Electromagnetic Scattering from Conductors: the Recursive Iterative Surface Equivalence Approach

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Abstract

A recursive iterative surface equivalence technique is proposed to solve electromagnetic scattering from arbitrary conducting bodies. The technique has advantages over existing methods based on integral equations, e.g. the method of moments (MoM), as well as methods based on finite differences. These advantages are particularly apparent for large scatterers, as the technique is inspired by the scattering from an infinite conducting plane. Also, trivial changes to MoM codes are needed to implement the technique. The generalization of the one-dimensional treatment is verified with two-dimensional problems.

1 Introduction

Accelerating and minimizing memory requirements of electromagnetic (EM) scattering problem solvers is still an area of great interest to the community. Solvers based on the method of moments (MoM) have memory requirement problems, require singularity extraction, and rely on computationally expensive matrix inversion [1]. On the other hand, solvers based on finite difference approximations have the problems of large numbers of unknowns and require domain termination (absorbing boundaries) [2, 3].

In this work, a recursive iterative surface equivalence approach is presented with the goal of simplifying the solution of the EM scattering problem by starting with a simple initial guess for the equivalent surface currents and iterating to their final values. The technique is inspired by the simple problem of oblique incidence on an infinite ground plane, and its first conception was verified for one canonical problem in [4]. Here it is verified with arbitrary 2D problems and its relation to existing MoM formulations is stated.

In the next section, the inspiration for the proposed method is explained to be the scattering from an infinite ground plane. In Section 3, the general procedure of the Recursive Iterative Surface Equivalence (RISE) approach is presented. Section 4 provides the treatment of the 2D problem of scattering from a conducting cylinder, while Section 5 is where the results are presented. The paper is concluded in Section 6.

2 One Dimensional Scattering from Perfectly Conducting Ground

If a uniform plane wave is incident on an infinite conducting ground plane (Fig. 1a), it is completely reflected and an electric current is induced on the plane surface $\mathbf{J} = \hat{\mathbf{n}} \times \mathbf{H}^t$. The surface equivalence principle ensures that the fields scattered by $\mathbf{J}$ after the ground is removed, and the incident fields, constitute the same total fields of the original problem everywhere (Fig. 1b). Now if the current was not known, choosing:

$$\left.\mathbf{J}^i = \hat{\mathbf{n}} \times \mathbf{H}^i, \quad \mathbf{M}^i = -\hat{\mathbf{n}} \times \mathbf{E}^i \;\right\} \quad \text{on } S, \tag{1}$$

one can be sure that these currents will annihilate the incident field in the region below the ground in the original problem. However, these currents together scatter zero fields in the region above the ground where the illumination originates. By direct inspection, a good guess at the equivalent current on the ground surface is (Fig. 1c and Fig. 1d):

$$\tilde{\mathbf{J}} \leftarrow \mathbf{J}^i + \frac{1}{Z_w}\hat{\mathbf{n}} \times \mathbf{M}^i \tag{2}$$

with Z_w being the wave impedance. In fact, $\tilde{\mathbf{J}}$ is equal to $\mathbf{J}$; the sought electric current of Fig. 1b.

To mathematically illustrate the mentioned procedure for reaching the induced current, the following operators (and their duals for $\mathbf{M}(\mathbf{r})$) can be used:

$$(\mathbf{E}_\nu^J \circ \mathbf{J})(\mathbf{r}) = -j\omega\mu_\nu \int_V \mathbf{J}(\mathbf{r}')g_\nu(\mathbf{r},\mathbf{r}')dv' - \frac{j}{\omega\varepsilon_\nu}\nabla \int_V \nabla' \cdot \mathbf{J}(\mathbf{r}')g_\nu(\mathbf{r},\mathbf{r}')dv',$$
$$(\mathbf{H}_\nu^J \circ \mathbf{J})(\mathbf{r}) = \nabla \times \int_V \mathbf{J}(\mathbf{r}')g_\nu(\mathbf{r},\mathbf{r}')dv', \tag{3}$$

with $g_\nu(\mathbf{r},\mathbf{r}')$ being the Green's function of an unbounded medium of permittivity and permeability ε_ν and μ_ν, respectively.

If a conducting plane is at $z = 0$ and the plane wave is incident from the $z < 0$ half space, with perpendicular (TM) polarization, and at an incidence angle θ_i with the z-axis, the incident fields are $\mathbf{E}^i(\mathbf{r}) = \hat{\mathbf{y}}E_0 e^{-jk_0(x\sin\theta_i + z\cos\theta_i)}$, and $\mathbf{H}^i(\mathbf{r}) = \frac{-E_0}{\eta_0}e^{-jk_0(x\sin\theta_i+z\cos\theta_i)}\left(\hat{\mathbf{x}}\cos\theta_i + \hat{\mathbf{z}}\sin\theta_i\right)$. As a result of this excitation, a current is induced on the ground surface $\mathbf{J} = \hat{\mathbf{y}}\frac{2E_0}{\eta_0}\cos\theta_i e^{-jk_0 x\sin\theta_i}$. The field scattered by this current can be found using the operators of (3):

$$\mathbf{E}^s(\mathbf{r}) = (\mathbf{E}_0^J \circ \mathbf{J})(\mathbf{r}) = -j\omega\mu_0 \frac{2E_0}{\eta_0}\cos\theta_i e^{-jk_0 x\sin\theta_i}\frac{e^{-jk_0\cos\theta_i|z|}}{2jk_0\cos\theta_i}\hat{\mathbf{y}}. \tag{4}$$

This scattered field is seen to annihilate the incident field in the region $z > 0$, as well as to produce the well known standing wave pattern of this problem in the region $z < 0$. The currents $\mathbf{J}^i$ and $\mathbf{M}^i$ of (1) scatter the field:

$$\mathbf{E}^s(\mathbf{r}) = (\mathbf{E}_0^J \circ \mathbf{J}^i)(\mathbf{r}) + (\mathbf{E}_0^M \circ \mathbf{M}^i)(\mathbf{r}) = -j\omega\mu_0\frac{E_0}{\eta_0}\cos\theta_i\frac{e^{-jk_0\cos\theta_i|z|}}{2jk_0\cos\theta_i}\hat{\mathbf{x}} - jk_0\cos\theta_i E_0 \mathrm{sgn}(z)\frac{e^{-jk_0\cos\theta_i|z|}}{2jk_0\cos\theta_i}\hat{\mathbf{x}}, \tag{5}$$

which can be seen to annihilate the incident field in the region originally occupied by the ground $z > 0$, and to equal zero in the other half space $z < 0$. Now if the electric current is updated as in (2), a current $\tilde{\mathbf{J}}$ is obtained, which scatters the same fields as $\mathbf{J}$. *One iteration* was needed to reach the final value of the induced surface current. The same procedure for finding the current can be applied to the TE polarization.

3 The RISE Procedure

The proposed RISE technique seeks to solve the problem of scattering from an arbitrarily shaped conductor (Fig. 2a) by requiring that the equivalent surface current satisfy the condition of *annihilating the fields*

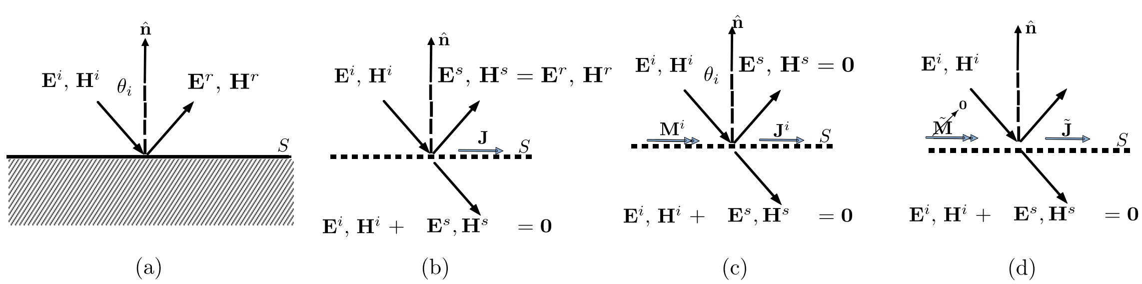


Figure 1: Infinite conducting ground excited by an obliquely incident plane wave: (a) original problem and (b) equivalent problem, and the RISE procedure applied to 1.5D problem: (c) initial guess of surface currents and (d) currents after first iteration.

within the conducting scatterer iteratively. Surface equivalence and uniqueness guarantee that if a current distribution satisfies this condition, then it radiates the same scattered fields of the original problem in the region outside the scatterer. The procedure followed is shown in Fig. 2b, with the initial guess being that

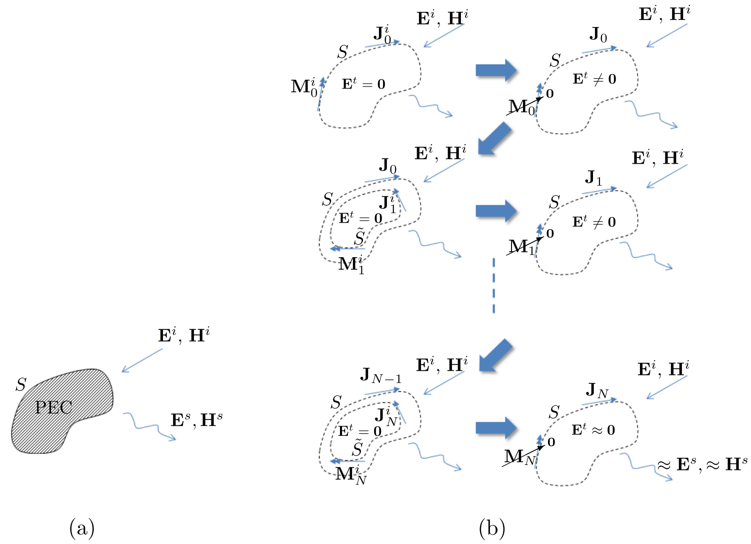


Figure 2: (a) A conducting body under arbitrary illumination, (b) The RISE procedure applied to a conducting body of arbitrary shape.

of (1). The electric current $\mathbf{J}$ is updated as in (2), and the magnetic current $\mathbf{M}$ is zeroed. In the next iteration, new electric and magnetic currents are introduced on the fictitious surface $\tilde{S}$. $\tilde{S}$ is then allowed to expand so $S \leftarrow \tilde{S}$, the currents on $\tilde{S}$ are added to that on S, and $\mathbf{J}$ is updated as in (2), while $\mathbf{M}$ is zeroed. Iterations continue until the surface current converges to its final value. The nth iteration can be mathematically stated as:

$$\left.\begin{array}{l}\mathbf{J}_n^i = \hat{\mathbf{n}} \times (\mathbf{H}^i + (\mathbf{H}_0^J \circ \mathbf{J}_{n-1})) \\ \mathbf{M}_n^i = -\hat{\mathbf{n}} \times (\mathbf{E}^i + (\mathbf{E}_0^J \circ \mathbf{J}_{n-1}))\end{array}\right\} \text{ on } \tilde{S}$$
$$\left.\begin{array}{l}\mathbf{J}_n \leftarrow \mathbf{J}_{n-1} + \mathbf{J}_n^i + \frac{1}{Z_w}\hat{\mathbf{n}} \times \mathbf{M}_n^i \\ \mathbf{M}_n \leftarrow 0\end{array}\right\} \text{ on } S \leftarrow \tilde{S} \tag{6}$$

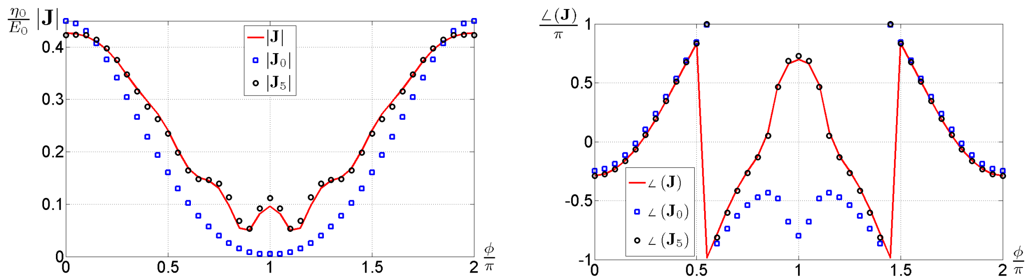
where $n = 0, 1, 2, ..., N$, $\mathbf{J}_{-1}^i = \mathbf{M}_{-1}^i = \mathbf{0}$, and the Nth iteration satisfies the criterion $\frac{\|Z_w \mathbf{J}_N^i + \hat{\mathbf{n}} \times \mathbf{M}_N^i\|}{E_0} \leq \mathcal{E}$.

4 Two-Dimensional Scattering from Infinite Conducting Cylinders

The problems of scattering from an infinite conducting circular cylinder of radius a under different electromagnetic wave excitations, and of scattering from an infinite cylinder of arbitrary, uniform cross section will now be solved using the RISE procedure. It is clear from (6) that RISE requires the calculation of the tangential $\mathbf{E}$ and $\mathbf{H}$ on S due to $\mathbf{J}$ on S. This means that the RISE solution uses *the same operator formulation* as that used by the MoM, particularly the MoM for solving the combined field integral equation.

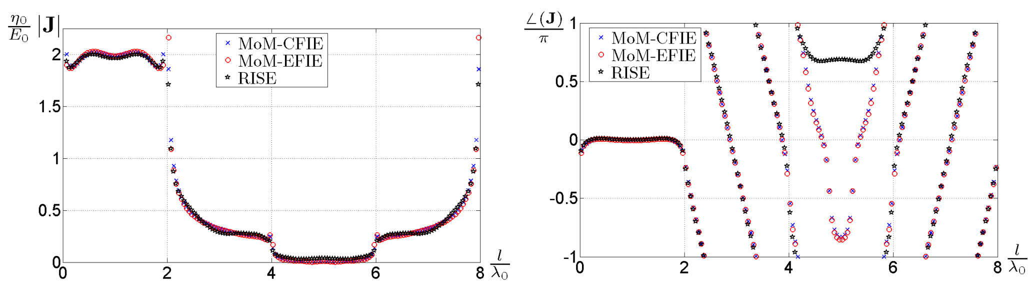
5 Results and Discussion

A cylinder with $a = \frac{\lambda 0}{2}$ was used for various excitation cases. It can be seen that the RISE solution converges to the direct solution for incident cylindrical waves from a magnetic line source (Fig. 3). The value of the error $\mathcal{E}$ was 4%. Similar results were obtained with $\mathcal{E}$ of 3% for TE plane waves, and 5% for TM cylindrical waves. The RISE and MoM solutions for the induced surface current on a $2\lambda_0 \times 2\lambda_0$ square conducting cylinder excited by a plane TM wave are shown to be in excellent agreement in Fig. 4. The phase disagreement for normalized arc length $\frac{l}{\lambda_0}$ between 4 and 6 is unimportant because of vanishing magnitude.



(a) (b)

Figure 3: The current induced on a $\lambda_0/2$ PEC cylinder excited by a magnetic line source located at $\rho_s = 5a$, $\varphi_s = 0$, and the RISE current at $n = 0, 5$: (a) magnitude and (b) phase.



(a) (b)

Figure 4: The MoM solution for the current induced on a $2\lambda_0 \times 2\lambda_0$ PEC square cylinder excited by a TM plane wave, and the RISE current at $n = 15$: (a) magnitude and (b) phase.

6 Conclusion

A new technique for solving electromagnetic scattering problems from conducting bodies was introduced. The technique was shown to have the main advantages of becoming less expensive as the electrical size of the scatterer increases, and of easy implementation through MoM code adaptation.

References

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