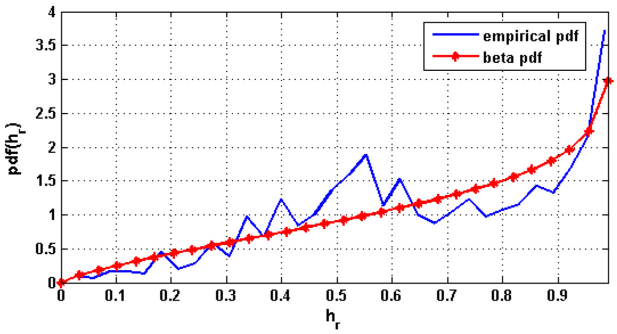

Fig. 3. Empirical probability density function of the coupling factor h_r and its approximation by the beta pdf

factor h_r is determined in the following. For that purpose, a Monte-Carlo simulation is used, which mainly consists of the following six steps:

1) Choose a random angle of incidence and a random polarization
2) Excite the transmission line with CW signals in order to get its transfer function for this angle
3) Multiply the transfer function with the pulse spectrum
4) Perform inverse discrete Fourier transformation IDFT
5) Note the coupled peak voltage
6) Return to step 1) e.g. 1000 times

The result of the Monte-Carlo simulation for a 20 cm long transmission line excited by a double exponential pulse (with the same parameters as the pulse used in section 2 to determine the *BFP*) is shown in Fig. 3. In the same figure the empirical probability density function (*pdf*) is approximated by the beta distribution using the maximum likelihood method [3, p. 523]. The beta distribution was chosen here because it only needs two parameters (α and β) and is supported on the bounded interval $[0, 1]$.

For a random angle of incidence and polarization the coupled voltage V_c can be calculated as $V_c = E_{inc} \cdot h_r \cdot h_{WC}$, where h_r is a random variable. The distribution of the coupled voltage can then be described by the generalized beta distribution:

$$\begin{aligned} pdf_{V_c}(V_c) &= \frac{1}{B(\alpha,\beta)} \frac{V_c^{\alpha-1}(E_{inc}h_{WC} - V_c)^{\beta-1}}{(E_{inc}h_{WC})^{\alpha+\beta-1}} \\ &= \frac{1}{B(\alpha,\beta)} \frac{(V_c/h_{WC})^{\alpha-1}(E_{inc} - V_c/h_{WC})^{\beta-1}}{h_{WC}E_{inc}^{\alpha+\beta-1}}, \end{aligned} \qquad (6)$$

where $B(\alpha, \beta)$ is the beta function. Till now the distribution of the coupled voltage V_c is determined in this section and the distribution threshold voltage V_T is known from section 2. Building on that, both distributions need to be combined in order to determine the failure probability for a random angle of incidence and polarization.

4. Combination of the threshold distribution and coupling distribution

Equation (5) describes the cumulative distribution function of the threshold voltage (*strength*) and eq. (6) describes the probability density function of the coupled voltage (*load*). Based on the knowledge of these distributions, the breakdown failure probability of the system for a random angle BFP_{ra} can be determined by using the load-strength analysis which is commonly used in structural reliability analysis [3, p. 315]. Hence, we are looking for the probability that the load exceeds the strength. This probability can be determined in the following way:

$$P(V_c(E_{inc}) > V_T) = BFP_{ra}(E_{inc}) = \int_0^\infty cdf_{V_T}(V) \cdot pdf_{V_c}(V)\, dV. \qquad (7)$$

Inserting eq. (5) and eq. (6) into eq. (7) and substituting $z = V/h_{WC}$ leads to:

$$BFP_{ra}(E_{inc}) = \int_{E_0}^{E_{inc}} (1 - \mathrm{e}^{-\left[\frac{z-E_0}{E_c-E_0}\right]^b}) \cdot \frac{1}{B(\alpha,\beta)} \frac{z^{\alpha-1}(E_{inc} - z)^{\beta-1}}{E_{inc}^{\alpha+\beta-1}} dz\ . \qquad (8)$$

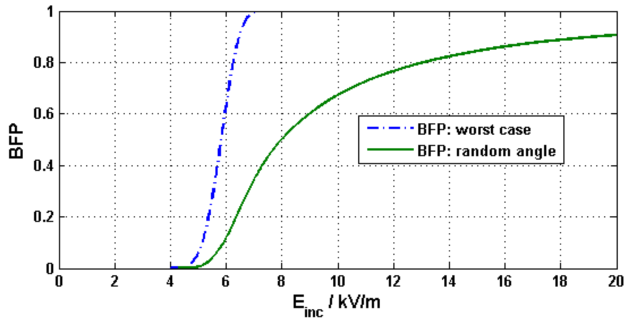

Fig. 4. Comparison of the BFP in the worst case and BFP for a random angle

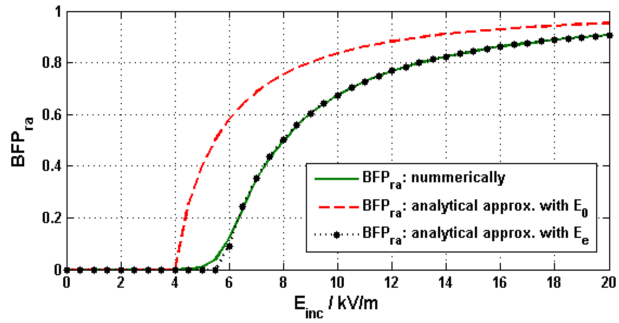

Fig. 5. Comparison of the nummerically calculated BFP for random angles and its analytical approximations

Unfortunately this integral cannot be solved analytically. Hence, two approaches are suggested:

1) Solve the integral numerically.
2) Replace the Weibull distribution by a single threshold value. This value can be the minimum threshold E_0 or the expected value of the threshold: $E_e = E_0 + (E_c - E_0) \cdot \Gamma(1 + 1/b)$. Then the integral in eq. (8) is simply the complementary cumulative beta distribution function:

$$BFP_{ra}(E_{inc}) = 1 - B_x(x, \alpha, \beta)/B(\alpha, \beta) \tag{9}$$

where $B_x(x, \alpha, \beta)$ is the incomplete beta function with $x = E_0/E_{inc}$ or $x = E_e/E_{inc}$, respectively.

5. Results and Conclusion

Fig. 4 shows the results of the numerical calculation of $BFP_{ra}(E_{inc})$ in comparison with the *BFP* for the worst case. From the comparison in Fig. 4 it can be seen that by taking the random angle of incidence into consideration the failure probability of the component decreases significantly. It is also observable that the *BFP* approaches the 100% probability at much higher amplitude levels. From this observation it can be concluded that by using redundant devices which are placed independently from each other, low failure probabilities can be reached even for very high field strengths.

Next the numerical calculation of the BFP_{ra} is compared to the analytically expressed approximations of the BFP_{ra} using eq. (8). The results are shown in Fig. 5. It can be seen that by using the breakdown-free field strength E_0 as the threshold level the BFP_{ra} is overestimated, especially for lower values of E_{inc}. Thus, using E_0 as the threshold determines the upper bound of the BFP_{ra}. The use of the expected value E_e as the threshold shows a very good agreement with the numerical calculation. Thus the expression

$$BFP_{ra}(E_{inc}) = 1 - B_x(x, \alpha, \beta)/B(\alpha, \beta) \quad \text{with} \quad x = \left[E_0 + (E_c - E_0) \cdot \Gamma(1 + 1/b)\right]/E_{inc} \tag{10}$$

can be used to describe the failure probability of electronic devices to a field incident from a random direction.

The main difference between eq. (9) and the Weibull distributed *BFP* in the worst case is the behavior towards higher amplitudes. While the BFP_{WC} reaches the "almost 100%-probability" very fast, the BFP_{ra} approaches the final value much slower. This can be easily explained if the the angles of incidence and polarization are considered where almost nothing couples into the system. This means, that in order to make the device fail much higher amplitudes are needed.

7. References

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