Joint study of PAPR reduction and digital predistortion

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Abstract
Within the issue of transmission of signals with no constant envelope as Orthogonal Frequency Division Multiplexing (OFDM) for example, the Peak-to-Average Power Ratio (PAPR) reduction and the linearization are nowadays the two solutions proposed to deal the effects of the nonlinearities of the power amplifiers. In spite of their interdependence, these two solutions are separately studied and optimized. This can degrade seriously their common performance once stakes together because of opposite effects. In this paper, based on Rapp’s memoryless amplifier, we demonstrate that predistortion increase the signal’s PAPR and then proposed a joint combination scheme of “clipping & filtering” and polynomial predistortion. Computer simulations and analysis show that signal keeps a good level of linearity improvement measured with Modulation Error Ratio (MER) while decreasing the predistortion’s complexity.

1. Introduction
The power efficiency and the distortions level of the amplified signal are both important in any multi carrier context, especially in a cellular base station. Unfortunately, the most usual amplifiers, class-A and class-AB, are not perfectly linear and have modest efficiencies. Considering their transfer characteristics, AM/AM and AM/PM, two regions can roughly draw our attention. A linear region where the amplifier transfer characteristic is ideal and the output waveshape is identical to that of the input. Then a saturation region where the output waveshape is no longer identical to that of the input, the amplifier introduces amplitude and phase distortions. These regions are illustrated in Figure 1. An intuitive approach to avoid distortions is to achieve the power amplification in the linear region but in this case the amplifier has a low efficiency. The increasing of hand-portable equipments and the debate around power saving have focused attention on methods to improve equipments power efficiencies. The solution is then a compromise between distortions level and power efficiency by amplifying as close as possible the saturation region.

Many PAPR reduction schemes and linearization methods are regularly proposed to reduce the signal’s dynamic and compensate the nonlinearities respectively. Most of them are described in [1]. Even if each of them is optimized according to its own criteria, their combination can not be optimal because of opposite effects. The objective of this paper is to propose a new approach that jointly combine PAPR reduction and linearization. This joint approach aims at an overall exchange between these two processes to improve their common performance and reduce the complexity. Several previous works have been done in [5, 6] where the performance of a simple combination is studied. In [3], a joint optimization of Tone Reservation (TR) and predistortion is also proposed for wireless LAN (Local Area Network) applications.

The remainder of this paper is organized as follows. In Section 2, we consider the predistortion method and demonstrate that linearization can get opposite effects on PAPR reduction. In Section 3, we investigate a methodology to achieve a polynomial predistortion and propose a joint combination scheme of ”clipping & filtering”[1, 2] as PAPR reduction method and polynomial predistortion[4] as linearization technique. Simulations results and analysis are shown in Section 4 and a conclusion is given in Section 5.

2. Effects of Predistortion on PAPR reduction

PAPR reduction and linearization are complementary to deal the effects of the HPA nonlinearities and improve the power efficiency but they have other opposite effects. From predistortion[4] as linearization method, we demonstrate mathematically that the predistorted signal has a larger PAPR.

Let us consider $\tilde{x}(t)$ and $y(t)$ the input and output of a predistorter’s function $h$ respectively; so $y(t) = h(\tilde{x}(t))$ with $t \in [0; T_s]$. The nonlinear function of HPA (Figure 1) is a concave function thus we can conclude
that $h$ is convex because the predistortion basically performs an inverse of amplification. Since $h$ is convex, we can write:

$$P_x = E[\tilde{x}(t)] \quad \text{and} \quad P_y = E[y(t)] = E[h(\tilde{x}(t))].$$

(1)

$$0 < P_x < \max_t |\tilde{x}(t)|^2 \quad \Rightarrow \quad \frac{P_y}{P_x} \leq \frac{\max_t |y(t)|^2 - P_y}{\max_t |\tilde{x}(t)|^2 - P_x}$$

$$\Rightarrow \quad P_x \max_t |y(t)|^2 - P_y \max_t |\tilde{x}(t)|^2 \geq 0$$

$$\Rightarrow \quad PAPR_{y(t)} - PAPR_{\tilde{x}(t)} \geq 0$$

(2)

Throughout these equations, we denote that the predistortion increases the PAPR of the signal contradictory to the PAPR reduction. This is an example of opposite effects and a proof that a joint study of both methods is necessary.

### 3. Joint combination scheme of "clipping & filtering" as PAPR reduction and polynomial predistortion as linearization method

Ideally, a predistorter performs the inverse of HPA transfer characteristic but the problem is to identify the required predistorter and to design a circuit with the same transfer characteristic. Moreover the predistortion increases the PAPR as shown in Section 2. An approach to solve these problems is to use a polynomial predistortion [4] which is easy to design and allows to control the deliberate level of linearity improvement depending to the degree of the polynomial. Furthermore, the complexity of predistortion becomes polynomial $O(N^d)$ where $d$ is the degree of the polynomial and $N$ the number of samples. As illustrated on Figure 2, we propose a joint combination scheme that consist first to reduce the PAPR using "clipping & filtering"[1, 2] and then predistort the signal using the polynomial. The degree of the predistorting polynomial is calculated according to the clipped signal PAPR and the Input Back-Off (IBO). The working region of the amplifier is controlled by the IBO criteria expressed in $dB$ and defined in [1] as:

$$IBO = 10 \log_{10} \frac{P_{in,sat}}{P_{in}} [dB]$$

(3)

where $P_{in}$ is the input mean power and $P_{in,sat}$ the input saturation power which represents the power’s threshold from which the HPA does not amplify any more. The polynomial predistortion can be time-invariant as in our computer tests in Section 4 or adaptive according to temperature, aging, etc. In this case, the polynomial coefficients are generated in real-time from the feedback of the HPA output. The joint combination’s steps can be described as follows:

- **Initialization.** This step is once executed and consists in choosing a set of parameters which are:
1. the Clipping Ratio \( CR = 20 \log_{10} \frac{A}{\sqrt{P_{in}}} \) [dB] where \( A \) is the desired clipping level.
2. the desired IBO (Equation 3) which determine the working point of the HPA.
3. and the maximum mean error allowed after predistortion \( \Delta Err_{max} \). This error represent the gap between the perfect predistorsion and that with the chosen degree.

**Run time.** These steps are executed for each multi carrier symbol. Basically, the first step is PAPR reduction and the second one is the polynomial predistortion using information from the first.

1. Proceed to "clipping & filtering" with the clipping level given by CR.
2. Initiate the polynomial degree \( d = d_0 \) where \( d_0 \) is the initial degree (\( d_0 = 1 \)).
3. Predistort the clipped signal using the polynomial at the chosen degree \( d \).
4. Evaluate \( \Delta Err = E[|z - \tilde{x}|^2] \), if \( \Delta Err \leq \Delta Err_{max} \) \( d \) is optimal else go back to the step 2 and increment the degree \( d = d + 1 \).
5. Amplify the signal with the predefined IBO and transmit it.

The main objectives in this combination scheme are the predistorter complexity, the power efficiency and the distortions compensation. By changing the objectives and using the PAPR reduction scheme, we can defined some other optimization procedures.

4. **Tests results and analysis**

The purpose of a joint approach is to improve the performance of the power amplification. This takes place through an information exchange between the PAPR reduction scheme and the linearization scheme to avoid opposite effects and improve their common performance. Three types of joint combination can be distinguished. The PAPR reduction scheme is optimized according to the performance of linearization method as in [3] or vice versa as we present in Section 3 but the most interesting would be to optimize both jointly.

The proposed combination scheme in Section 3 and depicted in Figure 2 has been simulated using an IEEE standard 802.11g. OFDM signal is modulated with 16-QAM and the number of used sub carriers is \( N = 64 \). A memoryless Solid State Power Amplifier(SSPA) is implemented using Rapp model. The AM/AM transfer function of the HPA and the associated polynomial predistotrer are given:

- **HPA AM/AM transfer characteristic:**
  \[
  f_A[x] = \frac{x}{1 + \left( \frac{x}{A_{sat}} \right)^{2p}}, \quad x \in [0, +\infty[. \tag{4}
  \]

- **Predistortion function:**
  \[
  f_A^{-1}[x] = \frac{x}{1 - \left( \frac{x}{A_{sat}} \right)^{2p}}, \quad x \in [0, A_{sat}[. \tag{5}
  \]

- **Polynomial predistortion obtained by Taylor series expansion of \( f_A^{-1} \) with \( p = 2 \) and \( A_{sat} = 1 \):**
  \[
  h(x) = x + \frac{1}{4} x^5 + \frac{5}{32} x^9 + \frac{15}{128} x^{13} + \frac{195}{2048} x^{17} + \frac{663}{8192} x^{21} + \frac{4641}{65536} x^{25} + \frac{16575}{262144} x^{29} + \ldots \tag{6}
  \]

We can notice that the predistorter of Equation 5 is defined for \( x < A_{sat} \). This means that over \( A_{sat} \) the nonlinearities cannot be compensated. Thus, a PAPR reduction is imperative. In our tests simulations, the OFDM signal is first clipped with clipping ratio \( CR = 5dB \) and then IFFT/FFT out-of-band filter [2] is applied. After that, the output signal is predistorted using the polynomial predistorter (Equation 6) with an optimal degree. Without the joint approach, the predistortion is always done with a maximal complexity using the function \( f_A^{-1} \) even if the HPA is working in the linear region and the input signal is backed-off to avoid saturation. With joint approach, the predistortion’s complexity (Equation 5) becomes polynomial and
depends on the clipped signal’s dynamic and the IBO. A compromise must be found between PAPR reduction and predistortion which increases the PAPR.

The Figures 3 and 4 compare the performance of the proposed joint scheme and the simple combination of "clipping & filtering" method and predistortion. The level of linearity improvement is measured between the input signal $x(t)$ and the amplified signal $z(t)$ by the MER[dB]. The complexity $O(N^d)$ of the predistorter is represented by the polynomial’s degree in Figure 4. If the HPA is working in linear region, the predistorter will denote a good compensation with large MER and small polynomial’s degrees as shown in Figure 4. However, when the IBO decrease and the HPA start working near the saturation region, the MER degrades and larger degrees are required for the predistortion polynomial. It’s illustrated that for $\Delta Err_{max} = -10dB$, a minimum $IBO = 4dB$ is required to have a gain of reduction of the predistortion complexity and for $\Delta Err_{max} = -5dB$, the minimum $IBO = 3dB$.

5. Conclusion and acknowledgment

In this paper, we have described the joint combination of PAPR reduction and linearization. Having shown that the predistorsion increases the PAPR, we have proposed an example of joint scheme which combined clipping & filtering method and polynomial predistortion. Overall, the tests of the proposed method show a good performance of linearity improvement and complexity reduction.

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References