Veselago-Pendry Superlens Imaging Modeled with a Spectral Waveguide Approach

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Abstract

A compact spectral waveguide model of a superlens imaging system based on the Fourier decomposition of the source field into waveguide modes is presented. This model maps the electromagnetics of the superlens into that of the well-understood waveguide with concomitant advantages in the analysis of dynamics and reality effects and in accurate numerical simulation. Insights into the dynamic response of the superlens, gained from both theoretical and numerical studies, are presented. The effect of loss on the dynamic properties is investigated. In addition, the proposed model leverages a wealth of expertise available for the design of filters, artificial dielectrics and backward wave structures and could possibly aid in the engineering of practical super-resolution imaging systems that will be an enabling technology for future nanoelectronics systems.

1. Introduction

Subwavelength imaging/lithography with visible light is a critical enabling technology for emerging nanoelectronic system with fast decreasing device length scales. The subwavelength imaging problem involves the interaction of electromagnetic waves with objects comparable in size to the wavelength. Microwave engineering models and tools are thus perfectly suited to analyze and design such systems. The proposed waveguide model employs spectral domain waveguide theory to model the imaging process. The Veselago-Pendry superlens, consisting of a negative refractive index slab, transmits both propagating and evanescent fields from the object to the image location with perfect fidelity [1], [2]. Fig. 1 (a) shows a traditional convex dielectric lens side-by-side with the superlens, to illustrate the essential differences between them [2], [3]. Since the superlens (Fig. 1 (b)) extends to infinity in the *y*-*z* plane, all its geometrical and electromagnetic properties are independent of *y* and *z*, suggesting that the transmission of electromagnetic fields in *x*-direction can, in principle, be described by a one-dimensional model. However, a full space-and-time discrete electromagnetic analysis of a *single* sub-wavelength object is not straightforward, as Fig. 1 (c) suggests. To capture the information that travels at large angles with respect to the shortest path, and to minimize the effect of lateral boundaries, the computational domain must be several times the lens thickness *d*. At the same time, the resonant nature of the fields

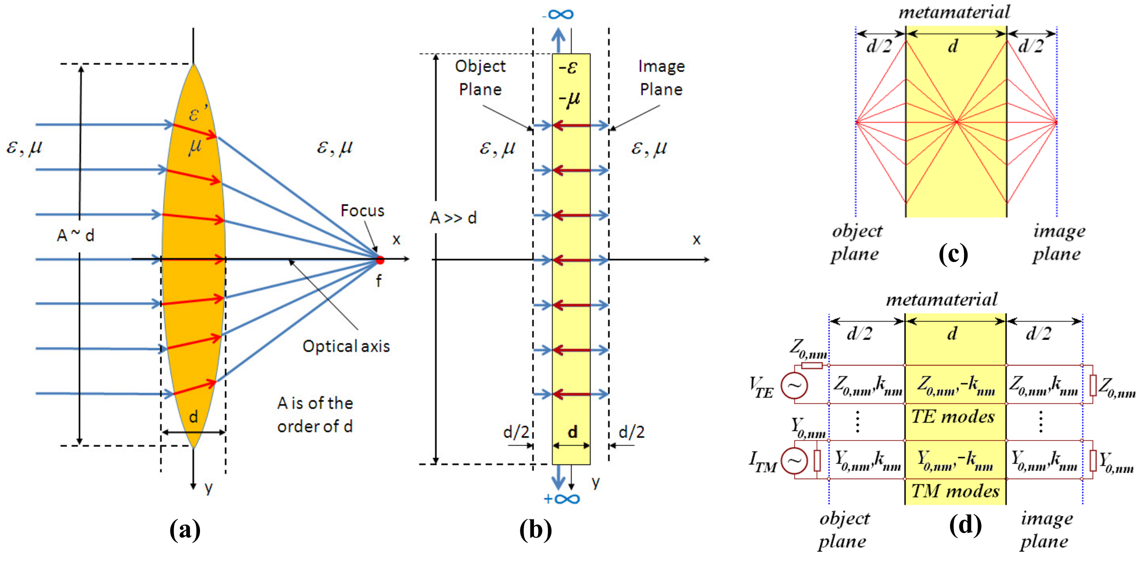

Fig. 1 (a) A traditional convex dielectric lens focuses only the propagating wave content of the image onto the focal region, situated on the optical axis (x-axis) of the lens. Evanescent fields do not reach the focus. (b) The Veselago-Pendry planar superlens transmits the field in a given point in the object plane to the opposite point in the image plane, as indicated by the parallel arrows which indicate the direction of the phase velocity. Two models of super-resolution imaging with the superlens are the classic multi-path model (c) in the spatial domain and the proposed multi-mode waveguide model (d) in the spectral domain.

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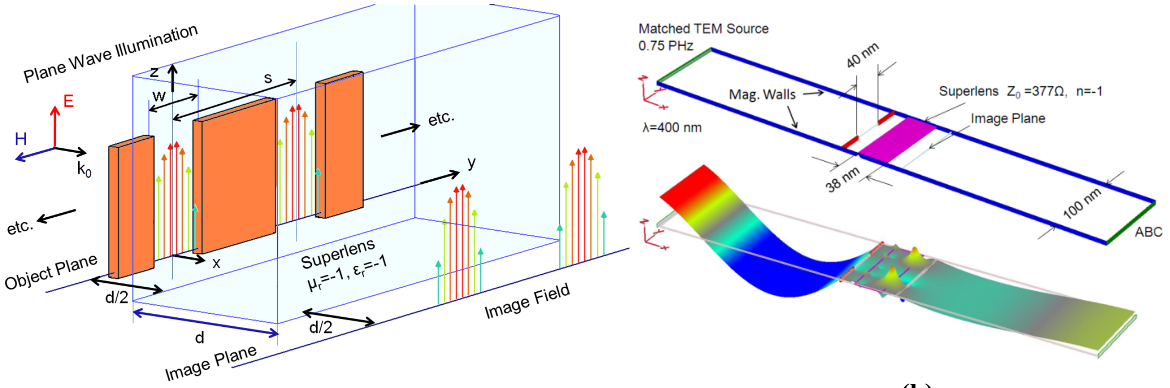
in the lens requires a very large number of time steps, typically several millions, for a transient simulation to converge. It is thus highly desirable to reduce the computational burden by making the problem periodic in transverse direction, choosing a periodicity *s* such that neighbouring objects do not strongly interact. This allows the introduction of symmetry boundary conditions (electric or magnetic walls, depending on field polarization), leading to a compact waveguide model of image transmission [4]. This approach resembles the spectral domain technique that has been applied to a variety of planar microwave devices [5] and also to electromagnetic bandgap structures [6]. It has the following advantages:

a. The spatial spectrum of the periodic array of objects is now discrete, representing a sampling of the continuous spectrum of a single object at intervals of 2π/s in k-space. The harmonically related spectral terms can be interpreted as modes in a single waveguide which thus accommodates the total field and enables nonlinear time domain analysis.
b. This waveguide model requires two or three orders of magnitude less memory than a full 3D simulation of a superlens imaging system. More importantly, having a smaller computational domain allows to simulate the dynamic response of the system within correspondingly shorter CPU time.



(a) **(b)**

Fig 2. (a) Object and image fields of two elements of a periodic array of parallel slots in a conducting sheet, uniform in *z*-direction (propagation is in the x direction), illuminated from behind by a z-polarized plane TEM wave of *400 nm* wavelength. *w = 40 nm*, *s = 100 nm*. A perfect superlens transfers the magnitude and phase of the object field exactly into the image plane. (b) MEFiSTo implementation of the spectral waveguide model for the simulation of a periodic series of slots fields imaged by a perfect lens. The longitudinal waveguide axis points in *x*-direction. The magnetic sidewalls ensure the spatial periodicity of the slit array, requiring the inclusion of a single slot in the simulation model.

To illustrate the features and advantages of the waveguide model, consider the periodic slits in a conducting screen shown in Fig. 2. Ideally, the field in the slots (the object field) will transfer to the image plane through the superlens and the two adjacent air sections with a total zero phase shift and an amplitude transmission factor of unity. Given the z-polarization of the electric field, a parallel-plate waveguide model with magnetic sidewalls will be able to carry this field as TE_{m0} modes. The *p-th* spectral term of the object field has *p* spatial wavelengths along the waveguide width *s*. Therefore the waveguide mode index *m* is not identical with the spectral Fourier term index *p*, because the former designates the number of *half-wavelengths* along *y*. We must therefore substitute the mode index *m* with *2p*. The mode propagation is now fully described by the modal propagation constant $k_{x,m}$ and the characteristic impedance $Z_{0,m}$ of the waveguide. *m=0* designates the dominant *TEM*-mode of the guide. The superlens imaging system can thus be modeled by a single parallel-plate waveguide which simultaneously transfers all spectral components of the image from the object plane to the image plane. Naturally, the waveguide, partially filled with metamaterial, can also be excited by a single mode source to study the behaviour of a single spectral component, as presented in [7] and [8].

2. Effects of Loss on the Dynamic Response

Numerous objections have been raised in the literature claiming that Pendry's concept was flawed (see [9] and [10] for a summary of the controversy and references). It was shown in [9] that the causality of the superlens response is ensured by the time delay in the transmission of the evanescent spectral terms that increases with increasing transverse wavenumber. Hence, perfect resolution requiring the attainment of the steady-state can never be reached in finite time, especially when the lens has no loss and is perfectly matched to the surrounding space ($\varepsilon_r = \mu_r = -1$). Furthermore, the limited dynamic range of any real or computational super-resolving system restricts its resolution [11]. Other reality effects, such as electric (and/or magnetic) losses in the lens material, small deviations from the ideal ε_r and μ_r, finite cell size of the metamaterial, or imperfections in the lens boundary shape, have been considered in the literature. A complete discussion of these issues can be found in [10]. While these studies deal mainly with the steady-state case, [7] and [8] report numerical studies demonstrating the effect of loss on the dynamics of *single* spectral components. All these reality effects conspire against the transmission of the higher evanescent terms and effectively curtail the spatial resolution of the superlens. At first glance it would appear that the deleterious effect of losses on resolution could be overcome by introducing gain in the lens material [12], [13]. However, this would not remove the fundamental limitation on resolution imposed by the long settling time of the higher spectral terms. The waveguide model not only validates the various results published so far, but it also provides a numerically efficient approach to modeling situations that cannot be treated by spectral decomposition.

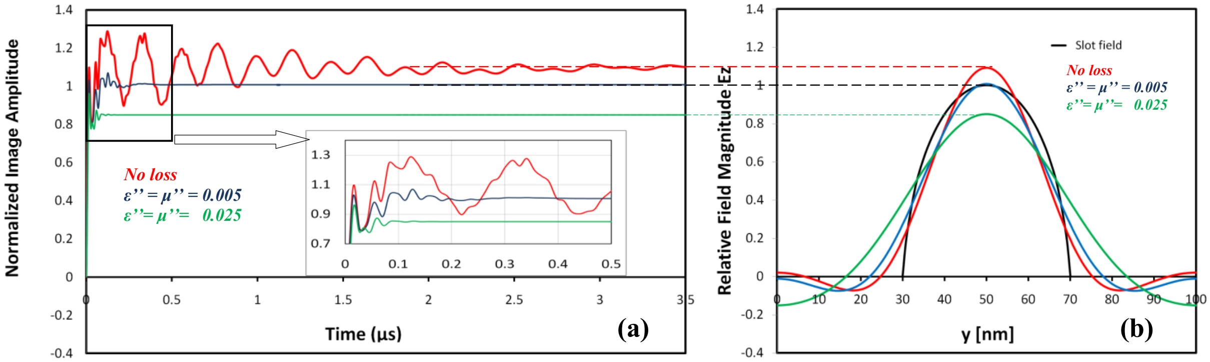
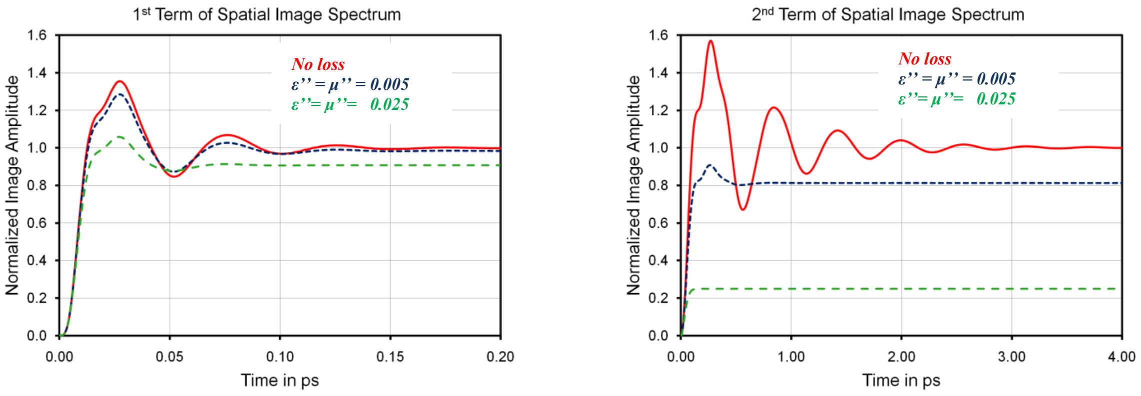


Fig 3. Image formed for the configuration shown in Fig. 2. (a) The normalized electric field of the image and the object are shown as a function of time for three different values of the absorptive loss in the metamaterial lens (inset shows time evolution during initial onset) (b) For the same losses as in (a), the images (one period) of the slit object are shown.



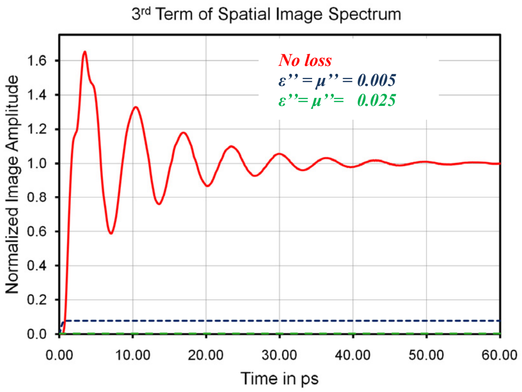


Fig. 4: Normalized electric field envelopes of the first three (excluding the 0th dc component) Fourier spectral terms in the image plane for three different values of the absorptive loss in the metamaterial lens.

Waveguide model simulations were performed using the model shown in Fig 2(b). The field amplitudes at the image plane were monitored as a function of time (t = 0 corresponds to the instant when the plane wave source is switched on). Fig. 3 (a) shows the envelope of the normalized electric field amplitude at the center of the image plane as a function of time for a lossless lens and for two values of absorptive loss in the metamaterial lens (ε’’ = µ’’ = 0 , 0.005 and 0.025, respectively). The formation of the image due to transfer of the evanescent fields is not instantaneous, and the transient behavior shown in fig 3(a) is quite complicated. It is observed that the settling time (time taken for the normalized amplitude of the electric field at the image plane to reach its final value 1.0 within 1 percent) clearly decreases with the addition of the absorptive losses. However, the presence of absorptive loss also confirms the decrease in resolution of the image once it has settled, as shown in Fig. 3 (b). To further clarify the behavior demonstrated in Fig. 3, the object field has been decomposed into the constituent waveguide modes and simulated individually using the same waveguide model. In Fig. 4, the time evolution of the envelopes of the normalized image amplitudes is shown for the first three evanescent components. Again, the same amounts of loss in the metamaterial lens were used. Note that the Heaviside condition $\varepsilon'' = \mu''$ avoids reflection loss due to impedance mismatch at the air-metamaterial interface. These results illustrate how absorptive losses simultaneously affect both the settling time and the amplitude transmission factor of the individual spectral components (waveguide modes.).

3. Conclusion

The sub-wavelength imaging of small objects with the Veselago/Pendry superlens has been modeled using a waveguide equivalent. By replacing single small objects by periodic arrays of such objects, the object field can be decomposed into discrete spatial harmonics which form a modal set of the equivalent waveguide. The compactness of the waveguide model and its well-defined boundary conditions significantly simplify its treatment with electromagnetic simulators. In particular, the waveguide model is not restricted to the simulation of single spectral Fourier terms of the image, but allows the full-wave simulation of arbitrary periodic object fields in a single run, which is particularly important when nonlinear effects are to be investigated. The time evolution of the image formation in the Veselago-Pendry lens was studied. The effect of absorption losses on the transient dynamics was clearly visualized using the waveguide model. The results not only confirm the well-documented reduction in image resolution due to loss, but also show the concurrent reduction in the image settling time. Both effects become more pronounced as the spectral order of the evanescent modes increases. This result has implications for loss-compensated superlens imaging systems presently under study. Furthermore, the compactness of the waveguide model makes it suitable for studying nonlinear effects in such systems.

4. Acknowledgments

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