GAIN SENSITIVITIES OF A MICROWAVE AMPLIFIER WITH RESPECT TO THE MICROSTRIP PARAMETERS

Salih DEMİREL¹ and Filiz GÜNES²

¹,²Electronic and Communication Engineering Department, Yıldız Technical University
Beşiktaş, Istanbul/TURKEY
¹salihd@yildiz.edu.tr, ²gunes@yildiz.edu.tr

Abstract

This paper presents a method for the sensitivity analysis of the gain with respect to the width and length of a microstrip line used in any position of the input/output matching circuit of a microwave amplifier. The method is applied to a low-noise amplifier and the resultant typical sensitivity variations are also given. This work shows which microstrip lines are sensitive with respect width/length changes, therefore extra care should be taken for them during the practical realization stage.

1. Introduction

Sensitivities, or partial derivatives of certain network functions with respect to circuit parameters are very important in the design of microwave circuits [1]. Sensitivity analysis let us estimate the simulation errors; and by using the results of the sensitivity analysis, we can arrange network parameters according to the increasing values of the sensitivity measures. In consequence, the procedure can lead to the identification of circuit parameters with excessive values of the sensitivity or help us to simplify the model by eliminating the elements for which sensitivities are negligible.

In this paper, analytical formulations of gain sensitivities for a low-noise amplifier are obtained with respect to the width and length of any microstrip line used in the matching circuits and is compared to the numerical counterparts. Identification is made among these microstrip elements and the most sensitive microstrip elements are determined with their sensitivity variations with respect to the frequency. Thus, these elements must be realized within the tolerance limits subject to their sensitivities in the practice. Here, in order for a F network function not to be sensitive to circuit parameter “x”, the limits of the normalized sensitivity $S_x(F)$ function should obey the following criteria:

$$0 \leq \left| S_x(F) \right| \leq 1, \quad S_x(F) = \frac{\partial (\ln F)}{\partial (\ln x)} = \frac{x}{F} D_x(F) \quad (1)$$

In the next section, analytical derivation of the gain sensitivities with respect to any microstrip width $W$ and length $\ell$ utilized in the matching circuits will be given, in the third section a worked example will be presented for the low-noise amplifier with the T-type matching circuits in the accompanied paper [2]. Finally, conclusions end the paper.

2. Gain Sensitivities with respect to the microstrip parameters for a microwave amplifier

Sensitivity of the Characteristic impedance with respect to the microstrip width

The characteristic impedance $Z_c(W, T, \varepsilon_r, H)$ and effective dielectric coefficient $\varepsilon_{eff}(W, T, \varepsilon_r, H)$ functions of microstrip lines for the quasi-static case are taken derivative with respect to the width $W$ for a given set ($\varepsilon_r, H, T$) in Fig. 1:

$$Z_c = \frac{60}{\sqrt{\varepsilon_{eff}}} N_1, \quad N_1 \triangleq \ln \left( 8 \frac{h}{W} + 0.25 \frac{W}{h} \right) \quad W/h \leq 1 \quad (2.1)$$

$$\frac{\partial Z_c}{W} = \frac{60}{\sqrt{\varepsilon_{eff}}} \left[ \frac{-8h}{W^2} + \frac{1}{8h} \right] - \frac{30\pi}{N_1 \sqrt{\varepsilon_{eff}}^3} \frac{\partial \varepsilon_{eff}}{\partial W} \quad (2.2)$$

$$Z_c = \frac{120\pi/\sqrt{\varepsilon_{eff}}}{N_2}, \quad N_2 \triangleq \left( \frac{W}{h} + 1.393 + 0.667 \ln(W/h + 1.444) \right) \quad W/h \geq 1 \quad (2.3)$$
\[
\frac{\partial z_c}{\partial W} = \frac{120\pi}{N_z^2} \frac{\epsilon_{\text{eff}}(\frac{1}{N_z^2} + 0.667 \frac{1}{W + 1.444h})}{\epsilon_{\text{eff}}} - \frac{60\pi}{N_z^2} \frac{\partial \epsilon_{\text{eff}}}{\partial W} \tag{2.4}
\]
\[
\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{12H}{W}\right)^{-\frac{1}{2}} + F(\epsilon_r, H, W) - 0.217(\epsilon_r - 1) \frac{T}{\sqrt{W/H}} \tag{2.5}
\]
\[
F(\epsilon_r, H, W) = 0.02(\epsilon_r - 1) \left(1 - \frac{W}{H}\right)^2 \quad W/H \leq 1 \tag{2.6}
\]
\[
F(\epsilon_r, H, W) = 0 \quad W/H > 1 \tag{2.7}
\]
\[
\frac{\partial \epsilon_{\text{eff}}}{\partial W} = \frac{\epsilon_r-1}{4} \left(1 + \frac{12H}{W}\right)^{-\frac{3}{2}} \frac{12H}{W^2} - 0.04(\epsilon_r - 1) \left(1 - \frac{W}{H}\right) \frac{1}{W} + 0.217(\epsilon_r - 1) \frac{T}{2\sqrt{H} \sqrt{W^3}} \quad W/H \leq 1 \tag{2.8}
\]
\[
\frac{\partial \epsilon_{\text{eff}}}{\partial W} = \frac{\epsilon_r-1}{4} \left(1 + \frac{12H}{W}\right)^{-\frac{3}{2}} \frac{12H}{W^2} + 0.217(\epsilon_r - 1) \frac{T}{2\sqrt{H} \sqrt{W^3}} \quad W/H > 1 \tag{2.9}
\]

**Gain Sensitivities with respect to the microstrip width and length:**

In this work, gain sensitivity matrix is obtained using the chain sensitivity matrix method \[1-2\] which can be applied to any network in cascaded configuration. This method calculates the overall chain matrix by dividing the network into the three groups: The two-port with the sensitivity parameters in question, the two-ports at the left-hand and right-hand sides (Fig.2). Thus the overall chain T matrix can be expressed as

\[
T = T_L \cdot T_I \cdot T_R \tag{2.10}
\]

where only \( T_I \) is needed to be taken derivatives with respect to the circuit parameters \( \vec{x} \) which in our case \( \vec{x} = [W_i \ \ell_i]^T \), \( i = 1, \ldots, \kappa \), \( \kappa \) is number of the microstrip line in the design. Thus, we have sensitivity of the overall T chain matrix as follows:

\[
\frac{\partial T}{\partial x_n} = T_L \frac{\partial T_I}{\partial x_n} \cdot T_R \tag{2.11}
\]

Gain of the overall network can be expressed in terms of the chain parameters as

\[
G_T = \frac{4R_sR_l}{P}, \quad P = \left|\Delta Z_L + B + Z_s(CZ_L + D)^2 \right| \tag{2.12}
\]

Where the gain sensitivity with respect to the \( x_n \) can be shown to be equal to \[3\]

\[
\frac{\partial G_T}{\partial x_n} = \sum_{i=1}^{\Delta} \frac{\partial G_T}{\partial T_i} \frac{\partial T_i}{\partial x_n} + \sum_{i=1}^{\Delta} \frac{\partial G_T}{\partial T_i} \frac{\partial T_i}{\partial x_n} \tag{2.13}
\]

In the next section gain sensitivities are obtained with respect to the width and length of microstriplines in the T-configurations at both input and output ports (Fig.3).

**3. Worked Examples**

In the worked examples, NE3511S02 is chosen as a high technology low-noise transistor to be used in the front-end amplifier and the synthesized gain and noise figure performance variations with respect to the frequency at the bias condition \( V_{DS}=2V, I_{DS}=5mA \) are given in the accompanied paper. Typical sensitivity variations with frequency are given in Figure 4 for the microstrip line at the input and output matching circuits. In these Figures, furthermore
another method called “Gain Factorization” is also employed from [3] for the purpose of comparison. Besides, numerical derivatives are applied too.

5. Conclusion

In this work, a method is proposed that is directly applicable to sensitivities of gain with respect to the width and length of a microstrip line in any position in a low-noise amplifier. Results show us that gain performance is very sensitive to width / or length of some microstrip lines in the circuit. Thus, particularly, these microstrip lines should be cared during the practical realization stage.

Figure.1 Microstripline used in the front-end low-noise amplifier

Figure.2 Block Diagram of the m Cascaded Two-Ports

Figure.3 Low-Noise Amplifier with T- type Microstripline Matching Networks at both Input and Output Ports
TABLE – I  Solution Space for the (T - T) IMC & OMC Elements

<table>
<thead>
<tr>
<th>W_1(mm)</th>
<th>W_2(mm)</th>
<th>W_3(mm)</th>
<th>W_4(mm)</th>
<th>ℓ_1(mm)</th>
<th>ℓ_2(mm)</th>
<th>ℓ_3(mm)</th>
<th>ℓ_4(mm)</th>
<th>ℓ_5(mm)</th>
<th>ℓ_6(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.15</td>
<td>2.36</td>
<td>4.99</td>
<td>1.65</td>
<td>0.78</td>
<td>1.72</td>
<td>4.23</td>
<td>2.98</td>
<td>0.51</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Figure 4 Typical Sensitivity variations with frequency w.r.t length and width for the microstrip lines numbered 1 and 4 at input and output ports, respectively.

6. References

