Spreading of Radial Gaussian Beam Array in Turbulent Atmosphere

Bin Luo,[1*] Hua Tang[2*], Baolin Ou[2**], and Hong Guo[1**]

[1]CREAM Group, State Key Laboratory of Advanced Optical Communication Systems and Networks,
School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, P.R. China
*E-mail: lbcream@pku.edu.cn; **E-mail: hongguo@pku.edu.cn

[2]School of Electronics and Information Engineering, Beihang University, Beijing 100191, P. R. China
*E-mail: huatang@buaa.edu.cn; **E-mail: baolinou@gmail.com

Abstract

The laser beam array is often encountered and widely used in many fields, and its propagation through the turbulent atmosphere is a very important subject in the case of remote sensing, imaging and communication systems. We investigate the spreading of radial Gaussian beam array under non-Kolmogorov model. The results indicate that the beam spreading width depends greatly on the beam number and the generalized exponent of the non-Kolmogorov model. The optimum ring radius, which leads to a minimum beam spreading width, is suggested. When the turbulence is strong, the optimization effect by the ring radius is weakened.

1. Introduction

The optical propagation through the turbulent atmosphere is a very important subject in the case of remote sensing, imaging and communication systems and has attracted considerable theoretical and practical interests in the past decades. For a long time, Kolmogorov's power spectrum of the refractive-index fluctuations has been widely accepted and used. However, recent studies showed that, in the real atmosphere, for the propagation along the vertical path, the turbulence indicates non-Kolmogorov characteristics and its spectral exponent greatly depends on the altitude [1, 2]. Thus, a non-Kolmogorov model is presented [3, 4], which reduces to the Kolmogorov model only when the generalized exponent $\alpha = 11/3$.

Meanwhile, the laser beam array consisting of more than one beam is often encountered and widely used in many fields, such as the high-power system and the inertial confinement fusion, and its propagation through atmosphere is also important. The average spreading of a *linear* Gaussian beam array propagating in Kolmogorov turbulence and non-Kolmogorov turbulence are studied [5, 6], respectively. To the best of our knowledge, there have been no reports on the average spreading of a *radial* Gaussian beam array propagating in non-Kolmogorov turbulence.

For laser beam array, the geometric distribution is very important and will affect the system performance. In this paper, we investigate the influences of the beam number, the generalized exponent and the ring radius on rms beam width. The results indicate that the rms beam width depends greatly on the generalized exponent and the beam number. Further, an optimum ring radius, which leads to a minimum beam width, is proved to exist within certain travelling distance and the optimum ring radius increases with increasing the beam number. With the presence of the turbulence, the optimizing effect will be weakened.

2. Basics of Radial Gaussian Beam Array

We assume that a radial Gaussian beam array is composed of N equal Gaussian beams, which are located symmetrically on a ring with the radius r_0 and the separation angle between two adjacent Gaussian beams is $2\pi/N$, as shown in Fig. 1. For the coherent combination, the rms beam width of the radial Gaussian beam array, which represents the beam spreading, after propagating distance z in non-Kolmogorov turbulence is defined by [7]

$$w_{\mathrm{turb}} = \left[\iint r^2 \langle I(\mathbf{r}, z)\rangle \, \mathrm{d}^2 r \Big/ \iint \langle I(\mathbf{r}, z)\rangle \, \mathrm{d}^2 r\right]^{1/2},$$

and can be expressed as

$$w_{\mathrm{turb}} = \left(A + B\frac{z^2}{k^2} + \frac{4}{3}\, T\, z^3\right)^{1/2},$$

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where k is the wave number and

$$\mathrm{A} = \frac{w_0^2}{2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\left\{\left(\frac{r_0}{w_0}\right)^2[1+\cos(\alpha_m-\alpha_n)]+1\right\}S_{mn}\Big/\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}S_{mn},$$

$$\mathrm{B} = \frac{2}{w_0^2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\left\{1-\left(\frac{r_0}{w_0}\right)^2[1-\cos(\alpha_m-\alpha_n)]\right\}S_{mn}\Big/\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}S_{mn},$$

$$S_{mn} = \exp\left\{\left(\frac{r_0}{w_0}\right)^2[1+cos(\alpha_m-\alpha_n)]\right\}, \qquad T = \pi^2\int_0^\infty \kappa^3\phi_n(\kappa,\alpha)d\kappa.$$

Consider the inner- and outer-scale effects, the non-Kolmogorov spectrum is expressed as [8]

$$\phi_n(\kappa,\alpha) = A(\alpha)\tilde{C}_n^2\cdot\exp\left(-\frac{\kappa^2}{\kappa_m^2}\right)\cdot(\kappa^2+\kappa_0^2)^{-\frac{\alpha}{2}}, \quad 0\le\kappa\le\infty, 3\le\alpha\le4,$$

where $A(\alpha) = \Gamma(\alpha-1)\cdot\cos(\alpha\pi/2)/(4\pi^2)$, $\tilde{C}_n^2$ is the generalized refractive-index structure parameter to describe the turbulence. $\kappa_0 = 2\pi/L_0$ and $\kappa_m = \{\Gamma[(5-\alpha)/2]\cdot \mathrm{A}(\alpha)\cdot 2\pi/3\}^{1/(\alpha-5)}/l_0$, where l_0 and L_0 are the inner and outer scale of turbulence, respectively.

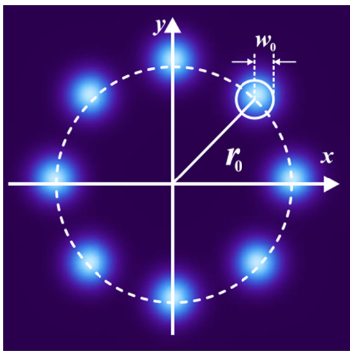

Fig. 1 Schematic diagram of the radial beam array.

3. Optimal Ring Radius

In numerical simulations, for simplicity, we choose $\lambda = 850$ nm, $w_0 = 1$ cm, $z = 10$ km, $L_0 = 1$ m, and $l_0 = 1$ cm. The influence of N is depicted in Fig. 2 (a). It is obvious that the use of beam array will definitely decrease the beam width. The variation of w_{turb} with r_0 for different N is plotted in Fig. 2 (b). For multiple beams ($N\ge2$), there is an *optimum* ring radius r_{0m} which leads to minimum w_{turb} and this r_{0m} depends strongly on N.

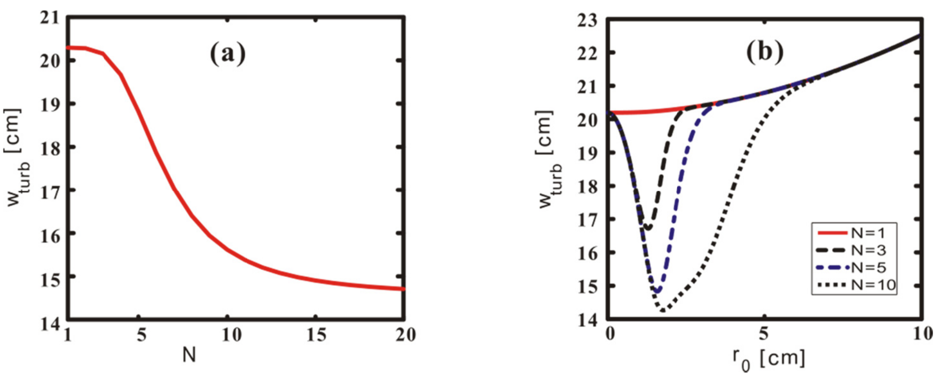

Fig. 2 (a) w_{turb} as the function of N with $r_0 = 2$ cm. (b) w_{turb} as the function of r_0 for different N. For both, $\tilde{C}_n^2 = 1\times10^{-15}\ m^{3-\alpha}$, $\alpha = 3.8$.

The optimal ring radius r_{0m} depends strongly on N. The dependence of optimum ring radius r_{0m} on N is also manifested in Fig. 3. The specific relation between r_{0m} and N is shown in Fig. 3, which indicates that r_{0m} increases with N and tends to reach its asymptotical value when N is large enough.

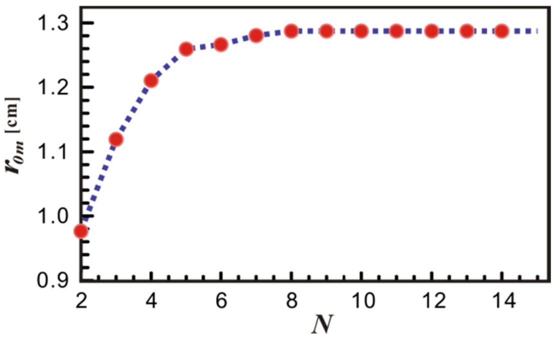

Fig. 3 The optimum ring radius r_{0m} as the function of N. $\tilde{C}_n^2 = 1 \times 10^{-15}\ m^{3-\alpha}$, $\alpha = 3.8$.

The optimal ring radius does not increase when $N \geq 9$. As is shown in Fig. 4, the beams distribute closely at the transmitter when the beam number is large. When $N \geq 9$, it is very similar to hollow beams, and the generation of this kind of beam can be much easier.

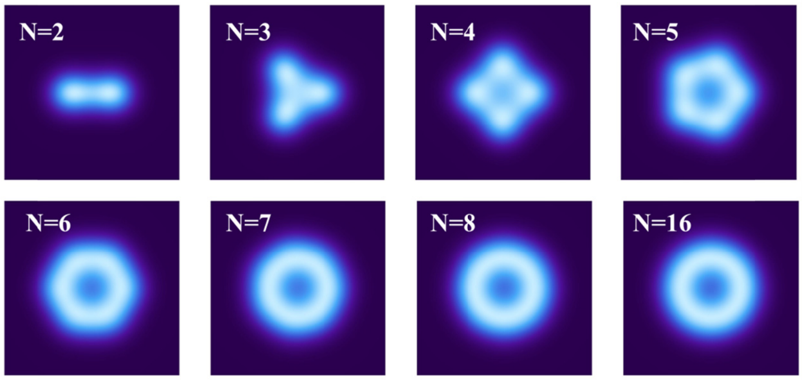

Fig. 4 Geometric distributions of the beam array with optimal ring radius.

The beam spreading under different non-Kolmogorov generalized exponent α is shown in Fig. 5. It is clear that α greatly affects the beam spreading. The optimization effect by the ring radius is weakened when the turbulence is strong.

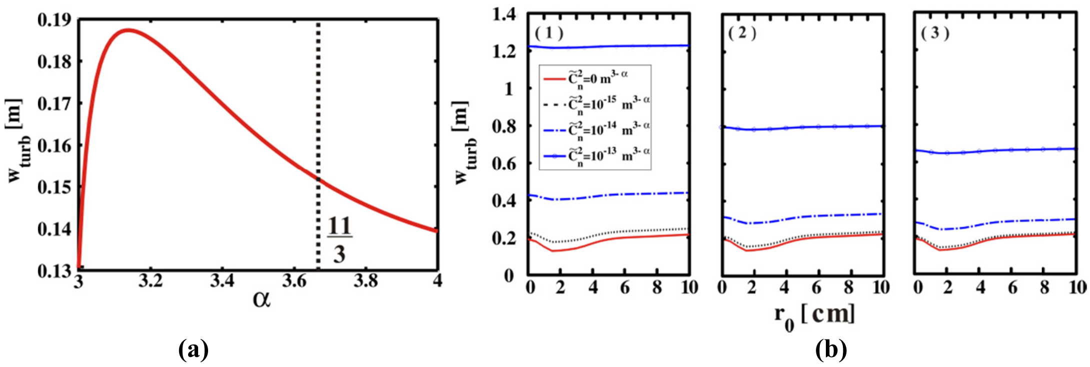

Fig. 5 (a) w_{turb} as the function of α, $\tilde{C}_n^2 = 1 \times 10^{-15}\ m^{3-\alpha}$, $N = 10$, $r_0 = 2$ cm. (b) w_{turb} as the function of r_0 for different $\tilde{C}_n^2$ with $N = 10$, (1) α = 3.3, (2) α = 11/3, (3) α = 3.8.

4. Conclusion

In conclusion, we investigate the spreading of radial Gaussian beam array under non-Kolmogorov model. The influences of the beam number, the generalized exponent and the ring radius on rms beam width are studied. We show that the rms beam width depends greatly on the generalized exponent and the beam number. Further, an optimum ring radius, which leads to a minimum beam width, is proved to exist within certain travelling distance and the optimum ring radius increases with increasing the beam number. When $N \geq 9$, it is very similar to hollow beams, and the generation of this kind of beam can be very different with the beam arrays. It is also shown that the optimization effect by the ring radius is weakened when the turbulence is strong.

5. Acknowledgments

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