On the Minimization of Side-Lobes in Large Antenna Arrays for Microwave Power Transmission

B. Shishkov[(1)], N. Shinohara[(2)], H. Matsumoto[(3)], K. Hashimoto[(2)], T. Mitani[(2)]

[(1)] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria, bshishkov@math.bas.bg

(2) Research Institute for Sustainable Humanosphere, Kyoto University, Kyoto, Japan, shino@rish.kyoto-u.ac.jp; kozo@rish.kyoto-u.ac.jp; mitani@rish.kyoto-u.ac.jp

[(3)] Kyoto University, Kyoto, Japan, matsumoto@hq.kyoto-u.ac.jp

Abstract

Recent advances in space exploration have shown a great need for antennas with high resolution, high gain and low side lobe level (SLL). The last characteristic is of paramount importance especially for the Microwave Power Transmission (MPT) in order to achieve higher transmitting efficiency (TE) and higher beam collection efficiency (BCE). In order to achieve low side lobe levels, statistical methods play an important role. Various interesting properties of a large antenna arrays with randomly, uniformly and combined spacing of elements have been studied, especially the relationship between the required number of elements and their appropriate spacing from one viewpoint and the desired SLL, the aperture dimension, the beamwidth and TE from the other. We propose a new unified approach in searching for reducing SLL by exploiting the interaction of deterministic and stochastic workspaces of proposed algorithms. Our models indicate the side lobe levels in a large area around the main beam and strongly reduce SLL in the entire visible range. A new concept of designing a large antenna array system is proposed. Also, we have proposed a new technique to the beam-control in MPT using syclostationarity. Our theoretic study and simulation results clarify how to deal with the problems of side lobes in designing a large antenna array, which seems to be an important step toward the realization of future SPS/MPT systems.

1. Introduction

For the conventionally designed arrays where all elements are spaced uniformly, there exists an upper limit to the spacing if the grating lobes are not permitted to appear in the visible region.

The deterministic non-uniformly spaced algorithms are numerically difficult to implement for large antenna arrays.

The randomly spaced algorithms (the concept of "thin" arrays) are easier to implement, but need of further study in order to determine their merits and drawbacks.

In this paper we develop further the existing algorithms [1-3] and our previous research [4-6], proposing a new techniques to deal with side lobes and grating lobes.

2. Randomly and Uniformly Spaced Arrays

Consider a linear array along the X axis in Cartesian coordinate system and suppose we are given N+1 equally excited antenna elements by isotropic radiation to be placed at random within an aperture defined by $\left|X\right| \leq a/2$ in wavelength, in accordance with a common probability density function (pdf) $f(x)$.

Assume that the random positions $\{X_n\}$ are independent. Then for each sample vector $\{X_n\}$, $X_n \in R^{N+1}$, there is a sample radiation pattern function given by the magnitude of [2]

$$P(u) = \frac{1}{N+1} \sum_{n=-N/2}^{N/2} \exp(jux_n) \tag{1a}$$

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$$P(\theta) = \frac{1}{N+1} \sum_{n=-N/2}^{N/2} \exp\{j2\pi(\sin\theta - \sin\alpha)\} X_n \qquad (1b)$$

θ - the observation angle measured from the normal to the array axis
α - the scan angle measured from the normal to the array axis
$u = a\pi(\sin\theta - \sin\alpha)$- the observation angle parameter
{ $x_n = 2X_n / a$ }- normalized workspace
$a = Nd_x$ - the aperture, measured in wavelength.

We can determine the array factor (AF) $= |P(u)|^2$ or $|P(\theta)|^2$ as a random function. In (1) if $\{X_n\}$ is considered as positions of conventional uniform spacing $\{X_n = nd_x\}$ the model is automatically transformed in the deterministic one – see (2).

$$P(\theta) = \frac{1}{N+1} \sum_{n=-N/2}^{N/2} \exp\{j2\pi(\sin\theta - \sin\alpha)\}\ nd_x = \frac{1}{N+1} \sum_{n=-N/2}^{N/2} \exp(j\psi_n) \qquad (2)$$

- $\psi_n = \frac{2\pi}{\lambda}(\sin\theta - \sin\alpha) nd_x$. Normalized work space equals $x_n = 2nd_x / N$. We can determine the array factor AF $= |P(\theta)|^2$ as a periodic function [4].

The model (1) is pure stochastic model and will be coded as RA (random array). The model (2) is well known deterministic model of uniform spacing and will be coded as UA (uniform array). In [2] was found the distribution of maximum of SLL outside of the main beam region and that at any "*u*" the probability of antenna response being less than any level *r* is given by

$$\Pr\{\ |P(u)|\ < r, all\ u : \delta < |u| < 2\pi a\} \approx \left[1 - \exp^{-Nr^2}\right]^{[4a]} \qquad (3)$$

$$\varphi(u) = E\{\exp(jux)\} = \int_{-\infty}^{\infty} f(x)\exp(jux)dx \qquad (4)$$

Here δ is the first positive zero of characteristic function and the bracket $[4a]$ is the integer part of $4a$. This is a chi-squared distribution with two degrees of freedom.

This expression gives the number of elements required to achieve the desired SLL (maximum, not average) with predetermined confident probability of success such as 0.9, 0.95, etc.

3. Combined Stochastic Algorithm [4]

Let's consider the sample radiation pattern function given by the magnitude

$$P(\theta) = \frac{1}{N+1} \sum_{n=-N/2}^{N/2} \exp\{j2\pi(\sin\theta - \sin\alpha)(nd_x + X_n)\} \qquad (5)$$

The positions are a linear combination of deterministic one and random one. The AF $|P(\theta)|^2$ consists of two parts: The first part is a periodic function and if $d_x > \lambda / 2$ grating lobes can not be avoided.

Unlike, periodicity of the second part is strongly destroyed from random positions of x_n. It is equivalent of non-uniform spacing and there is no grating lobes, but on the price of relatively high SLL.

The basic role of the algorithm for optimization both side lobes and grating lobes play positions $\{X_n\}$ of antenna elements. This set $\{X_n\}$, $X_n \in R^{N+1}$, or its normalized version $\{x_n\}$ creates the work space which plays a fundamental role. Generally $\{x_n\} = \{x_{n\det}\} + \{x_{nrand}\}$. In section 2, we have considered the models $\{x_{nrand}\}$ and $\{x_{n\det}\}$ separately that were called RA and UA respectively.

Let's consider the model (5) over

$$\{x_n\} = \{x_{n\det}\} + \{\varepsilon_{nrand}\}. \quad (6)$$

It operates over two times larger workspace and $\{\varepsilon_{nrand}\} = \{x_{nrand}\}/N$ is a small random perturbation of the deterministic workspace and N is the number of elements. It is non-uniform spacing stochastic algorithm, which suppresses grating lobes, but with respect to SLL it is almost the same as RA algorithm, even with two time larger aperture. This model will be called Combined Stochastic Algorithm (CSA) and will compete with existing UA and RA algorithms. Our algorithm is a non-uniform spacing stochastic algorithm, where random workspace is strongly reduced, and positions $\{x_{nrand}\}/N$ play the role of very small random perturbations around the deterministic positions $\{x_{n\det}\}$. This algorithm as we will see has any advantages with respect to both UA and RA algorithms.

4. Further Development of the Problem of Minimization of SLL [5]

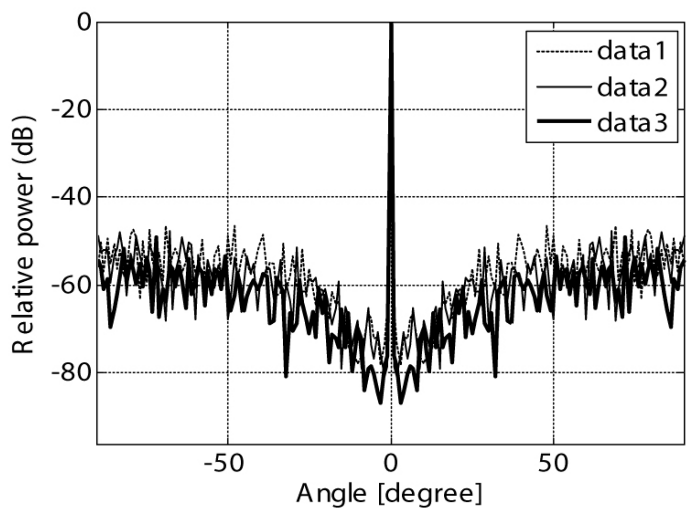

Fig.1 Radiation characteristics of the ICSA, *N*=16000 (data1), *N*=32000, (data2) and *N*=64000 (data3), $d_{av} = 2\lambda$

Let's rewrite Eqn. (6) as follows:

$$\{x_n\} = c_1 \{x_{n\det}\} + c_2 \{x_{nrand}/N\} \quad (7)$$

The influence of these two constants c_1 and c_2 over the behavior of the workspaces $\{x_{n\det}\}$ and $\{x_{nrand}\}$ reveals an interesting properties of our algorithm, which was studied in previous section with $c_1 = 1$ and $c_2 = 1$. We have found experimentally the following set of constants { $c_1 = 0.93,\ c_2 = 0.1$ } to improve minimization of SLL and no gratings lobes appear – see Fig. 1. Let's call this model improved combined stochastic algorithm (ICSA). From now on into entire visible range will be used suitable amplitude excitation function which concerns the beamwidth and influence to transmitting efficiency.

In the Table 1 are presented the features of the transmitting part of the system for MPT. η_{1D} and η_{2D} are one dimensional and two dimensional transmitting efficiency of linear array. The diameter of transmitting antenna in [m] is indicated for different number of antenna elements.

Table 1 Characteristics of the transmitting part of the system for MPT

Number of PCM	Number of Antenna Elements	Number of elements in Sub-array	Power [GW]	Diameter[m]	TE $d_{av}=c_1\lambda$ η_{1D} η_{2D}
1600	16000	10	1.02	769	0.9067 0.8221
2134	32010	15	1.82	1538	0.8831 0.7799
2560	64000	25	2.62	3076	0.8580 0.7362

5. Conclusion

1. The pure stochastic algorithms with average spacing more than λ (thinned arrays) really suppress grating lobes but on the price of sufficiently high SLL. So a large amount of elements need to decrease a little SLL.

2. We propose a new combined stochastic algorithm. It is a new unified approach in searching for reducing SLL by exploiting the interaction of deterministic and stochastic workspaces. It is a non-uniform spacing stochastic algorithm. So we have succeeded to reduce SLL considerably with out grating lobes appear and achieve high TE. We have proposed a new technique to the beam-control in MPT using syclostationarity.

3**.** Our study and simulation results clarify how to deal with the problems of side lobes and grating lobes in designing a large antenna array, which seems to be an important step toward the realization of future SPS/MPT systems.

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