Modeling of the lower atmospheric electric field due to thundercloud

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Abstract

Electric field generation in the lower atmosphere by thunderclouds with a suitable charge distribution profile has been modeled. The responses of the atmosphere are presented through Maxwell's equations together with a time-varying source charge distribution. Different conductivities of the medium are taken as exponentially graded function of altitude. The expression of electric potential has been deduced from which the variation of electric field at different heights is numerically analyzed.

1. Introduction

The distributions of electric charge in the electrified clouds introduce important effects in the ionosphere and into the region between the ionosphere and the Earth. Various models are there to examine electrical coupling between the earth's upper and lower atmospheric region. The main sources of electric current in these regions are the thunderstorms which are distributed statistically in geographic areas [1, 2, 3, 4].

A model for the penetration of DC thundercloud electric field at these regions has been presented. The model deals with the electromagnetic responses of the atmosphere which are simulated through the Maxwell's equations together with a time-varying source charge distribution. For the charge distribution of the electrified cloud, the modified ellipsoidal-Gaussian profile has been taken [4]. The conductivity profile of the medium is taken isotropic below 70 Km height and anisotropic above 70 Km [4].

The Earth's surface is considered to be perfectly conducting. A general form of equation representing the thundercloud electric field component is deduced. In spite of assumptions for axial symmetry of thundercloud charge distribution considered in the model, the results are obtained giving the electric field variation in the upper atmosphere.

The vertical component of the electric field is related to the global electric circuit while the radial component would show the electrical coupling between the lower atmosphere and the ionized Earth's environment.

2. Mathematical formulations

The transmission of electric field during thunderclouds has been investigated through the following equations:

$$\nabla^2\Phi = \frac{-\rho_T}{\varepsilon_0}(r,z) \text{ at } 0 \le z \le z_E \tag{1}$$

$$\nabla.\bar{j} = 0 \text{ at } 0 \le z \le z_L \tag{2}$$

$$\bar{j} = (\sigma_0 - \sigma_1)(\bar{E}.\hat{e}_0)\hat{e}_o + \sigma_1\bar{E} - \sigma_2\bar{E}\times\hat{e}_0 + \bar{j}_s \text{ at } z_L \le z \le z_E \tag{3}$$

$$\bar{j}_s = \frac{Q(t)}{2\pi r}\delta(r)\xi(z - z_e)\hat{e}_s \tag{4}$$

Φ is the electric potential of the electric field of the electrified cloud; $\hat{e}_0$, the unit vector in the direction of the external magnetic field; $\bar{j}_s$, the source current density assumed to generate a centered charge at the height z_c in the cloud; σ_0, σ_1, σ_2, the longitudinal, Pedersen and Hall conductivities; ξ and δ the unit Heaviside and Dirac function; Q(t), the source function which gives the total current injected at time *t*; $\hat{e}_s$, the vertical unit vector; $z = 0$, taken to

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be surface level; z, the axis of symmetry of the charge density distribution; z_E, the height between 100 Km to 120 Km; z_L and z_U are the lower and upper boundaries of the electrified cloud. The electric conductivity is isotropic in the atmospheric region $(0, z_L)$. z_L = 70 Km and anisotropic above $z = z_I$. Schematic heights are shown in Figure 1. z_B is above the E-region of the ionosphere. Above 70 Km height, the conductivity is tensorial in nature.

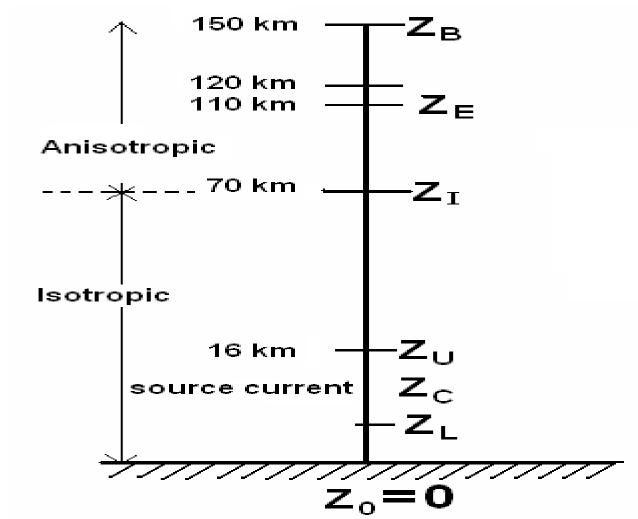

Figure 1. Schematic view of the upper atmosphere with height

For 110 < z_E < 125 Km, the longitudinal conductivity σ_0 becomes very large compared to transverse conductivities. The charge is assumed to have an ellipsoidal Gaussian distribution given by (4). Thus,

$$\rho_T = \rho_{model} + \rho_S$$

Where
$$\rho_{model} = \gamma \exp[\frac{r^2}{\alpha^2} - \frac{(z - z_0)}{\beta^2}], \quad z_L \le z \le z_U$$

and
$$\rho_S = \frac{Q(t)}{2\pi r}\delta(r)\xi(z - z_0), \quad 0 \le z \le z_U$$

$$z_0 = (z_L + z_U)/2.$$

γ is the normalization constant. α^2 and β^2 are the vertical dispersions of the charge density. The charge is higher for small α^2 and β^2. The maximum density is reached at $r = 0, z = z_0$.

From (2), in the region $0 < z < z_L$ and $z_U < z < z_E$,

$$\nabla(\sigma_1).\nabla(\sigma_2) + \sigma_1\nabla^2\Phi + \nabla(\sigma_2)(\nabla\Phi \times \hat{e}_0) + \sigma_2\nabla.(\nabla\Phi \times \hat{e}_0) + \nabla.j_s = 0 \qquad (5)$$

Now, $\hat{e}_0 = \alpha(r,z)\hat{e}_r + \beta(r,z)\hat{e}_z$ and let $\nabla.\bar{j}_s = f(z)$,

The eq (5) yields,

$$\begin{aligned}
&[\sigma_1 + \alpha^2(\sigma_0 - \sigma_1)]\frac{\partial^2\Phi}{\partial r^2} + 2\alpha\beta(\sigma_0 - \sigma_1)\frac{\partial^2\Phi}{\partial r\partial z} + [\sigma_1 + \beta^2(\sigma_0 - \sigma_1)]\frac{\partial^2\Phi}{\partial z^2} + \\
&+ [\frac{\sigma_1}{r} + \alpha\beta\frac{\partial}{\partial z}(\sigma_0 - \sigma_1)] + (\sigma_0 - \sigma_1)[\frac{\alpha^2}{r} + 2\alpha\frac{\partial\alpha}{\partial r} + \beta\frac{\partial\alpha}{\partial z} + \alpha\frac{\partial\beta}{\partial z}]\frac{\partial\Phi}{\partial r} + \\
&+ [\frac{\partial\sigma_1}{\partial z} + \beta^2\frac{\partial}{\partial z}(\sigma_0 - \sigma_1) + (\sigma_0 - \sigma_1)\frac{\alpha\beta}{r} + 2\beta\frac{\partial\beta}{\partial z} + \alpha\frac{\partial\beta}{\partial r} + \beta\frac{\partial\alpha}{\partial r}]\frac{\partial\Phi}{\partial z} + f(r,z) = 0
\end{aligned} \qquad (6)$$

For isotropic region, $0 < z < z_I$, $\qquad \sigma_0 = \sigma_1 = C_1 \exp(\frac{z}{h})$

The eq (6) yields,
$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial\Phi}{\partial r}+\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{h}\frac{\partial\Phi}{\partial z}+f(z,r)=0 \tag{7}$$

For the anisotropic region with $\sigma_0 = C_A \exp(\frac{z}{h_0})$ and $\sigma_1 = C_A \exp(\frac{z}{h_1})$ one can deduce

$$\begin{aligned}&\frac{\partial^2\Phi}{\partial r^2}-\frac{r}{a}[1-\exp(\frac{z}{l})]\frac{\partial^2\Phi}{\partial r\partial z}+\exp(\frac{z}{l})\frac{\partial^2\Phi}{\partial z^2}+[\frac{1}{r}-\frac{r}{2a}\{\frac{1}{h_1}-\frac{1}{h_0}\exp(\frac{z}{l})\}\frac{\partial^2\Phi}{\partial z^2}+\\&[\frac{1}{r}-\frac{r}{2a}\{\frac{1}{h_1}-\frac{1}{h_0}\exp(\frac{z}{l})\}]\frac{\partial\Phi}{\partial r}+[(\frac{1}{h_0}+\frac{r}{a})\exp(\frac{z}{l})-\frac{r}{a}]\frac{\partial\Phi}{\partial z}+g(r,z)=0\end{aligned} \tag{8}$$

3. Solutions

For isotropic region, $\Phi(r,z)=\int_0^\infty J_0(rk)[a_i(k)\exp(c_1 z+b_i(k)\exp(c_2 z)]dk$

For anisotropic region,

$$\Phi(r,z)=[f(z)]^n\int_0^\infty\left[J_0(rk)\{A(k)I_n[kf(z)]+B(k)K_m[kf(z)]\}+[(r^2+\frac{k^2}{r_k^2})J_0\{kf(z)\}-k^2]\right]dk$$

For the thundercloud region,

$$\Phi(r,z)=\int_0^\infty J_0(kr)[C(k)\exp(-kz)+D(k)\exp(+kz)+\upsilon(z,k)]dk$$

Hence electric field around the electrified cloud would yield,

$$E_r=[f(z)]^n\int_0^\infty kJ_1(rk)\{A(k)I_n[kf(z)]+B(k)K_n[kf(z)]\}dk$$

$$\begin{aligned}E_z &= n(z-z_l)\exp(-\frac{z-z_l}{g})[f(z)]^{n-1}\int_0^\infty J_0(rk)\{A(k)I_n[kf(z)]+B(k)K_n[kf(z)]\}dk+\\&[f(z)]^n\int_0^\infty J_0(rk)\{A9k)kf'(z)I_{n-1}[kf(z)]+B(k)kf'K_{n-1}[kf(z)]\}dk\end{aligned}$$

4. Results

The variations of electric fields at latitude 23° N are plotted in Figure 2 for different heights with their source positions. The vertical component of electric field is related to global electric circuit whereas the radial component is related to the electric field during coupling between the lower atmosphere and the ionized Earth's environment. Different current parameters are given as: Maxwell current [5],

$$\bar{J}_M=\bar{J}_E+\bar{J}_C+\bar{J}_L+\bar{J}_P+\frac{\partial\bar{D}}{\partial t}$$

$$J_M=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\sigma E_z+\varepsilon\frac{\partial E_z}{\partial t})\partial y\partial z$$

Plot of J_M vs time for different heights and for various source centres are depicted in Figure 3.

During sudden removal of thundercloud charges at low latitude by lightning discharge, the quasi-electromagnetic fields are supposed to heat up the mesospheric electrons producing ionization and light, which are consistent with the observed features of red sprite type of discharges.

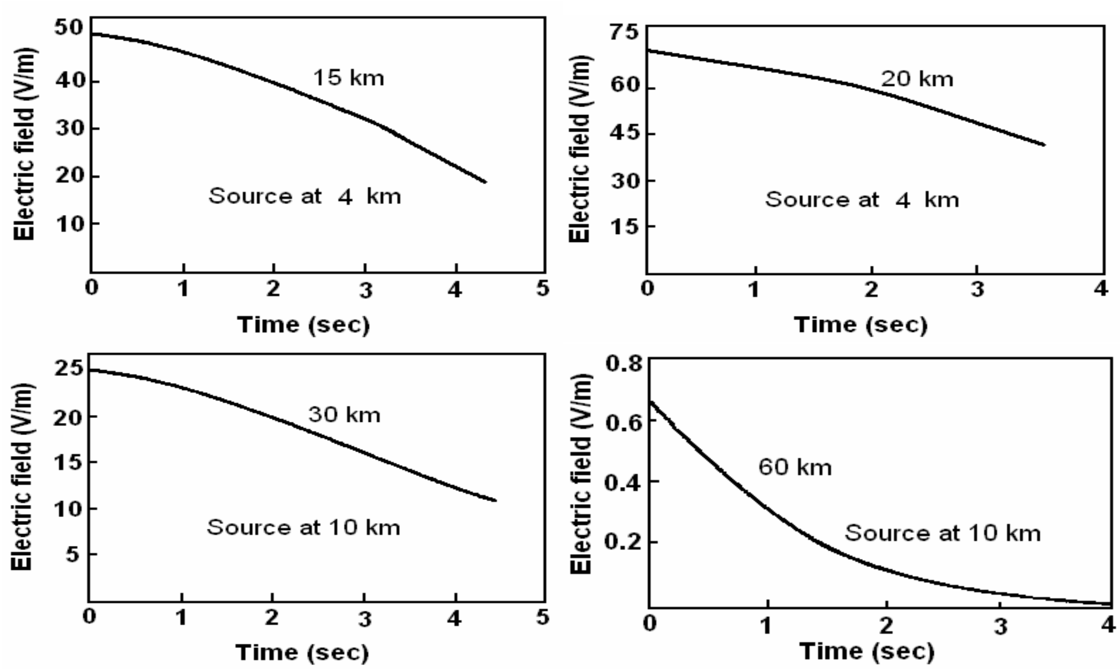

Figure 2. Variation of Electric field at different heights for 23° N Latitude (Kolkata)

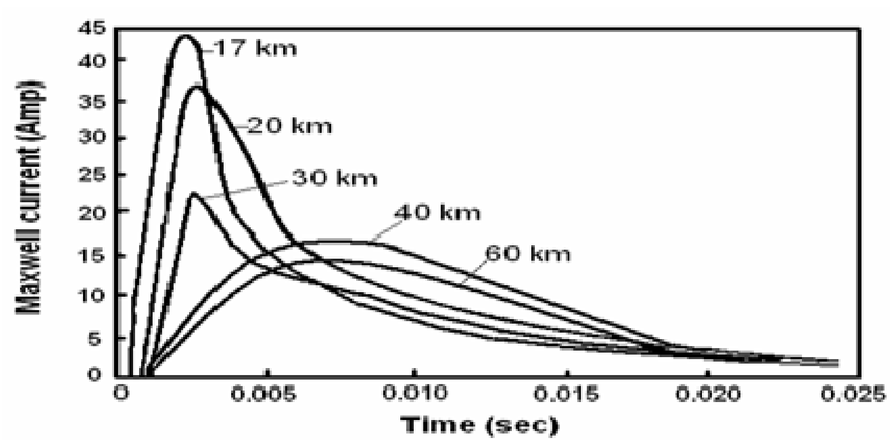

Figure 3. Maxwell current Vs time for different heights

5. References

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