Optical rogue waves and localized structures in nonlinear fiber optics

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Abstract

We review our recent work in the field of optical rogue wave physics. Beginning from a brief survey of the well-known instabilities in optical fiber, we trace the links to recent developments in studying the emergence of high contrast localized breather structures in both spontaneous and induced nonlinear instabilities.

1. Introduction

A central challenge in understanding extreme events in physics is to develop rigorous models linking the complex generation dynamics and the associated statistical behavior. Quantitative studies of extreme phenomena, however, are often hampered in two ways: (i) the intrinsic scarcity of the events under study and (ii) the fact that such events often appear in environments where measurements are difficult. A particular case of interest concerns the oceanic rogue or freak waves that have been associated with many catastrophic maritime disasters. Studying rogue waves under controlled conditions is problematic, and the phenomenon remains a subject of intensive research. On the other hand, there are many qualitative and quantitative links between wave propagation in optics and in hydrodynamics, and it is thus natural to consider how insights from studying instability phenomena in optics can be applied to other systems. The field of "optical rogue wave physics" began in 2007 [1] and has since become a major international research effort involving many international groups and consortia [2]. The purpose of this paper will be to discuss these results that have been obtained in optics, and to consider the precise nature of optical rogue wave statistics and to examine in detail the dynamics leading to the formation of extreme events in the context of noise-driven supercontinuum generation in highly nonlinear fibers [3-4]. In addition, we will report on recent experimental results on the observation in a fiber-based system of novel classes of nonlinear structure predicted theoretically over 25 years ago. The particular structure that we study is the Peregrine soliton [5], a localized nonlinear wave which has not to date been experimentally observed in any physical system [6]. It is of fundamental significance because it is localized in both time and space, and because it defines the limit of a wide class of solutions to the nonlinear Schrödinger equation. In showing that Peregrine soliton characteristics appear with initial conditions that do not correspond to the mathematical ideal, our results may impact widely on studies of hydrodynamic wave instabilities where the Peregrine soliton is considered a freak wave prototype.

2. Akhmediev breathers

The nonlinear Schrödinger equation governs the propagation of ultra-short pulses in optical fibers:

$$i\frac{\partial A}{\partial z}+\frac{|\beta_2|}{2}\frac{\partial^2 A}{\partial T^2}+\gamma |A|^2 A=0\,, \tag{1}$$

where $|A|^2$ has dimensions of instantaneous power in W and the dispersion and nonlinearity parameters β_2 (< 0) and γ have dimensions of ps^2 km^{-1} and W^{-1} km^{-1} respectively. Modulation instability (MI) describes the evolution along z of a plane wave with initial constant amplitude on which there is a small periodic perturbation in T. Growth conditions for this perturbation are generally derived using linear stability analysis, however, the full dynamics can in fact be described analytically in terms of Akhmediev breather (ABs) solutions which consists of an evolving train of ultra-short pulses periodic in time and that exhibits a recurrence (growth-return) cycle along propagation direction [7,8]. In contrast to soliton solutions which are localized in T, the ideal AB is "localized" in the z direction so that the perturbed plane wave grows into a train of pulses which then decay again and never reappear. The AB solution is given by

$$A(z,T)=\sqrt{P_0}\;\frac{(1-4a)\cosh(bz/L_{NL})+ib\sinh(bz/L_{NL})+\sqrt{2a}\cos(\omega_{\mathrm{mod}}T)}{\sqrt{2a}\cos(\omega_{\mathrm{mod}}T)-\cosh(bz/L_{NL})}, \tag{2}$$

where the variable independent parameter ω_{mod} is the (perturbation) frequency of the initial temporal modulation. Coefficients a and b are related to ω_{mod}, by: $2a = [1 - (\omega_{mod}/\omega_c)^2]$ and $b = [8a\,(1-2a)]^{1/2}$ with $\omega_c^2 = 4\,\gamma\,P_0\,/\,|\beta_2|$ and P_0 the CW power at large $|z|$. The scaling (nonlinear) length $L_{NL} = (\gamma P_0)^{-1}$. The AB solution is valid over modulation frequencies that experience MI gain: $\omega_c > \omega_{mod} > 0$ such that a varies in the interval $0 < a < 1/2$ while the parameter $b > 0$ governs the MI growth. Maximum gain $b = 1$ occurs for $a = 1/4$, i.e. $\omega_{mod} = \omega_c / \sqrt{2}$. The solution in Eq. (2) describes an evolving periodic train of ultrashort pulses with temporal period $T_{mod} = 2\pi/\omega_{mod}$. Of course in practice small non-ideal initial conditions lead to recurrence in the AB solution, but even without perfect initial conditions the analytic AB solution still describes extremely accurately the initial phase of evolution up to the first point of initial compression and subsequent decay and this for a large range of initial conditions.

The AB characteristics depend strongly on modulation frequency. As the modulation parameter a increases, the temporal separation between adjacent peaks increases at the same time as the compressed temporal width of each individual peak decreases. This leads to a greater temporal localization. The evolution at $a = 0.25$ is associated with maximum MI gain and the limiting solution for a $\rightarrow$0.5 derived by Peregrine has a particular fractional form which has led this class of solution to be described as a "rational soliton" [5,7]. Figure 1 shows a selection of evolution plots for different values of the a-parameter, highlighting both periodic evolution and the increasing localization as we approach the Peregrine soliton regime.

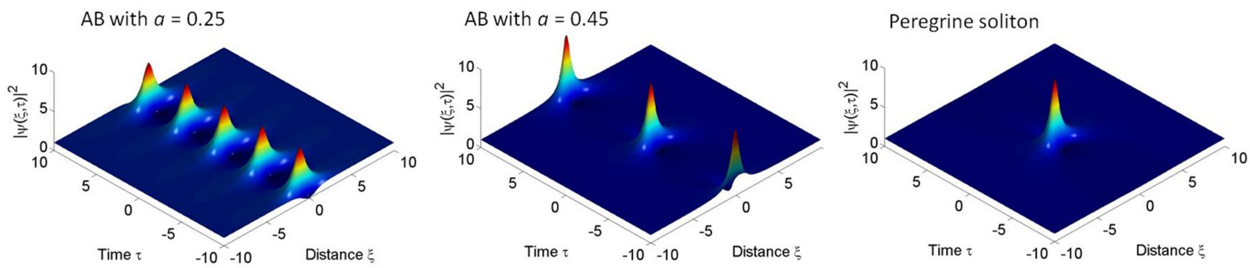

Figure 1. Plotted ABs solutions for modulation parameter: $a = 0.25$; $a = 0.45$ as indicated, as well as the ideal Peregrine soliton, the limiting case of the Akhmediev breather as $a \rightarrow 0.5$. Maximum temporal compression occurs at normalized distance $\xi = 0$. Characteristic length $L_{NL} = (\gamma\,P_0)^{-1}$ and timescale $T_0 = (|\beta_2|\,L_{NL})^{1/2}$, dimensional distance z [m] and time T [s] are related to the normalized parameters used in the figure by $z = \xi\,L_{NL}$ and $T = \tau T_0$.

3. The Peregrine soliton

The AB formalism provides a convenient way to experimentally approach the Peregrine soliton limit. Our experimental setup is shown in Fig. 2. Initial signals were generated from two external cavity lasers around 1550 nm with linewidths < 200 kHz. The fiber used was 900 m of highly nonlinear fiber with $\beta_2 = -8.85 \times 10^{-28}\ \mathrm{s^2\,m^{-1}}$ and $\gamma = 0.01\ \mathrm{W^{-1}\,m^{-1}}$. The fiber was dispersion-flattened to have low third-order dispersion around 1550 nm and loss was 1 dB/km. A phase modulator was used to broaden the ECL linewidths to ~100 MHz to suppress Brillouin scattering in the fiber at the power levels used in our experiments. Both pump and seed were then amplified by means of an Erbium Doped Fiber Amplifier. A low noise amplifier was used so as to favour the induced breather dynamics over spontaneous broadband MI. Indeed, the limiting factor in reducing the modulation frequency between pump and signal so as to approach the ideal case of $a \rightarrow 0.5$ is the decreasing gain for stimulated relative to spontaneous MI. A key aspect of the setup is the characterization using FROG permitting retrieval of the intensity and phase of the underlying field.

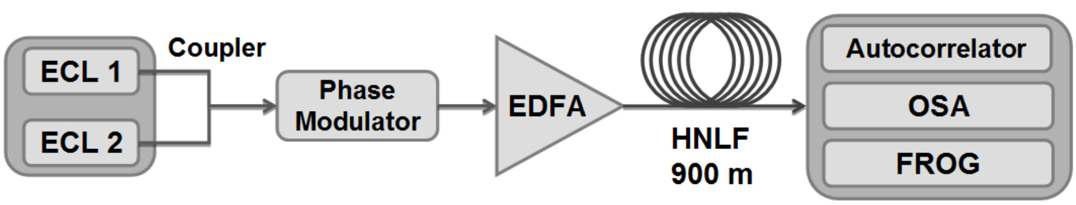

Figure 2. Experimental setup. ECL: External Cavity Laser; OSA: Optical Spectrum Analyser; FROG: Frequency Resolved Optical Gating. See [6] for full details.

The results from the experiments are shown in Fig. 3. With the frequency detuning and power of the lasers adjusted to an effective modulation frequency parameter $a = 0.42$ and a point of maximum temporal compression at the fiber output, we characterized the field characteristics using FROG as shown in Fig. 3(a). The retrieval shown in (b) yielded the experimentally measured intensity and phase profile in (c). The experimental results are compared with numerical simulations for the experimental parameters and also with the exact analytic solution for the Peregrine soliton. There is excellent agreement between experiment and simulation, and the agreement with the analytic prediction confirms that our experiments reproduce the characteristic features of the limiting solution of strong localization on a non-zero background.

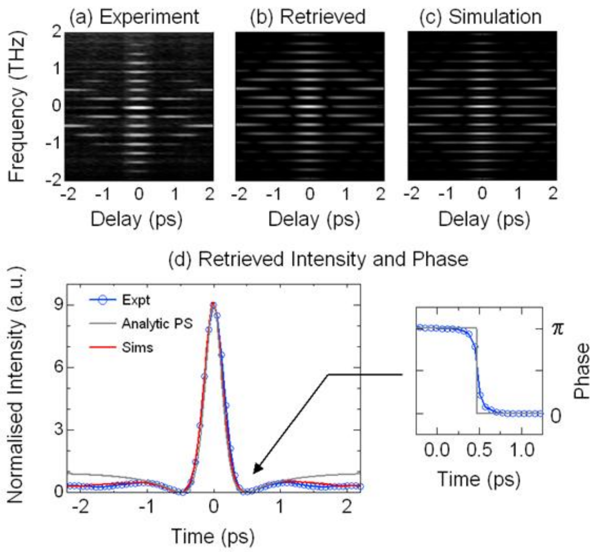

Figure 3. Results from characterisation of the Peregrine soliton. (a)-(c) show the measured, retrieved and simulated FROG traces. (d) shows the intensity and phase of the retrieved optical field, highlighting the dislocation as the optical field amplitude changes sign. The use of the FROG technique is essential in being able to unambiguously characterise this effect which is not seen directly in intensity measurements using other techniques.

4. Breathers and noise-driven supercontinuum generation

We have also recently shown that the characteristics of both the modulated temporal profile and the associated spectrum in noise-driven emergent SC generation can be explained in terms of the development of high amplitude ABs sub-pulses [3]. This interpretation represents an important step in applying the formalism of breather evolution to the interpretation of supercontinuum generation. But of course this is not a complete explanation of how these sub-pulses subsequently separate from the extended MI spectrum and temporal pulse train and evolve towards higher-amplitude pulses that exhibit rogue wave statistics. In this context, however we note that the extended MI regime where the AB theory works well is essentially described by the integrable NLSE with noise, but any realistic optical fiber system would contain symmetry-breaking physical effects due to odd orders of chromatic dispersion and Raman scattering. In fact, both these effects are important in that they can induce collisions between the emergent breathers, but for illustrative purposes, we consider only the effect of third order dispersion in the results that follow.

This is illustrated in Fig. 4 where we model 100 W, 5 ps FWHM Gaussian pulses at 1060 nm propagating in 20 m of photonic crystal fiber with zero dispersion at 1040 nm. Dispersion coefficients at the pump wavelength are: $\beta_2 = -4.10 \times 10^{-1}$ ps^2 km^{-1}, $\beta_3 = 6.87 \times 10^{-2}$ ps^3 km^{-1}and the nonlinearity parameter is: $\gamma = 0.011$ W^{-1} m^{-1}. The input pulse soliton number for these parameters is $N \sim 147$, and we see clearly from Fig. 4 how the dynamics develops. Specifically, we see the clear development of a high contrast modulation at early stage of propagation corresponding to the formation of a large number of breathers corresponding to the whole range of frequency within the MI gain [3]. However, with further propagation the modulation is not stationary relative to the reference frame of the pump group velocity, but rather drifts towards larger co-moving time i.e. towards the pulse trailing edge. This is a consequence of the TOD. In fact, we see that the combination of the TOD and noise for this case actually results in a non-uniformity in the drift behavior such that collisions can occur between different ABs. As a result of these collisions, we see large amplitude localization and the emergence of precursor soliton structures with peak powers ~800 W in comparison with the 200 W peak powers seen for the individual ABs [9]. Statistical analysis performed over a large ensemble of realizations with different random noise seeds reveals that the collisions are actually the most intense events and can be interpreted as optical rogue waves [4].

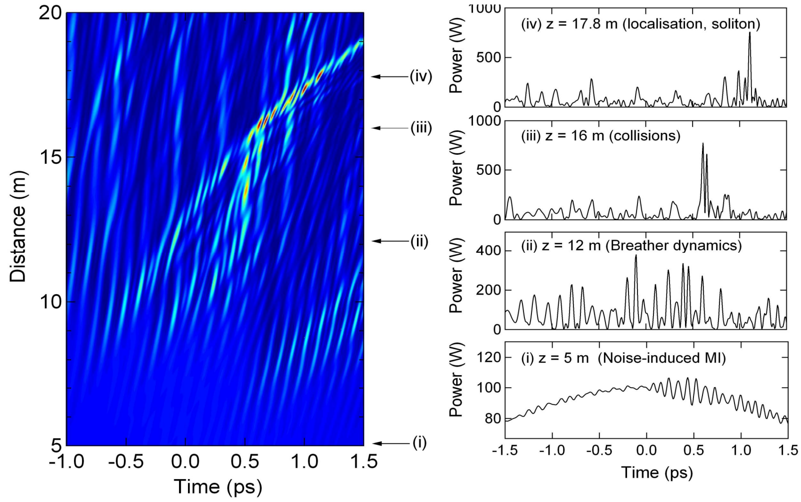

Figure 4. Detailed view of propagation in the NLSE with noise and TOD. Initially, noise-induced MI generates high-contrast AB-like pulses, but the effect of dispersion induces an additional phase of interactions, collisions and localization.

5. Conclusions

The experiments on the Peregrine soliton represent the first amplitude and phase measurements of a nonlinear breather structure in any continuous system described by the NLSE. The measurements of a strongly localized temporal peak upon a non-zero background confirm Peregrine's theoretical predictions of a rational soliton envelope. These results highlight how experiments in optics can be used to conveniently test more general theories of nonlinear waves and we anticipate future applications in establishing links between optical and hydrodynamic extreme events. In addition, numerical studies of the temporal and spectral properties of spontaneous MI have shown that the characteristics of both the modulated temporal profile and the associated spectrum can be explained in terms of the development of high amplitude AB sub-pulses. Other results have shown that SC generation described by an NLSE model perturbed only by TOD can lead to large amplitude localized optical rogue wave structures.

6. References

1. D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, "Optical rogue waves," Nature **450**, 2007, pp. 1054-1057.
2. J. M. Dudley, C. Finot, G. Millot, J. Garnier, G. Genty, D. Agafontsev, and F. Dias, "Extreme events in optics: Challenges of the MANUREVA project," *Eur. Phys. J. Special Topics* **185**, 2010, pp. 125-133.
3. J. M. Dudley, G. Genty, F. Dias, B. Kibler and N. Akhmediev, "Modulation instability, Akhmediev Breathers and continuous wave supercontinuum generation," *Opt. Express* **17**, 2009, pp. 21497-21508.
4. M. Erkintalo, G. Genty, and J. M. Dudley, "On the statistical interpretation of optical rogue waves," *Eur. Phys. J. Special Topics* **185**, 2010, pp. 135-144.
5. D. H. Peregrine, "Water waves, nonlinear Schrödinger equations and their solutions," *J. Austral. Math. Soc. Ser.* B **25**, 1983, pp. 16-43.
6. B. Kibler, J. Fatome, C. Finot, G. Millot, F. Dias, G. Genty, N. Akhmediev, and J. M. Dudley, "The Peregrine Soliton in Nonlinear Fiber Optics," *Nature Physics* **6**, 2010, pp. 790-795.
7. N. Akhmediev and A. Ankiewicz, *Solitons, Nonlinear Pulses and Beams*, Chapman and Hall, London, 1997.
8. N. Akhmediev and V. I. Korneev, "Modulation instability and periodic solutions of the nonlinear Schrödinger equation," *Theor. Math. Phys.* **69**, 1986, 1089-1093.
9. G. Genty, C. M. de Sterke, O. Bang, F. Dias, N. Akhmediev, J. M. Dudley , "Collisions and turbulence in optical rogue wave formation," *Phys. Lett. A* **374**, 2010, pp. 989-996.