Resource allocation algorithm for heterogeneous services in decode-and-forward OFDM relay system

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Abstract

This paper studies the resource allocation problem for heterogeneous services in decode-and-forward orthogonal frequency division multiplexing system. A resource allocation algorithm is proposed to maximize the data rate of non-realtime service (NRTS) while satisfying realtime service (RTS) requirements. The problem to be solved is decomposed into two sub-problems: power allocation for RTS and NRTS. The problem of RTS is solved with Lagrangian method. A two-step method is proposed for NRTS. Simulation results show that the proposed algorithm has a much better performance than the fixed algorithm and its complexity is less than the exhaustive algorithm.

1 Introduction

Multihop relay systems with orthogonal frequency division multiplexing (OFDM) schemes have attracted much research attention because of resource allocation problem and their application in wireless communications [1-4]. Relay strategies are categorized into two major types: amplify-and-forward (AF) and decode-and-forward (DF). Capacity maximization algorithm is studied in DF OFDM relay system under separate power constraints for the non-realtime service [2]. A power minimized algorithm for fix data rate has been proposed for realtime service [3]. A capacity maximization resource allocation algorithm for heterogeneous services is proposed in AF OFDM system under total power constraint in [4]. Future wireless communication systems expects heterogeneous services under separate power constraints at source and relay. This motivates us to further investigate the problem in DF OFDM relay system with heterogeneous services under separate power constraints.

2 System Model and Problem Formulation

We consider a two-hop DF OFDM relay system which has N subcarriers. The source transmits a signal during the first time slot while the relay and destination receive. The relay receives in the first time slot, and decodes, recodes and retransmits in the second time slot. Each subcarrier used by the source in the first time slot is paired with the same subcarrier used in the second time slot. We use \(h_{0,i}, h_{1,i}, \) and \(h_{2,i}\) to denote the channel gains of S-D, S-R and R-D link on subcarrier \(i\). \(P_{s,i}\) and \(P_{r,i}\) are the power at source and relay on subcarrier \(i\). All channel gains are assumed to be known at source.

The channel capacity of the \(i\)-th subcarrier between the source and the destination can be written as [2],

\[
R_i = \min \left\{ \frac{1}{2N} \log_2 \left( 1 + \frac{P_{s,i}h_{1,i}}{\Gamma \sigma^2} \right), \frac{1}{2N} \log_2 \left( 1 + \frac{P_{s,i}h_{0,i} + P_{r,i}h_{2,i}}{\Gamma \sigma^2} \right) \right\}
\]

(1)

where \(\Gamma = - \ln(5\mu)/1.5\) is the signal to noise ratio (SNR) gap, \(\mu\) is bit error rate (BER), \(\sigma^2\) is noise power.

Using \(P_t (P_t = P_{s,i} + P_{r,i})\) and \(h_i\), \(R_i\) is maximal and (1) is simplified as

\[
R_i = \frac{1}{2N} \log_2 \left( 1 + \frac{P_t h_i}{\Gamma \sigma^2} \right), \quad h_i = \begin{cases} \frac{h_{1,i}h_{2,i}}{h_{0,i} + h_{2,i} - h_{0,i}}, & h_{1,i} > h_{0,i} \\ \frac{h_{0,i}}{h_{0,i}}, & h_{1,i} \leq h_{0,i} \end{cases}
\]

(2)
where $h_i$ is the equivalent channel gain on subcarrier pair $i$. The power on subcarrier $i$ at source and relay is

$$P_{s,i} = \begin{cases} \frac{h_{2,i}}{h_{1,i} + h_{2,i} - h_{0,i}} P_{r,i}, & h_{1,i} > h_{0,i} \\ h_{1,i} \leq h_{0,i} & 0, \end{cases} \quad P_{r,i} = \begin{cases} \frac{h_{2,i} - h_{0,i}}{h_{1,i} + h_{2,i} - h_{0,i}} P_{s,i}, & h_{1,i} > h_{0,i} \\ h_{1,i} \leq h_{0,i} & 0 \end{cases}$$ (3)

The resource allocation problem for heterogeneous services in DF OFDM relay system is formulated as,

$$R_b = \max \sum_{i \in \Omega_b} R_i, \quad s.t. \sum_{i \in \Omega_a} R_i = R_a, \sum_{i \in \Omega} P_{s,i} \leq P_s, \sum_{i \in \Omega} P_{r,i} \leq P_r, P_{s,i}, P_{r,i} \geq 0, \forall i \in \Omega$$ (4)

where $\Omega_a$ and $\Omega_b$ are the subcarrier set of RTS and NRTS. $R_a$ and $R_b$ are the data rate of RTS and NRTS. $P_s$ and $P_r$ are the total power at source and relay.

The above nonlinear optimal problem can be solved with exhaustive algorithm by calculating $P_{s,i}$ and $P_{r,i}$ for every $\Omega_a$ and $\Omega_b$, and obtaining the maximal value of $R_b$ as the solution with a complexity of $o(2^N)$. Here, we propose a suboptimal algorithm. First, we compute $h_i$, and sort as $h_1 \geq h_{i+1}$. Then, the number of $k$ ($k = 1, \cdots, N$) subcarriers which have the maximal value of $h_i$ are allocated to RTS $\Omega_a = \{1, 2, \cdots, k\}$, the other subcarriers are allocated to NRTS $\Omega_b = \{k + 1, \cdots, N\}$. The complexity of the proposed algorithm is $o(N)$.

### 3 Resource Allocation Algorithm

The problem (4) is decomposed into two sub-problems which are the power allocation for RTS and NRTS for fixed $\Omega_a$ and $\Omega_b$.

**Sub-problem 1:** $P_a = \min \sum_{i \in \Omega_a} (P_{s,i} + P_{r,i}), \quad s.t. \sum_{i \in \Omega_a} R_i = R_a, P_{s,i}, P_{r,i} \geq 0, \forall i \in \Omega_a$ (5)

**Sub-problem 2:** $R_b = \max \sum_{i \in \Omega_b} R_i, \quad s.t. \sum_{i \in \Omega_b} P_{s,i} \leq P_{sb}, \sum_{i \in \Omega_b} P_{r,i} \leq P_{rb}, P_{s,i}, P_{r,i} \geq 0, \forall i \in \Omega_b$ (6)

Sub-problem 1:

Taking $h_i$ and $P_i$ into (5), sub-problem 1 can be solved with Lagrangian method [5].

$$P_i = \Gamma_a \sigma^2 \left[ \left( \frac{4^N R_a}{\prod_{i \in \Omega_a} h_i} \right)^{1/N_a} - \frac{1}{h_i} \right]^+, \quad i \in \Omega_a$$ (7)

where $[x]^+ = \max(0, x)$, $N_a$ is the number of subcarriers of $\Omega_a$. $P_{s,i}$ and $P_{r,i}$ are solved with (3). $P_a = \sum_{i \in \Omega_a} P_{s,i}$ and $P_{ra} = \sum_{i \in \Omega_a} P_{r,i}$, are defined as the total power for RTS at source and relay. The total power for NRTS at source and relay are: $P_{sb} = P_s - P_{sa}$ and $P_{rb} = P_r - P_{ra}$.

Subproblem 2:

The sub-problem 2 can be solved with two steps: initial power allocation and power adjustment.

1. **Initial power allocation:** We take separate power constraints ($P_{sb}, P_{rb}$) as total power constraint ($P_b = P_{sb} + P_{rb}$). Similar to that of sub-problem 1, the sub-problem 2 can be simplified with $P_i$ and $h_i$, and solved with Lagrangian method.

$$P_i = \left[ \frac{1}{2N \rho \ln 2} - \frac{\Gamma_b \sigma^2}{h_i} \right]^+, \quad \rho = \frac{N_b}{2N \ln 2 \left( P_b + \Gamma_b \sigma^2 \sum_{i \in \Omega_b} 1/h_i \right)}$$ (8)
The initial power allocation $\mathcal{P}_{s,i}$ and $\mathcal{P}_{r,i}$ are obtained with (3). The required power is $P_{sb} = \sum_{i \in \Omega_b} \mathcal{P}_{s,i}$ and $P_{rb} = \sum_{i \in \Omega_b} \mathcal{P}_{r,i}$. $\Delta P_{sb} = P_{sb} - P_{sb}$ and $\Delta P_{rb} = P_{rb} - P_{rb}$ are the difference between the actual power and the required power.

2. Power adjustment: If $\Delta P_{sb} > 0$ and $\Delta P_{rb} < 0$, the relay subcarrier power is reduced to satisfy: $\sum_{i \in \Omega_b} \tilde{P}_{r,i} \leq P_{rb}$. And, the source subcarrier power is increased to satisfy: $\sum_{i \in \Omega_b} \tilde{P}_{s,i} \leq P_{sb}$. If $\Delta P_{sb} < 0$ and $\Delta P_{rb} > 0$, we just reduce the subcarrier power at source, while the subcarrier power are remained at relay. According to channel gains, the set of $\Omega_b$ is decomposed into two sets: the subcarrier set for direct link $\Omega_{b1} = \{i|h_{1,i} \leq h_{0,i}, i \in \Omega_b\}$ and the subcarrier set for relay link $\Omega_{b2} = \{i|h_{1,i} > h_{0,i}, i \in \Omega_b\}$.

If $\Delta P_{sb} > 0$ and $\Delta P_{rb} < 0$, the process for power adjustment is as follows.

1. The subcarrier power at relay is reduced, and the subcarrier power at source is $\mathcal{P}_{s,i}$. The problem is solved with Lagrangian method.

\[
\tilde{P}_{r,i} = \left[ \frac{1}{2N\lambda \ln 2} - \frac{\sum_{i \in \Omega_{b2}} \tilde{P}_{s,i} h_{0,i}}{h_{2,i}} \right]^{+}, \quad \lambda = \frac{N_{b2}}{2N\ln 2} \left( \sum_{i \in \Omega_{b2}} (\Gamma_b \sigma^2 + \tilde{P}_{s,i} h_{0,i}) / h_{2,i} \right)
\]

The differences are $\Delta P_{r,i} = \tilde{P}_{r,i} - P_{r,i} \forall i \in \Omega_{b2}$. The subcarrier with the maximal value of $\Delta P_{r,i}$ is: $i^* = \arg \max \sum_{i \in \Omega_{b2}} \Delta P_{r,i}$. If $\Delta P_{r,i^*} > 0$, the final power of $i^*$ at relay is $\hat{P}_{r,i^*} = \tilde{P}_{r,i^*}$. Then, updating the parameters: $\Omega_{b2} = \Omega_{b2} - \{i^*\}$ and $P_{rb} = P_{rb} - \hat{P}_{r,i^*}$. The above process is repeated until $\Delta P_{r,i} \leq 0, \forall i \in \Omega_{b2}$ is satisfied, $\hat{P}_{r,i} = \tilde{P}_{r,i}, \forall i \in \Omega_{b2}$.

2. Increasing the subcarrier power at source, the subcarrier power at relay is $\tilde{P}_{r,i}$.

The problem is solved with Lagrangian method, and the process is similar to at relay.

\[
\tilde{P}_{s,i} = \left\{ \begin{array}{ll}
\frac{1}{2N\lambda \ln 2} - \frac{\Gamma_b \sigma^2}{h_{0,i}} & , i \in \Omega_{b1} \\
\frac{1}{2N\lambda \ln 2} - \frac{\Gamma_b \sigma^2 + \tilde{P}_{r,i} h_{2,i}}{h_{0,i}} & , i \in \Omega_{b2}
\end{array} \right.
\]

\[
\rho = \frac{N_b}{2N\ln 2} \left( P_{sb} + \sum_{i \in \Omega_{b1}} \frac{\Gamma_b \sigma^2}{h_{0,i}} + \sum_{i \in \Omega_{b2}} \frac{\Gamma_b \sigma^2 + \tilde{P}_{r,i} h_{2,i}}{h_{0,i}} \right)
\]

4 Simulation Results and Analysis

The OFDM relay system parameters are $B=1$MHz, the noise power spectrum density is $N_0=-80$dB/Hz, the bit error rate is BER=$10^{-3}$. The wireless channel is modeled as frequency-selective consisting of six independent Rayleigh paths. Three algorithms are studied: the proposed algorithm, the exhaustive algorithm and the fixed algorithm. The fixed algorithm is about allocating the fixed number of $G$ subcarriers to RTS, and $G_1 = N/4$ and $G_2 = N/2$ are considered in this paper.

Figure 1 gives the data rate of NRTS $R_b$ versus average SNR (SNR = $P_s/LN_0 B$) under the condition of $R_a=0.1$bps/Hz and $N=8$. The data rate of NRTS $R_b$ increases with SNR, and the proposed algorithm can achieve remarkable performance improvement over the fixed algorithm for $G_1 = N/4$ and $G_2 = N/2$, and is close to the exhaustive algorithm. The proposed algorithm has much higher spectral efficiency than the fixed algorithm, and the complexity of the proposed algorithm is much less than that of the exhaustive algorithm.

Figure 2 shows how $R_b$ varies with $R_a$ under the condition of $N = 16$ and SNR=16dB. It can be seen that $R_b$ decreases with $R_a$, and the proposed algorithm has the best performance. If $R_a$ increases, the required resource of RTS will be increased. The remaining resource for NRTS decrease, so $R_b$ decreases as shown in Fig 2. The $G_2 = N/2$ algorithm has the worst performance in the range of $R_a = 0.05 - 0.6$bps/Hz, and it allocates the most subcarriers in the three algorithms. $R_b$ of the $G_1 = N/4$ algorithm is the least value
in the range of \( R_a = 0.65 - 0.8 \text{bps/Hz} \), and outage occurs when \( R_a \) is 0.8 bps/Hz. Whereas, the proposed algorithm can adjust both subcarrier and power, the performance of the proposed algorithm is much better than that of the fixed algorithm.

![Fig. 1: the data rate \( R_b \) versus the average SNR](image1)

![Fig. 2: the data rate of \( R_b \) versus \( R_a \)](image2)

5 Conclusions

This paper studies the resource allocation problem for heterogeneous services in DF OFDM relay system under separate power constraints. A resource allocation algorithm is proposed to maximize the data rate of NRTS while satisfying the required data rate of RTS. We first calculate \( h_i \), and sort \( h_i \) in descending order. Then, \( k(k = 1, 2, \ldots, N) \) subcarriers are allocated to RTS \( \Omega_a = \{1, 2, \ldots, k\} \), and the remainders are allocated to NRTS \( \Omega_b = \{k + 1, \ldots, N\} \). For fixed \( \Omega_a \) and \( \Omega_b \), the power allocation problem is decomposed into two sub-problems: power allocation for RTS and power allocation for NRTS. The power allocation for RTS is solved with Lagrangian method. The power allocation for NRTS is calculated with the proposed two-step method: initial power allocation and power adjustment. The power allocation problem is solved with the proposed method for all possible cases of \( \Omega_a \) and \( \Omega_b \). The maximal value of \( R_b \) is the final solution of the problem. Simulation results show that the performance of the proposed algorithm is close to that of the exhaustive algorithm while the complexity is much less than that of the exhaustive algorithm, and it has much higher spectral efficiency than the fixed algorithm.

6 References


