Unified Model for Electrical and Optical Characteristics of a Transistor Laser with InGaAs Quantum Well and Dot in GaAs Base

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Abstract

A synthesis is made of the optical model for a quantum well (QW) and a quantum dot (QD) laser using Fermi golden rule and the electrical model based on continuity equation for a Transistor Laser. The calculated values of threshold base current and light power for InGaAs QW embedded in GaAs base agree with the experimental value. The predicted threshold base current for Quantum Dots is an order of magnitude lower.

1. Introduction

Laser action in a Transistor Laser (TL) [1-6]: a Heterojunction Bipolar Transistor, occurs due to population inversion in a Quantum Well (QW) embedded in the base due to sufficient injection and balance between optical gain and losses. Analytical models for TL developed so far [7, 8] rely on either charge control model or solution of continuity equation for currents and on rate equations for carriers and photons and could explain the gross behaviour of TLs only, with no attempt for quantitative agreement. A proper optical model considering quantized energy levels of strained QW, two-dimensional (2D) density-of-states, transition matrix elements, etc and connecting the optical model to terminal currents can predict the behaviour of TL for different materials, QW dimensions, degree of strain and recombination and loss mechanism. The present model, motivated accordingly, explains satisfactorily experimental results for the threshold base current and light power for $In_{0.2}Ga_{0.8}As$ single QW embedded in the intrinsic GaAs base of a TL. The model is applied for Quantum Dots (QDs) in the base and predicts lower threshold base current..

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2. Theory

The structure and band diagram of TL considered and some intrinsic physical processes are shown in Fig. 1. The QW is placed at a distance z_Q from the emitter-base junction in the base of width W_B in conformity with experimental situation, but in contrast to its placement in the middle of the base as considered in [7]. The solution of continuity equation in the base for diffusive motion gives the following expressions for excess electron density in the base in terms of emitter current density J_E

$$\delta N_1 = \left[\frac{N_{V.S} - (L_D / qD_n) J_E . e^{-z_Q / L_D}}{2\cosh(z_Q / 2L_D)}\right] e^{z / L_D} + \left[\frac{N_{V.S} + (L_D / qD_n) J_E . e^{z_Q / L_D}}{2\cosh(z_Q / 2L_D)}\right] e^{-z / L_D} \quad (1)$$

$$\delta N_2 = \frac{N_{V.S} e^{-W_B / L_D}}{2\cosh(z_Q - W_B / L_D)} e^{z / L_D} + \frac{N_{V.S} e^{W_B / L_D}}{2\cosh(W_B - z_Q / 2L_D)} e^{-z / L_D} \quad (2)$$

where δN_1 and δN_2 are the excess electron densities before and after the QW. Using the boundary condition $N_{vs} = \delta N_1(z = z_Q^+) = \delta N_2(z = z_Q^-)$ $J_{vs} = qD_n\left[\frac{\partial}{\partial z}(\delta N_1)_{z=z_Q} - \frac{\partial}{\partial z}(\delta N_2)_{z=z_Q}\right]$, where J_{VS} is the virtual state (VS) current density, the terminal currents are expressed as

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$$I_E = BN_{VS}\left[sinh(z_1) - coth(z_2)cosh(z_1)\right] - AJ_{VS}\ cosh(z_1) \tag{3}$$

$$I_C = BN_{VS}\left[tanh(z_1) - cosh(z_1)coth(z_2)\right] - AJ_{VS}\ cosh(z_1) \tag{4}$$

$$I_B = BN_{VS}\left[sinh(z_1) + coth(z_1) - 2\,cosh(z_1)coth(z_2)\right] - 2AJ_{VS}\ cosh(z_1) \tag{5}$$

$B = AqD_n / L_D;\ z_1 = z_Q / L_D\,; z_2 = z_Q - W_B / L_D$; L_D is the diffusion length, A is the junction area, D_n is the electron diffusion coefficient. The connection between the QW and virtual state (VS) carrier densities and between the virtual state and QW current densities are made through the relations [7]:

$$\frac{J_{VS}}{qd} = \frac{J_{QW}}{qd} - \frac{N_{VS}}{\tau_S} \quad \text{and} \quad \frac{J_{QW}}{qd} = \frac{N_{VS}}{\tau_{cap}} - \frac{N_{QW}}{\tau_{esc}} \tag{6}$$

The capture time τ_{cap} into and escape time τ_{esc} out of the QW appear in Eq. (6) [see Fig.1], where d is the QW thickness and τ_S is the recombination lifetime.

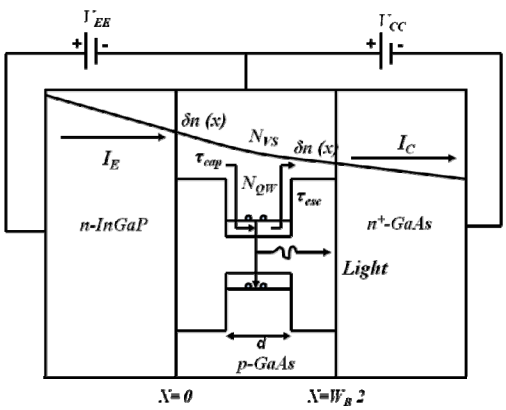

Fig.1 Structure of HBT. The band diagram for base and QW and physical processes are shown

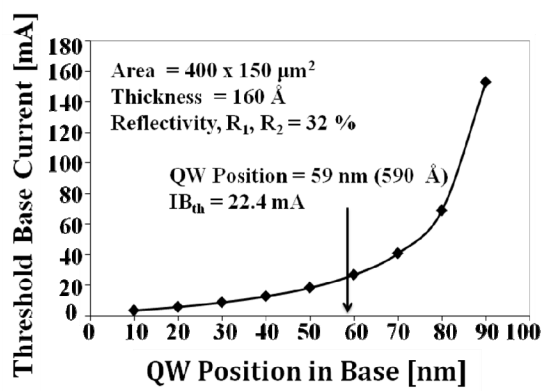

Fig.2. Variation of threshold base current for different position of QW in base

The optical gain G for 2D carrier density N, photon density N_P, and photon energy $\hbar\omega$ is expressed as [9-11]

$$G(N, N_P, \omega) = \frac{\omega}{\varepsilon_0 n_g} \int_{E_{1,c-hh}}^{E_{bar}} dE_{ch}\rho(E_{ch})\left|\mu(E_{ch})\right|^2 \frac{(f_e - f_{hh})\pi L(E_{ch,}, \omega)}{1 + N_P / N_{p,s}(E_{ch}, \omega)} \tag{7}$$

The dipole matrix element is expressed as,

$$\left|\mu(E_{eh})\right|^2 = \left[(1 + E_{c-hh} / E_{eh})q^2 / 12m_0\omega^2\right](m_0 / m_c - 1)\left[\frac{E_{c-hh}(E_{c-hh} + \Delta_{so})}{E_{c-hh} + 2\Delta_{so}./3}\right] \tag{8}$$

Here E_{ch} denotes the separation of energy between conduction and valence subbands satisfying **k**-conservation in 2D, ρ is the stair-case like density-of-states, f_e and f_{hh}, respectively, are the Fermi occupation probabilities in conduction and heavy hole subbands, L denotes the Lorentzian lineshape function, n_g is the group refractive index and Δ_{SO} is the split off energy due to spin-orbit interaction. The effect of strain on the band edges are determined by using the elastic constants and deformation potential constants for conduction and valence bands [10] and the subband energies are obtained for finite barrier offsets.

The plot of maximum gain g_{max} obtained from gain spectra versus 2D carrier density N gives the transparency carrier density for the QW which then is related to N_{VS}, J_{VS} and terminal currents through Eqs. (3-6). The slope of g_{max} - N plot gives the differential gain a, which yields the values of threshold current density expressed as $Jth = J_{tr} + qd / \Gamma\eta a\tau_s.\left[\alpha + (1/2L_B)ln(1/R_1R_2)\right]$. In this expression J_{tr} is the transparency current density, Γ is the mode confinement factor, η is the quantum efficiency, α is the material loss, L_B is the length of the cavity and the facet reflectivities are R_1 and R_2.

The model for QD given in [12, 13] is used in our work and dots having cylinder shape are embedded in the QW which acts as the wetting layer. The base layer represents the separate confinement heterostructure.

3. Results and Discussions

We have considered $In_{0.2}Ga_{0.8}As$ QW embedded in GaAs base of a TL, as in experimental samples. The values of parameters used in the calculation are given in [8-10] from which the values for alloys are obtained by linear interpolation.

Table 1 gives a comparison between our calculated values for threshold base current for QWs with experimental data along with the dimensions.

Table 1: Comparison of calculated (this work) and experimental values of threshold base current I_{Bth}

Sample no	Temp	Length, L_B (μm) , Width, W (μm) and Thickness, d (nm)	Area (μm^2)	I_{Bth} (calculated)	I_{Bth}(experiment) with [ref]
1	213	450, 10 μm and 12 nm	450 x 10	7.3 mA	7.5 mA [4]
2	288	400, 10 μm and 16 nm	400 x 10	21.4 mA	22 mA [5]
3	298	850, 2.2 μm and 12 nm	850 x 2.2	3.6 mA	40 mA [3]

Apart from Sample 3, the quality of which may not be high, the agreement is very good. We have chosen various data for sample 3 to present some of our findings.

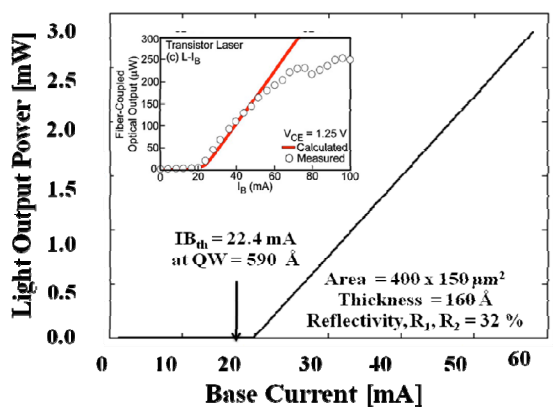

Fig. 3. Variation of light output with base current. The experimental values [6] are shown in the inset

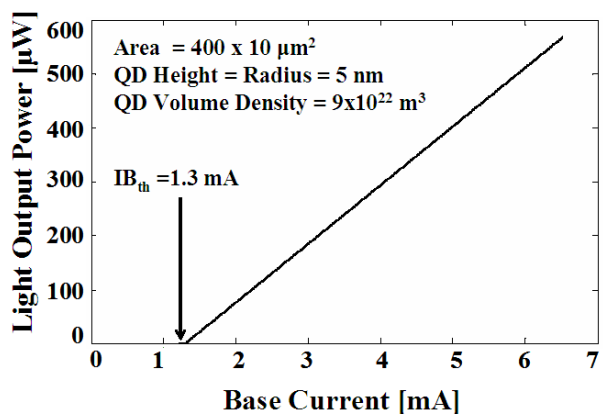

Fig. 4. Variation of light output with base current with one layer of QD inserted in 16 nm QW in base

The variation of injected electron density across the base with the QW located 59 nm away from the EB junction for $I_B > I_{Bth}$ (not shown here) is in complete agreement with the plot shown in Fig. 3 of [5] obtained from experimental I-V curve and charge control model. The variation of I_{Bth} for different position in base in Fig. 2 exhibits the same nature as given by Eq. (7) of [8] indicating higher I_{Bth} as the QW is placed closer to the collector. However the modulation bandwidth increases though at the cost of higher base current. Our calculated values of light power shown in Fig.3 are ~ 12 times higher than the experimental values (inset) which may be due to coupling loss between the TL and fiber in the experiment.

We consider one layer of $In_{0.2}Ga_{0.8}Ga$ QD with 5 nm for both height and radius embedded in QL of total thickness 16 nm. The parameters are chosen from [12,13]. The calculated light power versus base current shown in Fig. 4 indicate substantially low threshold base current. Further improvement in the performance may be achieved by including more than one layer of QDs and choosing other material combination.

4. CONCLUSIONS

We have derived expressions for terminal currents in a HBT in which a QW is placed at any position in the base by solving continuity equation in presence of diffusion. A synthesis is then made of analytical expressions for terminal currents and the expressions for gain coefficient, light output power etc of the TL. The calculated base current to attain laser threshold from the developed expressions for a strained InGaAs QW embedded in GaAs base agrees with the experimental threshold base current. The agreement between calculated and experimental light power variation is satisfactory in view of loss due to coupling of light to the fibre. The theory is then extended for QDs in base and a lower threshold base current is predicted. Further refinement of the theory can easily be incorporated in the current framework by considering band structure under **k.p** theory, multisubband occupancy, more layer of QDs in well, capture, escape and recombination lifetime calculated from first principles etc. The present model is applicable for different materials, QW and QD dimensions also.

ACKNOWLEDGEMENT

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