Evaluation of the Cooray-Rubinstein formula for distances farther than a few kilometers using the parallel FDTD method

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Abstract

In this paper, a parallel finite difference time domain (FDTD) code is developed for the calculation of the horizontal electric field above ground for three distances of 2 Km, 5 Km and 10 Km from the return stroke channel. An electromagnetic model of lightning is adopted for the purpose of simulation. The obtained results are compared with those simulated by Cooray-Rubinstein formula (CR). The effect of ground's conductivity on the electric field value is also investigated. The results show that the accuracy of CR formula decreases as the ground's conductivity decreases or distance of observation point from the lightning channel becomes larger.

1. Introduction

To investigate the induction effects of the lightning on the transmission lines, evaluation of the electromagnetic fields radiated by the lightning is necessary. The exact solution of the lightning fields needs calculation of the Sommerfeld [1] integrals for several elementary dipoles at many frequencies that is very time consuming. For evaluation of the above-ground horizontal electric fields radiated by the lightning, several approximate techniques have been presented. This component of the field can be obtained in the frequency domain by the so-called wave tilt formula [2] for observation points farther than a few kilometers. In [3] Rubinstein presented an approximate formula (known as the Cooray-Rubinstein (CR) formula) for evaluation of the horizontal electric field at a given height above the ground. In that paper he presented the horizontal electric field obtained by the CR formula with a ground conductivity of 0.01 S/m, and up to a distance of 1500 m from the lightning channel and compared those results with the ones obtained by the accurate approximations [4] to the exact Sommerfeld solution. Because of good agreement between the results of two methods he concluded that the CR formula is accurate for close and intermediate distances. Also he mentioned that because the CR formula reduced to the wave tilt formula for large distances, it could be used to calculate the horizontal electric field for long distances too. Also Rubinstein calculated the horizontal electric field using the CR formula with a ground conductivity of $10^{-4}\ S/m$ and at a distance of 500 m from the lightning channel and by comparing that result with the exact solution one, he showed that the CR formula was inadequate for poorly conducting grounds.

In this paper we investigate the accuracy of the CR formula for evaluation of the electromagnetic fields of lightning channel up to a distance of 10 km, farther than those were used in the [3]. Also we explore the effect of the ground's conductivity on the accuracy of the CR formula by doing calculations for two ground's conductivity.

This paper is organized as fallows. In section 2 the CR formula will be reviewed. In section 3 the numerical results of FDTD method and the CR formula will be compared with each other. Finally conclusion will be presented in section 4.

2. Cooray- Rubinsrein Formula

A. Nomenclature

$E_{r,\sigma}$ Horizontal electric field in the presence of a finitely conducting ground

E_{rp} Horizontal electric field in the presence of a perfectly conducting ground

$H_{\varphi p}$ Aazimuthal magnetic field in the presence of a perfectly conducting ground

ε_0 Permittivity of the free space

ε_{rg} Relative permittivity of the ground

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μ_0 Permeability of the free space

σ_g Ground conductivity

B. Geometry of the Problem

The geometry for the calculation of the underground and above-ground electromagnetic fields generated by the lightning stroke is shown in fig. 1.

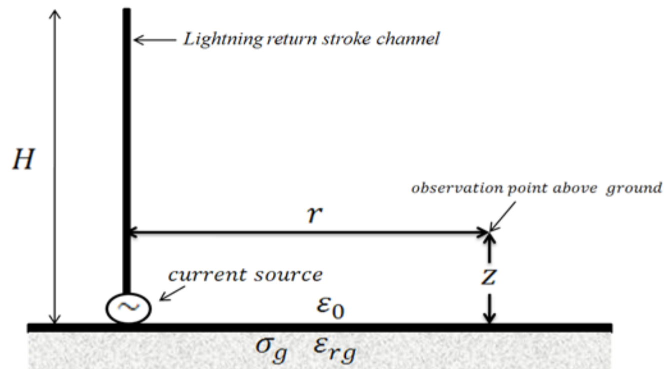


Fig. 1 Geometry for the calculation of above-ground electromagnetic fields radiated by a lightning channel.

C. Formulation

In 1996, Rubinstein [3] proposed an approximate formula to evaluate the horizontal electric field at a given height above the ground. Such formula, in the time domain states that the horizontal electric field at a distance r and at a height z is given by [5]:

$$E_{r,\sigma}(j\omega,r,z) = -H_{\varphi p}(j\omega,r,0)\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0\varepsilon_{rg}+\dfrac{\sigma_g}{j\omega}}} + E_{rp}(j\omega,r,z) \tag{1}$$

The above equation is known as the Cooray-Rubinstein (CR) formula. In the time domain the CR formula is specified as:

$$e_{r,\sigma}(t,r,z) = \int_0^t h_{\varphi p}(t,r,0)k(t-\tau)d\tau + e_{rp}(t,r,z) - \sqrt{\frac{\mu_0}{\varepsilon_0}}h_{\varphi p}(t,r,0) \tag{2}$$

Where the lowercase letters signify time domain quantities and $k(t)$ is the inverse Laplace transform of $\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0\varepsilon_{rg}+\frac{\sigma_g}{s}}} + \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{rg}}}$ and is given by [5]:

$$k(t) = -\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{rg}}}\frac{a}{2}e^{-\frac{at}{2}}\left[I_1(\frac{at}{2}) - I_0(\frac{at}{2})\right] \tag{3}$$

In which $a = \frac{\sigma_g}{\varepsilon_0\varepsilon_{rg}}$, and I_n is the modified Bessel function of first type and order n.

In this paper the validity of the CR formula is investigated using the parallel 3D FDTD [6] simulation method.

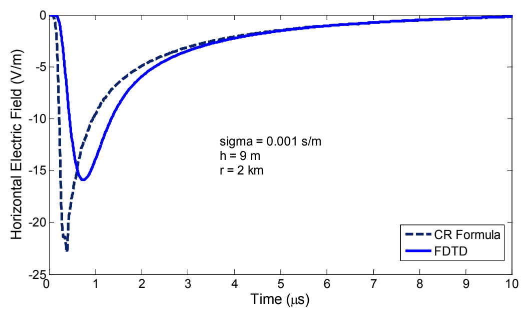
For simulations, we adopt the ATIL model with fixed inductive loading (ATIL-F) [7]. To model the lightning return stroke channel, we consider a monopole antenna above the perfect conducting ground. Then, we apply distributed inductances along the monopole antenna to control the propagation speed of current wave along it. Therefore, with this way, speed of current wave along the channel can be controlled consistent with optical observation ($\nu < 3\times10^8$

m/s)[7]. The radius of the wire is taken into account using the thin wire modeling [8]. The adopted distributed inductance and resistance, also radius of the channel are those proposed by Baba and Ishi [9], namely, $3\mu h/m$, $1\Omega/m$ and $0.3m$, respectively. The current at the channel base is the same as that was used in [10]. We adopt a 2km height lightning channel in our simulations. Calculations are made with two different ground conductivities (0.001 s/m and 0.01 s/m), while the permittivity of the ground is constant ($\varepsilon_r = 10$) throughout all the analyses. In the FDTD simulations the computational domain of the FDTD is truncated with the convolutional perfectly matched layer (CPML) [11]. We make calculations for three distance of r=2 km, r=5 km and r=10 km. The spatial discretization interval is set to 3 m. also we use a time step of 5 ns.

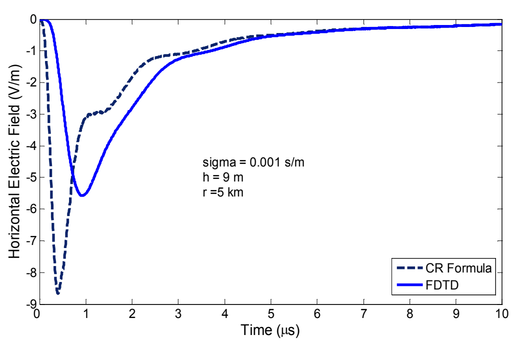
We execute our FDTD code on the three parallelized computer that each computer has a 2.83 GHz Quad CPU and 4GBs RAM.

3. Numerical Results

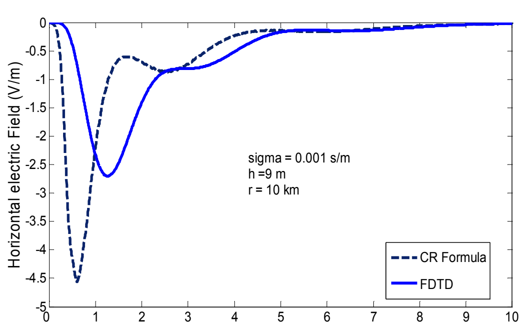
In order to test the validity of the CR formula we calculate the horizontal electric field at a height of 9 m from the lightning channel using tow procedures of FDTD method and the CR formula. The results obtained for $\sigma_g = 0.001$ and $\sigma_g = 0.01$ are shown in Fig. 2 and Fig. 3 respectively.



(a) r =2 km, h=9 m

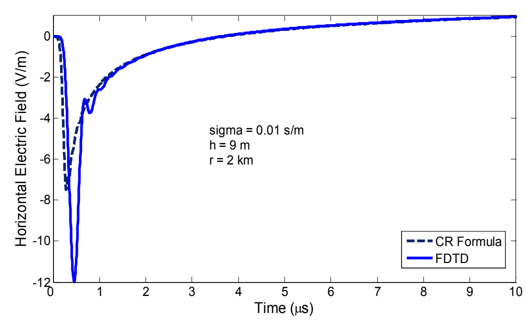


(b) r =5 km, h=9 m

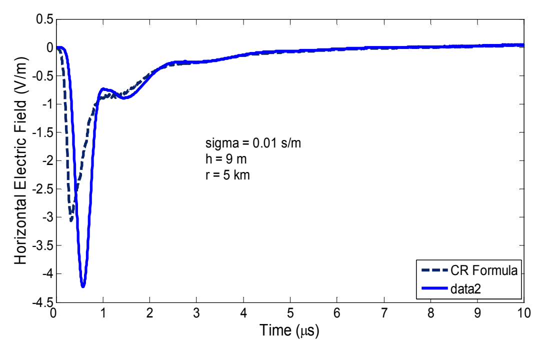


(c) r =10 km, h=9 m

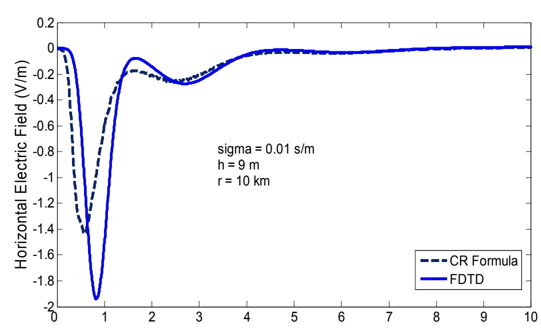
Fig. 2. Above-ground horizontal electric field calculated for $\sigma_g = 0.001$ and $\varepsilon_{rg} = 10$



(a) r =2 km, h=9 m



(b) r =5 km, h=9 m



(c) r =10 km, h=9 m

Fig. 3. Above-ground horizontal electric field calculated for $\sigma_g = 0.01$ and $\varepsilon_{rg} = 10$

In the both fig. 2 and fig. 3 we observe that the results of the CR formula deviate at the early time response from the FDTD method. Also, the difference between both the peak values and rise times of the plots related to the CR formula and the ones related to the FDTD method becomes larger as the distances from the lightning channel increases. On the other side we observe that the difference between the results of FDTD method and CR formula for the case of $\sigma_g = 0.01$ is less than the difference of these methods for the case of $\sigma_g = 0.001$. Therefore we can conclude that the accuracy of the CR formula increases as the ground's conductivity becomes larger.

4. Conclusion

From the results that are presented in this paper we can conclude that the CR formula doesn't have adequate accuracy for long distances. Also we observe that the accuracy of CR formula decreases as the conductivity of ground decreases. The low accuracy of the CR formula for poor conducting grounds can result in overestimation of lightning generated fields and this can lead to unnecessary protective arrangements and accordingly the costs increase unnecessarily.

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