Performance Assessment of Bi-scalar Beamformers in Practical Phased Array Feed Systems

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Abstract

We assess the sensitivity and polarimetric performance of a polarimetric phased array feed (PAF) system in which the two sets of nominally orthogonally polarized elements are beamformed separately. Out simulations of an actual PAF system indicate that such bi-scalar instead of full-polarimetric beamforming results in about 4% sensitivity loss and an XPD of about 45 dB. Our measurements confirm the sensitivity loss, but indicate worse polarimetric performance than the simulations. We indicate how the performance of a PAF system with bi-scalar beamforming can be improved by beamforming of the cross-polarization signals and polarimetric corrections to the beamformer outputs.

1 Introduction

The ability to perform large surveys is becoming increasingly important for radio telescopes. The radio astronomical community is therefore developing phased array feed (PAF) systems to extend the field-of-view (FoV) of reflector dishes [1]. These systems consist of two sets of feeds with orthogonal polarization to allow reconstruction of the polarimetric properties of the incoming signals. The signals from these two sets of feeds are typically processed separately as if the system consisted of two independent single polarization systems. We will refer to such an approach as the bi-scalar approach. Its main advantage is that the front-end processing, consisting of beamforming and correlation, is reduced from a system with $2N$ inputs to two systems with N inputs reducing the complexity of the system design. Here N is the number of elements in each set. However, a rigorous analysis of phased array antenna systems indicates that an optimal beamforming scheme, that achieves maximum sensitivity and preserves the polarimetric properties of the incoming signal, exploits the output signals from all antennas in the phased array feed [2]. In this paper, we discuss the limitations of the bi-scalar approach and use results from the APERTIF prototype system [3] to assess the impact of this design choice on system performance in terms of sensitivity and polarimetric quality of the beamformed signals.

In theory, any phased array system can be characterized by the voltage response of the receiving elements to two perfectly orthogonally polarized sources and the covariance of their output signals in the absence of a signal, the noise covariance matrix, $\mathbf{R}_{\mathrm{n}}$ [2]. The output voltages of the elements in response to an x-polarized signal can be stacked in a vector $\mathbf{v}_{\mathrm{x}}$ and similarly for the voltage responses to a y-polarized signal. A polarimetric phased array has two beamformer outputs, whose signals are obtained by applying weight vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ to the output voltages of the antennas. The output signal of the beamformer can thus be described by the 2×1 voltage vector

$$\mathbf{v}_{\mathrm{BF}} = \mathbf{W}^H \mathbf{v}, \tag{1}$$

where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2]$, $(\cdot)^H$ denotes the Hermitian transpose and $\mathbf{v}$ represents the output voltages of the receiving elements.

Associating the first beamformer output with the x-polarization and the second beamformer output with the y-polarization, the first beamformer output would be zero in response to a y-polarized signal for an ideal system and vice versa, i.e., an ideal system would have $\mathbf{W}^H \mathbf{V} = \mathbf{I}$, where $\mathbf{V} = [\mathbf{v}_{\mathrm{x}}, \mathbf{v}_{\mathrm{y}}]$. Maximizing the sensitivity under this constraint leads to the optimal beamformer weights for the system. It can be shown that these are given by [2]

$$\mathbf{W}_{\mathrm{opt}} = \mathbf{R}_{\mathrm{n}}^{-1} \mathbf{V} \left(\mathbf{V}^H \mathbf{R}_{\mathrm{n}}^{-1} \mathbf{V} \right)^{-1}. \tag{2}$$

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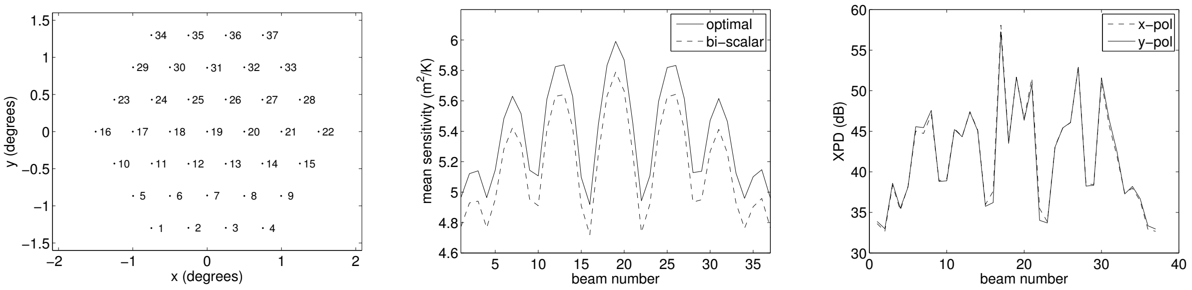

Figure 1: Center positions of the 37 beams produced by the phased array feed system (left), the average sensitivity of the two beam former outputs in the center of each beam for the optimal beamformer and the bi-scalar beamformer (middle) and the cross polarization discrimination for both polarizations of the bi-scalar beamformer (right). The XPD for the optimal method is not shown, because it is infinitely large.

If the phased array feed system consists of two sets of orthogonally polarized receiving elements, we can refer to the polarization of the matched field for one set as x-polarization and to the matched polarization for the other set as y-polarization. The array response to an x-polarized signal, can then be formulated as

$$\mathbf{v}_{\mathrm{x}} = \left[\begin{array}{c} \mathbf{v}_{\mathrm{co,x}} \\ \mathbf{v}_{\mathrm{xp,y}} \end{array} \right] \tag{3}$$

and similarly for $\mathbf{v}_{\mathrm{y}}$, where $\mathbf{v}_{\mathrm{co,x}}$ denotes the voltage response of the x-elements to a co-polarized field and $\mathbf{v}_{\mathrm{xp,y}}$ denotes the voltage response of the y-elements to a cross-polarized field. In words, the voltage response of the entire system to one polarization is given by the co-pol field of the set with a matched field pattern and the cross-polarized field of the other set. If we had an ideal instruments, the cross-polarized responses would be zero and the signals received by the two sets of antennas would be completely independent allowing fully separated processing. The rationale behind the bi-scalar approach is that the cross-pol fields are sufficiently small to be negligible. In the next section, we will assess the impact of this assumption on an actual system. We then explain how the optimal performance could be achieved using a bi-scalar beamformer by sacrificing half the beamformed bandwidth.

2 Performance assessment

For our assessment, we will use results from an APERTIF prototype system, which is a phased array feed system currently being developed for the Westerbork Synthesis Radio Telescope (WSRT) [3]. This system has been simulated using a full-wave EM package as well as tested in practice in one of the WSRT dishes. APERTIF will be able to form 37 beams on the sky, which will probably be arranged in a hexagonal pattern over the FoV of a single WSRT dish separated by the half power beam width as indicated in the left panel of Fig. 1.

In simulations, we can compare the sensitivity and polarimetric performance of the bi-scalar method with the optimal case described by (2). The results at a frequency of 1.4 GHz are shown in the middle and right panels of Fig. 1. The sensitivity plot shows the mean sensitivity of the two beamformer outputs. The sensitivity of the two outputs may differ slightly due to asymmetries in the system. This plot indicates that the bi-scalar approach, which ignores the power received by the cross-polarized field of the receiving elements, sacrifices about 4% sensitivity compared to the optimal beamformer. The optimal beamformer has an infinitely high cross polarization discrimination (XPD) in the noise free case and is therefore not plotted in the right panel of Fig. 1. This plot shows that the XPD for the bi-scalar beamformer is typically 45 dB, being better near the center of the FoV and deteriorating towards its edges. This XPD looks very promising, but our simulations use a simplified optical model, which does not, for example, contain the struts on which the feed box is mounted above the telescope [4]. Moreover, we have only shown the results for the center of

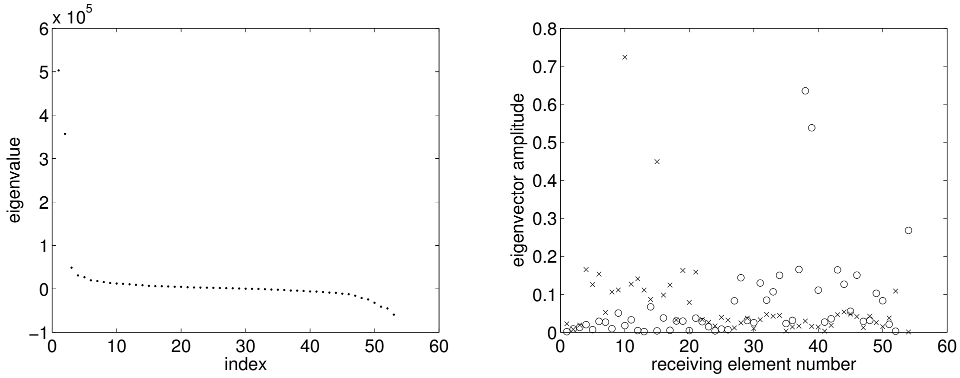

Figure 2: Eigenvalue structure (left) and amplitude of the elements of the two dominant eigenvectors (right) for an observation on a strong unpolarized source with a PAF prototype system in one of the WSRT dishes.

each beam, but that the sensitivity and polarimetric performance may vary over each beam area [5].

If the cross-polarized fields are negligible, the covariance matrix of the output signals of the receiving elements in response to an unpolarized source has a block diagonal structure with off-diagonal blocks containing only zeros. This covariance matrix should have two dominant eigenvectors, where one eigenvector has high values for the x-polarized elements and low for the y-polarized elements and vice versa for the other dominant eigenvector. We have tested this idea in practice using the APERTIF prototype system and show the results at 1.4 GHz in Fig. 2. The left panel shows the eigenvalues, showing only two dominant eigenvalues as expected. The right panel shows the amplitudes of the elements of the two dominant eigenvectors, which exhibit the expected structure. Based on the measured amplitudes associated with the cross-polarized field, we can estimate that ignoring them leads to about 2% loss in sensitivity, which is reassuringly close to the 4% estimated by our simulations. Based on our measurements, we can also calculate the covariance of the two beamformer output signals. This shows that the cross-polarized power measured on an unpolarized source is about -28 dB compared to the co-polarized power. Although this is worse than predicted by the simulations, as expected, this performance seems very acceptable for an actual system.

3 Achieving optimal performance with the bi-scalar approach

In Sec. 1, we explained that a good polarimetric beamformer has $\mathbf{W}^H\mathbf{V} = \mathbf{I}$. One can easily verify that the optimal beamformer satisfies this property. To see what happens in the bi-scalar beamformer, we expand $\mathbf{W}^H\mathbf{V}$ along the lines of (3) to get

$$\mathbf{W}^H\mathbf{V} = \begin{bmatrix} \mathbf{w}_{1,\mathrm{x}} & \mathbf{w}_{2,\mathrm{x}} \\ \mathbf{w}_{1,\mathrm{y}} & \mathbf{w}_{2,\mathrm{y}} \end{bmatrix}^H \begin{bmatrix} \mathbf{v}_{\mathrm{co,x}} & \mathbf{v}_{\mathrm{xp,x}} \\ \mathbf{v}_{\mathrm{xp,y}} & \mathbf{v}_{\mathrm{co,y}} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_{1,\mathrm{x}}^H\mathbf{v}_{\mathrm{co,x}} & \mathbf{w}_{1,\mathrm{x}}^H\mathbf{v}_{\mathrm{xp,x}} \\ \mathbf{w}_{2,\mathrm{y}}^H\mathbf{v}_{\mathrm{xp,y}} & \mathbf{w}_{2,\mathrm{y}}^H\mathbf{v}_{\mathrm{co,y}} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_{1,\mathrm{y}}^H\mathbf{v}_{\mathrm{xp,y}} & \mathbf{w}_{1,\mathrm{y}}^H\mathbf{v}_{\mathrm{co,y}} \\ \mathbf{w}_{2.\mathrm{x}}^H\mathbf{v}_{\mathrm{co,x}} & \mathbf{w}_{2,\mathrm{x}}^H\mathbf{v}_{\mathrm{xp,x}} \end{bmatrix}. \quad (4)$$

If the bi-scalar beamformer associates its first output with the x-polarization and the second output with the y-polarization, it will ignore the cross-polarization terms and set $\mathbf{w}_{1,\mathrm{y}} = \mathbf{w}_{2,\mathrm{x}} = \mathbf{0}$. This implies that the bi-scalar beamformer ignores the second term in the expansion forming beams that are described by the first term. The off-diagonal elements of the first term express the imperfect polarimetric performance of the bi-scalar beamformer due to the cross-polarization fields of the receiving elements.

Since the weights $\mathbf{w}_{2,\mathrm{x}}$ need to be applied to the x-polarized elements although they are associated with the other polarization, we can exploit the multi-beaming capability of a PAF beamformer to form a cross-polarization beam of the signals from the x-elements. A similar procedure can be followed for the y-elements. Once both co- and cross-polarization beams are formed, we can add the appropriate beams to reconstruct the same beam that would have been formed directly in a full polarimetric beamformer. However, such an approach doubles the number of beams to be formed. Since the total bandwidth of the beamformer is limited by the digital hardware, we can only apply this scheme by sacrificing half the bandwidth per beam reducing the sensitivity for continuum sources by a factor $\sqrt{2}$ and the survey speed by a factor 2. This may not seem attractive in view of the fact that both experiments and simulations indicate a sensitivity loss of only a few percent for the bi-scalar beamformer, but some observations, such as VLBI observations, may not need to full bandwidth of the system in which case this scheme may be applied without its drawbacks.

The polarimetric performance of the bi-scalar beamformer is another concern, especially for high dynamic range observations. Since each beam formed by the PAF system on the sky can be described as a weighted superposition of the voltage pattens of the individual receiving elements, we can associate an effective polarization with each formed beam. Although the two effective voltage beams in a given direction on the sky may not be as polarimetrically orthogonal as desired, we can still reconstruct the polarimetric properties of the source without much further loss in sensitivity provided that the effective polarization of the two voltage beams is sufficiently different. A good criterion for “sufficiently different” is provided by the intrinsic cross-polarization ratio over the FoV [6].

4 Conclusions

In this paper we assessed the impact of the design decision to use a bi-scalar beamformer for the APERTIF prototype system. Based on simulations,we concluded that this leads to about 4% sensitivity loss compared to an optimal beamformer and that the typical cross-polarization discrimination at the beamformer output will be about 45 dB. Our measurements confirm the sensitivity loss, but also indicate that the practicalities of an actual system deteriorate the polarimetric performance resulting in a cross-polarized power observed on an unpolarized source is still only -28 dB compared to the co-polarized power. This still seems very acceptable for an actual system. The cross-polarization level can be improved by applying an appropriate polarimetric correction to the beamformer output, while the lost sensitivity can be recovered by forming co- and cross-polarized beams in specific cases, such as VLBI observations.

References

[1] Wim van Cappellen et al. Phased Array Feeds for the Square Kilometre Array. In *XXXth General Assembly and Scientific Symposium of the Internation Union of Radio Science (URSI GASS)*, Istanbul, Turkey, 13-20 August 2011.

[2] Karl F. Warnick, Marianna V. Ivashina, Stefan J. Wijnholds, and Rob Maaskant. Polarimetry with Phased Array Antennas: Theoretical Framework and Definitions. *IEEE Transactions on Antennas and Propagation*, 2011. under review.

[3] W.A. van Cappellen, L. Bakker, and T.A. Oosterloo. Experimental Results of a 112 Element Phased Array Feed for the Westerbork Synthesis Radio Telescope. In *Proc. IEEE AP-S International Symposium*, pages 1522–3965, Charleston, USA, June 2009.

[4] M.V. Ivashina, O. Iupikov, R. Maaskant, Cappellen W. van, and T. Oosterloo. An Optimal Beamforming Strategy for Wide-Field Surveys With Phased-Array-Fed Reflector Antennas. *IEEE Transactions on Antennas and Propagation*, 2011. in press.

[5] V. Ivashina, Marianna, Stefan J. Wijnholds, Rob Maaskant, and Karl F. Warnick. Performance of Polarimetric Beamformers for Phased Array Radio Telescopes. In *XXXth General Assembly and Scientific Symposium of the Internation Union of Radio Science (URSI GASS)*, Istanbul, Turkey, 13-20 August 2011.

[6] T.D. Carozzi and G. Woan. A fundamental figure of merit for radio polarimeters. *Astronomy & Astrophysics*, 2009. Available: http://arxiv.org/PS_cache/arxiv/pdf/0908/0908_2330v1.pdf.