A Fast ICA Based Iterative Blind Deconvolution Algorithm

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ABSTRACT

Successful blind image deconvolution algorithms require the exact estimation of the Point Spread Function size, PSF. In the absence of any priori information about the imagery system and the true image, this estimation is normally done by trial and error experimentation, until an acceptable restored image quality is obtained. This paper, presents an exact estimation of the PSF size that yields the optimum restored image quality. The paper also describes a least squares PSF estimation, instead of the slowly iterative update, that is commonly used in Iterative Blind Deconvolution software, IBD. Moreover, a technique is also proposed to improve the sharpness of the de-convolved images using Independent Component Analysis techniques (ICA). Simulation examples are given to show that the proposed technique manages to accurately estimate the PSF size apart from competing very well with the existing approaches.

I. INTRODUCTION

The goal of blind deconvolution is to recover two convolved signals \( f \) and \( h \) from their convolved (and normally noisy), version \( g \). Neither \( f \) nor \( h \) is known. In image processing, \( f \) represents the true image, whereas \( h \) represents the Point Spread Function PSF, which is responsible for blurring \( f \). Even if we have a priori information about the PSF, recovering the original image by inverse filtering is usually counterproductive, as it involves noise amplification, [1-2]. In general, in the spatial domain this blurring effect amounts to

\[
g(m,n) = \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} h(k,l) f(m-k, n-l) + w(m,n) \quad 0 \leq (m,n) \leq N-1
\]

\[i.e \quad G = HF+W\]

\( w \) is the associated zero-mean noise, while the PSF size is \( N_1 \times N_1 \). \( f \), \( g \) and \( w \) are column vector of length \( N^2 \), constructed by stacking the rows of the matrices \( f \), \( g \) and \( w \). \( H \) is a block circulant matrix of size \( (N^2 \times N^2) \), constructed from \( h \) by first padding zeros to make its size \( NxN \) (instead of \( N_1 \times N_1 \)), [2]. Now, to have a unique restoration of \( f \) and \( h \), both of \( f \) and \( h \) must be irreducible. At this point it is worth mentioning that, not all blurring causes can be precisely determined. However, by central limit theory, the net result of all these independent blurring factors can be in general, approximated by Gaussian PSF. It is known that Gaussian PSF is one of the most difficult cases to deal with in blind deconvolution, as it can be factored into two Gaussian PSF.

Eqn.(1), can be solved iteratively by starting with an initial \( h \), then obtain the least squares solution \( f \) and reiterating until the residual error between both sides of the equation, is minimal. However, this approach is prohibitive as it requires an enormous amount of computations, even if we make use of the circulant properties of \( H \). Many efficient techniques have been proposed to solve this problem, [3-9]. In [3], an Iterative Blind Deconvolution technique IBD has been proposed by alternate updating the 2D FFT of \( f \) and \( h \), until the relation \( G(\omega_1, \omega_2) = F(\omega_1, \omega_2)H(\omega_1, \omega_2) \) is almost satisfied. In its initial versions, it suffered from poor convergence, yet in latter versions [4], its robustness to noise and convergence properties are highly improved. An alternate double iteration algorithm that has good anti-noise capability has also been described in [5-6]. It is known as the Richards-Lucy algorithm, and is characterized with robustness to either Poisson or Gaussian noise, [7-8]. In [4-9], a thorough treatment of different blind deconvolution techniques can be found. All these techniques require an exact estimation of the blurring PSF size. Poor estimation means that these blind deconvolution algorithms will fail to yield good quality images. To our knowledge, this is done on trial and error basis. This paper, addresses this problem. It presents an analytical method to accurately estimating the optimum PSF size. Then, it considers the IBD algorithm. It improves its performance, by using a least squares PSF estimation in conjunction with every updated \( f \) instead of the commonly used iterative PSF update. The paper also describes a technique for improving the sharpness of the deconvolved image, through using ICA techniques to free the restored image from any uncorrelated components. Simulation examples are given to verify that the proposed techniques manage to improve the quality of the restored images.
II. THE PROPOSED FAST ITERATIVE BLIND RESTORATION ALGORITHM

The constrained least squared error algorithm [1-2], amounts to obtaining a restored image \( \hat{f}(m, n) \) that is the solution of the following constrained optimization problem: Find the optimum \( \hat{f}(m, n), \hat{h}(m, n) \) that minimize the objective function \( J \).

Minimize  
\[
J = \left| Q(\omega_1, \omega_2) \hat{F}(\omega_1, \omega_2) \right|^2 \text{ for all } -\pi \leq (\omega_1, \omega_2) \leq \pi
\]
Subject to  
\[
|G(\omega_1, \omega_2) - H(\omega_1, \omega_2)F(\omega_1, \omega_2)|^2 \leq \varepsilon^2
\]

Using the Lagrange multiplier technique, this problem can be formulated as

Minimize  
\[
J = \left| Q(\omega_1, \omega_2) \hat{F}(\omega_1, \omega_2) \right|^2 + \lambda \left( |G(\omega_1, \omega_2) - H(\omega_1, \omega_2)\hat{F}(\omega_1, \omega_2)|^2 - \varepsilon^2 \right)
\]

The solution to this constrained minimization problem, can be shown to be

\[
\hat{f}(\omega_1, \omega_2) = \frac{\hat{H}^*(\omega_1, \omega_2)G(\omega_1, \omega_2)}{|\hat{Q}(\omega_1, \omega_2)|^2 + \lambda |\hat{H}(\omega_1, \omega_2)|^2}
\]

Now, the function \( Q(\omega_1, \omega_2) \) is chosen to boost the high frequency energies of \( \hat{F}(\omega_1, \omega_2) \). As all natural images have pre-dominant low frequency content, minimizing \( J \) means that the true image \( f(m, n) \) is obtained, or at least nearly obtained. \( Q \) can be chosen in many different ways. In [1], two formulas were given to \( Q(m, n) \) to approximate Laplacian function. In this paper, a simpler modification is proposed to \( Q(m, n) \) that satisfies the high frequency emphasis requirements, is proposed. It is chosen as \( Q(\omega_1, \omega_2) = \frac{1}{F(\omega_1, \omega_2)} \). Using this choice, leads to the following iterative restoration algorithm

\[
\hat{f}^{(k)}(\omega_1, \omega_2) = \frac{H^*(\omega_1, \omega_2)G(\omega_1, \omega_2)}{|\hat{F}^{(k-1)}(\omega_1, \omega_2)|^2 + \lambda |\hat{H}(\omega_1, \omega_2)|^2}
\]

This is precisely the update algorithm cited without proof in [4].

Now, it is clear that if this size is not correctly chosen, then part of the blurred image will be included in the associated noise \( w(m, n) \). Therefore, at the optimum size \( N_p \), the entropy of \( J \) defined by

\[
J_{er}(m, n) = g(m, n) - \sum_{r=0}^{N_p-1} \sum_{l=0}^{N_p-1} \hat{h}(r, l) \hat{f}^{(k+1)}(m - r, n - l)
\]

Where \( \hat{f}, \hat{h} \) are the estimated image and PSF. In [12], an efficient technique has been described to estimate density distribution and entropy of any random signal. In this paper, Shannon's entropy measure is found to be suitable.

Having estimated the blurring PSF, then to speed up convergence and enhance the algorithm performance we incorporate a least squares estimation of \( \hat{h}^{(k)}(m, n) \) in every iteration cycle of \( \hat{f}^{(k)}(m, n) \). The following modification is proposed to estimate \( \hat{h}^{(k)}(m, n) \), for every update \( \hat{f}^{(k)}(m, n) \). It is summarized as follows:

1. For an initial choice of \( \hat{h}^{(k)}(m, n) \), evaluate the 2-D FFT \( \hat{H}^{(k)}(\omega_1, \omega_2) \). Then, evaluate \( \hat{F}^{(k+1)}(\omega_1, \omega_2) \) of Eq.(5), using this value of \( \hat{H}^{(k)}(\omega_1, \omega_2) \). In all simulation examples considered, the initial choice of \( \hat{h}^{(k)}(m, n) \) is the ones-matrix.
2. Using the 2-D IFFT, evaluate \( \hat{h}^{(k+1)}(m, n) \).
3. Find the optimum \( \hat{h}^{(k+1)}(m, n) \) that minimizes, in a least squares sense, the energy of \( J_{er}(m, n) \) of Eqn.(6), over the whole image size, \( 0 \leq (m, n) \leq N - 1 \). The matrix energy is measured as its Frobenious norm.
4. Using this least squares estimation as the \( k+1 \) update of \( h \), i.e. \( \hat{h}^{(k+1)}(m, n) \), \( k=0,1,2, \ldots \), go to step 1 and repeat iterations until a prescribed tolerance \( \varepsilon \) is obtained, or there is no improvement in \( \hat{h}^{(k+1)}(m, n) \).

We denote the resulting algorithm that implements the order determination and incorporates the least squares estimation of \( \hat{h}^{(k+1)}(m, n) \), \( k=0,1,2, \ldots \), in the classical IBD algorithm, by the Fast Iterative Blind Deconvolution algorithm, FIBD.

The proposed algorithm has been tested for several illustrative examples. However, for space limitations only one example is cited to verify this algorithm. The Cameraman Matlab is blurred with Gaussian PSF of size \( 8 \times 8 \) and with \( \sigma = 5 \). The FIBD was run for different PSF sizes, and for both noiseless and noisy blurred images. The noisy images were constructed using AWGN with \( \sigma^2 = 0.0005 \). Fig.1 shows residual entropy of Eq. (6) behavior of these cases. Fig. (2), shows the blurred image as well as the deconvoluted images obtained using the IBD, Richards-Lucy and the proposed FIBD methods. The following table gives the resulting PSNR of these deconvolved images in the noiseless case when using a Gaussian blurring.

<table>
<thead>
<tr>
<th></th>
<th>IBD</th>
<th>R-Lucy</th>
<th>FIBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cameraman</td>
<td>24.8919</td>
<td>24.2398</td>
<td>25.0262</td>
</tr>
</tbody>
</table>
III. IMPROVING SHARPNESS OF THE RESTORATION QUALITY

As blurring affects the energies of the high frequency bands of the image, it is clear that deblurring should boost these high frequency energies. In [10], a wavelet based technique is proposed to boost the detail energies of the one-level wavelet decomposition of $\hat{f}$ while minimizing the residual energy of Eq. (6). In this section, an alternate efficient technique is proposed. It is based on applying ICA techniques [11] to remove the uncorrelated blurring effects from the reconstructed image $\hat{f}$ (obtained from the proposed FIBD algorithm). This is achieved by feeding $\hat{f}$ and the image $f_{IBD}$ (obtained using IBD technique), to the fast ICA algorithm, after converting each of these two images to column vectors. One of the outputs of the fast ICA algorithm is an improved reconstructed image. Fig. 3 shows the $IBD$, $FIBD$ and the ICA reconstructed images. The figure also shows the edge images, obtained using Bspline wavelet family [12], using a threshold that assigns the image derivatives greater than 0.06 to be 1 otherwise set them to zero. These results indicate that ICA approach manages to improve the quality of deconvolved image and minimize blurring.

IV. CONCLUSION

This paper, describes how, in blind deconvolution when there is no priori information about both the true image and the blurring PSF, the size of the blurring PSF can be accurately estimated even in the presence of noise. The paper also describes how to improve IBD performance by incorporating a least square PSF updated, instead of the slowly iterative update used for its determination. A novel method was also described to increase the sharpness of the deconvolved images by applying ICA techniques. Results have shown that the ICA techniques manages to improve the sharpness and visual quality of the restored images.

V. REFERENCES

Fig. 1: Shannon's Residual Entropy Error performance with PSF size, for the noisy and noiseless images.

Fig. 2: Original, Gaussian blurred and deconvolved Cameraman images, using 3 deconvolving techniques.

Fig. 3: IBD, FIBD and ICA blurring and restored images as well as its edge images.