On Further Discussion of Barkhausen Criterion

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Abstract

It has been pointed out that Barkhausen criterion cannot be used as a general oscillation condition, to evaluate (1) growth of oscillation, (2) steady state amplitude of oscillation in terms of circuit and active device parameters and (3) to as certain amplitude and frequency stability of oscillation, (4) to audit the nature of oscillation whether it is soft-self or hard-self excited. Here it has been shown that all these limitations can be over come through clubbing of quasi-state and quasi-linearization techniques and modified Barkhausen criterion can also be applied to four terminal as well as two terminal oscillators.

1. Introduction

During the last five or six years Barkhausen criterion which he developed in 1934 during his study on a vacuum tube feedback oscillator, is being actively reinvestigated [1-7]. Before initiating any further discussion it is important to recall that a sinusoidal (nearly so) is a device / system designed conceived to start oscillating to remain in an instable state in order to generate stable periodic waves with desired distortion at the finite output. Thus there is a process of transition from the initial state to the steady state. Naturally, Barkhausen criterion that was developed for pure sinusoidal oscillation in the steady cannot be considered as a full-proof criterion for oscillation in a practical oscillator. Thus the workers felt a need for discussion on this criterion on the following aspects, viz.,

(1) The possibility of applying this criterion during the transition from the initial to the steady state-i.e. non-steady part of the oscillations.
(2) The feasibility of applying the criterion at final part of the generation of oscillations where the growth of oscillation is restricted to a finite value - a manifestation of non-linear action producing distortion.
(3) Since Barkhausen developed the criterion for four terminal oscillators, it is felt that it cannot be applied for a two-terminal oscillator (Negative Resistance Oscillator). They differ only in form – only two sides of the same coin. Basically both are oscillators and can be studied from the same view points.
(4) It is sometimes felt that Barkhausen criterion can be only applied for a Lumped Parameter Localised Feedback Oscillator (like Barkhausen Oscillator). But the criterion can also be applied to the other two varieties of oscillators, viz., Distributed Parameter Localised Feedback Oscillator (Gunn oscillator) and Distributed Parameter Distributed Feedback Oscillator (Laser).
(5) It is interesting that not a single report has raised a question on the modification of Barkhausen criterion for soft self and hard self excited oscillators, particularly in respect of the modes of stability.
(6) Finally, physics of spectral purification of the output waveform during the growth part of the oscillation has been elaborated.
(7) In this paper we briefly touch upon these points which are yet to be clarified in spite of the efforts of many workers.

2. Barkhausen Modelling of FB and NR Oscillators and Spectral Processing

Referring to Appendix – I, it is seen that the following relations for the four-terminal and two-terminal oscillators can be written as

$$\left[\frac{v}{i_n}\right]_{4T} = \frac{G(s)}{1 - G(s)N(A)} \quad \left[4T\ Oscillator\right] \tag{1}$$

$$\left[\frac{v}{i_n}\right]_{2T} = \frac{G(s)}{1 - G(s)N(A)} \quad [2T\ Oscillator] \tag{2}$$

And the output v is taken as $v = A(t)\exp(j\omega_1 t + \theta)$ and the instantaneous frequency is defined by the relation:

$j\omega = \dfrac{1}{v(t)}\dfrac{dv}{dt}$. N(A) is the equivalent linearised gain of the nonlinearity. Typically it is given as
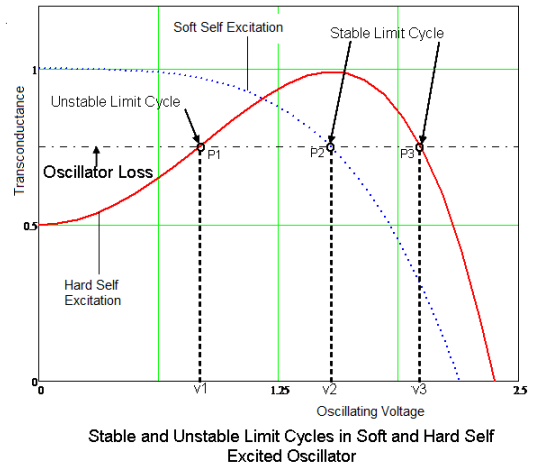
$$N(A) = \alpha - (3/4)\gamma A^2. \tag{3}$$

α and γ are constants of the nonlinearity. From the above it is easily seen that Barkhausen Criterion for both the varieties is expressed as $\qquad\qquad G(s)N(A) = 1 \tag{4}$

3. Soft and Hard Excitation

Consider the diagram which plots the reflected conductance of the oscillator appearing either due to internal or external feedback mechanism. The load of the oscillator is plotted on the same graph. It intersects the curves of reflected conductances (which are negative) at three points P1, P2 and P3. At these points, the energy balance principle holds, i.e. energy supplied equals the energy lost. Therefore, theoretically the system is capable of executing oscillations with amplitudes, V1, V2 and V3 respectively. Only one of the hard self excitation (P3 with V3; *Stable Limit Cycle*) and (P2 with V2; *Stable Limit Cycle*) of the soft self excitation are stable. These two aspects clearly demonstrate that growth of oscillations in these cases will differ in detail. As a result variation of dynamic bandwidth of the oscillators with the growth of oscillation for soft and hard self-excitations will differ in detail. As a result, *Spectral Processing* will be different in these two varieties. All these suggest that it is simply not possible to understand the mechanism of oscillations without taking into consideration of the nonlinear device-circuit interaction

Stable and Unstable Limit Cycles in Soft and Hard Self Excited Oscillator

4 Growth of Oscillations

G(s) is given as

$$\frac{1}{G(s)} = \left(\frac{1}{sL} + G + sC\right) \tag{5}$$

$$s = j\left(\omega_1 + \frac{d\theta}{dt}\right) + \frac{1}{A}\frac{dA}{dt} \tag{6}$$
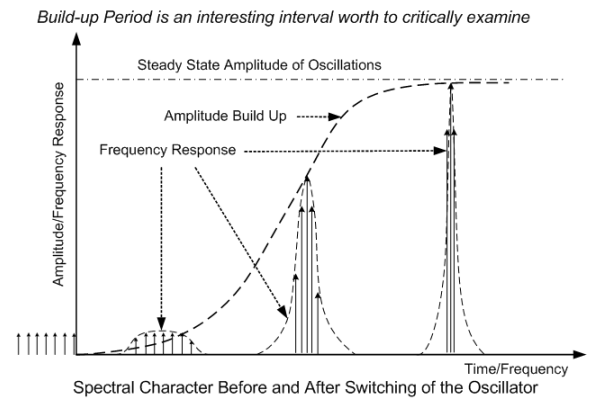
Barkhausen Criterion

$$G(s)N(A) = 1 \tag{7}$$

Using (5) to (7) and assuming that the A(t) and $\theta(t)$ are slowly varying function of time, it is easily shown that

$$\frac{da}{dt} = \frac{\omega_0}{2}(\alpha - G)\cdot(1 - a^2)a \tag{8}$$

Where

$$\omega_0 = \frac{1}{\sqrt{LC}}; a = \frac{A}{A_0}; A_0^2 = \frac{4}{3}\frac{\alpha - G}{\gamma} \tag{9}$$

Build-up Period is an interesting interval worth to critically examine

Spectral Character Before and After Switching of the Oscillator

The nature of variation of the amplitude of oscillation with time that can be obtained is shown in the adjoining figure by the dotted line.

5. Spectral Processing

Rewriting the equation (1) or (2) as

$$\frac{v}{i} = \frac{-1/(\alpha - G)}{\left(1 - A^2/A_0^2\right)} \cdot \frac{1}{1 - jQ(A)\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)} \tag{10}$$

Where   Q is given by

$$Q(A) = \frac{\omega_0 C}{\alpha - G} \cdot \frac{1}{1 - \left(A/A_0\right)^2} \tag{11}$$

It is interesting to note that the phase-shift between the voltage and current is obtained as

$$\varphi = \pi + \psi; \; \psi = \arctan\left(Q(A)\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right) \tag{12}$$

That is, an equivalent impedance of the form appears

$$Z_{eq} = -\frac{1}{(\alpha - G)\left(1 - A^2/A_0^2\right)}Cos\,\psi + j\frac{1}{(\alpha - G)\left(1 - A^2/A_0^2\right)}Sin\,\psi \tag{13}$$

Note that for a self-excited oscillator A is a growing (or decaying)   function of time and trying to approach a non-zero value (A_0) determined by the system parameter. Many things can be elaborated from the relation (10), but one important point is important to note. That is, as the amplitude grows the equivalent Q of the system becomes larger helping to clip off the unwanted part of the spectrum of the exciting source. But ultimately when it approaches the steady state the Q becomes very large, the spectral width of the oscillator output becomes narrow. But it never becomes a line spectrum for two reasons, namely the observation period is finite, i.e. what we observe a pulsed sinusoid not a sinusoid and infinite Q violates the law of thermodynamics (?). Distortion, a necessary evil, saves us from the catastrophic consequences. It tells us that the frequency of oscillation is not the resonant frequency ω_o and amplitude oscillation is not the steady state amplitude A_0. It is shown in [9] the presence of third harmonic distortion at the output indicates that the frequency of will be ω_a instead of ω_0

$$\omega_a = \omega_0 - \frac{1}{16\omega_0}\left(\frac{\alpha - G}{C}\right)^2 \tag{14}$$

6. Conclusion

The modified Barkhausen criterion, presented in this paper, takes into account the non-linear characteristic of the active circuit in a simple way that provides evaluation of the time-dependent growth of oscillation and the steady state amplitude of oscillations in terms of the circuit and device parameters. As a span off it also provides a method of ascertaining the properties of soft self and hard self excited oscillators. The finalising of this paper can also used to study the effect of device non-linearity on Nyquist Criterion [8]

7. Acknowledgement

8. References

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Appendix – I

Refer to adjoining figure depicting a 4-T and a 2-T terminal oscillator along with the different node voltages and currents. Note that i_n is the starting noisy excitation to the oscillator. **4-T Oscillator (Feedback)**

$$i_c + i_n = C\frac{dv}{dt} + Gv + \frac{1}{L}\int vdt$$

$$i_c = g_m(v) \cdot v_b; \quad v_b = M\frac{d}{dt}\left[\frac{1}{L}\int vdt\right]$$

Putting $N(v) = \frac{M}{L}g_m(v)$ it is easy show that

Feedback (4-T) and Negative Resistance (2-T) Oscillator

Negative Resistance Oscillator

$$\frac{v}{i_n} = \frac{G(s)}{1 - G(s) \cdot N(v)} \quad \text{(I.1)}$$

Where G(s) is given by

$$\frac{1}{G(s)} = sC + G + \frac{1}{sL}$$

FET Oscillator
Lumped Element Localized Feedback Oscillator

Gunn Oscillator
Distributed Element Localized Feedback Oscillator

Semiconductor Laser
Distributed Element Distributed Feedback oscillator

2-T Oscillator (Negative Resistance)

$$i_n = C\frac{dv}{dt} + Gv + \frac{1}{L}\int vdt - N(v)v$$

Using the notations indicated above it is easy to show that

$$\frac{v}{i_n} = \frac{G(s)}{1 - G(s) \cdot N(v)} \quad \text{(I.2)}$$

It is noted that N(v) denotes the equivalent linearised gain of the nonlinear element. The other important point to be carefully noted that although the above derivations have done for the lumped element localized feedback oscillators these derivations can be easily extended to the other varieties of the oscillators, namely distributed element localized feedback and distributed element distributed feedback.