A Multi-objective approach to Subarrayed Linear Antenna Arrays Design

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Abstract

In this paper we present a multi-objective optimization approach to subarrayed linear antenna arrays design. We define this problem as a bi-objective one. We consider two objective functions for directivity maximization and sidelobe level minimization. Two popular Multi-Objective Evolutionary Algorithms (MOEAs), the Generalized Differential Evolution (GDE3) and the Nondominated Sorting Genetic Algorithm-II (NSGA-II), are employed in this study. GDE3 and NSGA-II are applied to the synthesis of uniform and nonuniform subarrayed linear arrays, providing an extensive set of solutions for each design case. Depending on the desired array characteristics, the designer can select the most suitable solution. The results of the proposed method are compared with those reported in the literature, indicating the advantages and applicability of the multi-objective approach.

1. Introduction

In large antenna arrays, cost and complexity can be reduced by exciting a group of radiating elements with a single source, instead of exciting each element individually [1]. However, a main disadvantage of such a structure is the appearance of undesired secondary lobes. There are many approaches in the literature that address the problem of Sidelobe Level (SLL) reduction. Overlapped subarrays have been considered in [2] and subarray amplitude tapering has been proposed in [3]. In [4] a hybrid Genetic Algorithm (GA) has been used for subarray weight and size optimization. In [5] an excitation matching approach with SLL control has been applied to both linear and planar arrays design. An iterative technique has been employed in [6], in order to find the optimum element phases for SLL reduction. The main objective of the above mentioned works has been the SLL minimization. However, another important characteristic in antenna arrays synthesis is directivity. In order to reach an optimal design, a suitable trade-off between these characteristics has to be achieved.

In this work, the issue of uniform and nonuniform subarrayed linear arrays design is defined as a multi-objective one. Namely, two objective functions are considered, one for directivity maximization and the other for SLL minimization. To the best of the authors' knowledge, this is the first time that a paper addresses the specific problem with a multi-objective perspective. The paper is organized as follows: The problem formulation is described in Section II. In Section III the numerical results are presented and compared with respective ones reported in the literature. Finally, the conclusion is given in Section IV.

2. Formulation

We consider an *N*-element linear array spaced along the x-axis. The feed network is made of *Q* contiguous subarrays with real weights. The array factor is given by [5]:

$$AF(\theta) = \sum_{q=1}^{Q} w_q \sum_{n=1}^{N} \delta_{c_n q} I_n e^{j\frac{2\pi}{\lambda} x_n \sin\theta} \tag{1}$$

where λ is the wavelength, w_q is the weight of the q^{th} subarray, x_n and I_n stand for the position and the excitation of the n^{th} element respectively. $\overline{C} = \{c_n; n = 1,...N\}$ is an integer vector, where the element $c_n \in [1,Q]$ indicates the subarray membership and $\delta_{c_n q}$ is the delta Kronecker function given by: $\delta_{c_n q} = 1,\ c_n = q$ and $\delta_{c_n q} = 0, c_n \neq q$. If we assume that the elements are uniformly excited and no phase shifting is applied to the n^{th} element, then $I_n = 1, n = 1, 2, \ldots, N$. In the case of uniform subarraying (i.e. the subarray sizes are equal): $N = Q \times N_q$, where N_q is the number of elements in each subarray. When nonunifom subarraying (i.e. the subarray sizes are not equal) is considered: $N=\sum_{i=1}^{Q} N_q(i)$ [4], where $N_q(i)$ is the number of elements of the i^{th} subarray.

The subarray design problem is that of finding the optimum array characteristics (weight values, subarray sizes, element phases), so that the SLL is minimized and the directivity is maximized. Such a problem is inherently multi-objective and it can be defined by the minimization of the objective functions given below:

$$F_1(\overline{x}) = \max_{\theta\in S}\{AF_{dB}^{\overline{x}}(\theta)\},\quad F_2(\overline{x}) = -10\cdot\log D(\overline{x})\quad \text{subject to}\quad g_1(\overline{x}) = SLL_{dB}(\overline{x}) \le SLL_L \tag{2}$$

where $\overline{x}$ is the vector of the unknown array characteristics, S is the space spanned by the angle θ excluding the mainlobe, D is the array directivity and SLL_L is the maximum allowable sidelobe level in dB.

The above-mentioned problem is solved using Pareto optimization, which means the optimization of all the objectives simultaneously giving them equal importance. If for a given solution, none of the objective function values can be further improved without impairing the value of at least one objective, then this solution belongs to the set of non-dominated solutions called Pareto front. From these Pareto optimal solutions, optimal array designs that provide a suitable compromise between the objectives for the desired constraints can be realized. The Pareto front results from a Multi-Objective Evolutionary Algorithm (MOEA), like the a) Generalized Differential Evolution (GDE3) [7] and b) Nondominated Sorting Genetic Algorithm-II (NSGA-II) [8], which are used in this work. Differential evolution (DE) is a population-based stochastic global optimization algorithm [9]. GDE3 is a multi-objective DE algorithm that has outperformed other MOEAs for a given set of numerical problems [7, 10]. NSGA-II is a popular and efficient multi-objective GA, which has been used in several engineering design problems [11].

3. Numerical Results

It is assumed that the beamforming network is symmetric with respect to the physical center of the antenna. Furthermore the weights of the two subarrays located near the axis origin are set equal to 1. Due to this array symmetry, the number of unknowns to be found for uniform subarraying is $\frac{Q}{2}-1$, whereas for non uniform subarraying is Q-2. For all design cases we consider a linear array with N=128 elements, as in literature [4-6]. The inter-element spacing is $\lambda/2$. The algorithms are executed 20 times and the best results are compared. Both algorithms are set with a population size of 40 and run for 2000 iterations. The control parameters chosen for GDE3 are according to [7] F=0.5, CR=0.1, where F is the mutation control parameter and CR the crossover constant. For NSGA-II [8] we have set the crossover and mutation probabilities equal to 0.9 and 0.1, respectively.

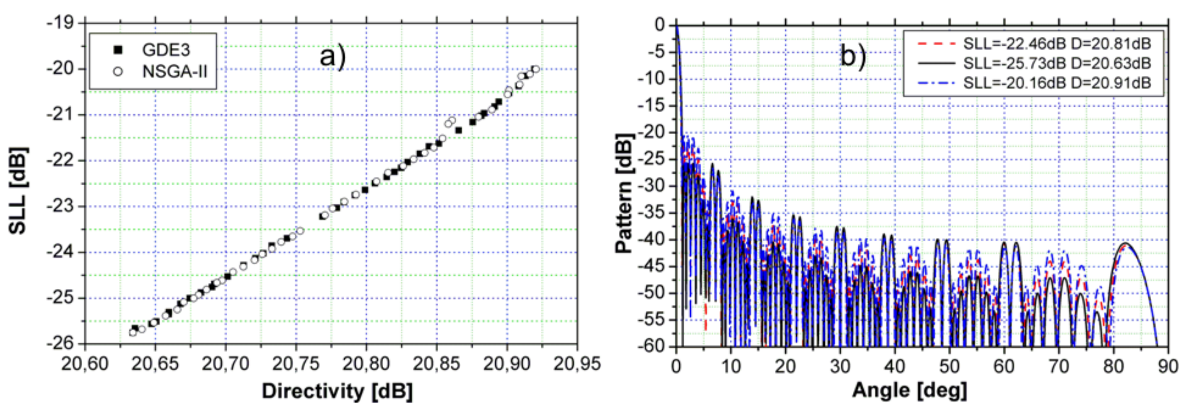

Fig 1. Uniform subarraying (N=128, Q=8). a) Pareto fronts found by GDE3 and NSGA-II algorithms, b) Optimized power patterns found by the GDE3 algorithm.

In the first example uniform subarraying with Q=8 is considered. Thus, each subarray consists of 16 elements. We set $SLL_L = -20dB$. The Pareto fronts for this case found by GDE3 and NSGA-II are shown in Fig. 1a. Each point of the Pareto front denotes a feasible design solution. Both algorithms perform similarly. Three example patterns produced by GDE3 are plotted in Fig.1b. Table I has the comparative results with the uniform array and an array with Taylor tapering ($SLL = -30dB, \bar{n} = 4$) [6]. The first design case has a high directivity value of 20.81dB and a relatively low SLL of -22.46dB. The second case has the lower SLL value than all and it has a directivity value of 20.63dB. It therefore outperforms the case from [6] in terms of both SLL and directivity. The subarray weights for this case are: w_1=0.411, w_2=0.576 and w_3=0.854. Case 3 presents the higher directivity value than all, which is very close to that of a uniform array. However, the high directivity value is at expense of the sidelobe level.

TABLE I
COMPARATIVE RESULTS

UNIFORM SUBARRAYING (N=128, Q=8)			NONUNIFORM SUBARRAYING (N=128, Q=16, N_{MIN}=4)		
Design case	SLL (dB)	Directivity (dB)	Design case	SLL (dB)	Directivity (dB)
Uniform	-13.27	21.07	Hybrid GA [4]	-35.90	19.89
Taylor [6]	-25.06	20.32	Hybrid CPM [5]	-36.90	19.69
GDE3 Case 1	-22.46	20.81	CPM [5]	-36.20	19.56
GDE3 Case 2	**-25.73**	**20.63**	**GDE3 Case1**	**-37.23**	**19.95**
GDE3 Case 3	-20.16	20.91	GDE3 Case2	-37.11	19.74
			GDE2 Case3	-35.98	20.08

The second example concerns non uniform subarraying, where both subarray size and weights are optimized. We select Q=16 and set $SLL_L = -30dB$, N_{min}=4 [4, 5]. The GED3 and NSGA-II Pareto fronts are shown in Fig. 2a. GDE3 performs slightly better. Figure 2b shows the radiation pattern plots for three cases that come from GDE3. Table I holds the performance indexes of the cases found by GDE3 and those from the literature. The first case outperforms the results cited in terms of both SLL and directivity. Case 2 has a SLL lower than -37dB and a relatively high directivity. It has outperformed the results from [5] found by CPM and hybrid CPM. The third case has the highest directivity value than all other results. The tradeoff for this case is that SLL is lower than the other cases, close to -36dB (-35.98db). Fig. 3 depicts the subarrays weights and the number of elements per subarray for each GDE3 case.

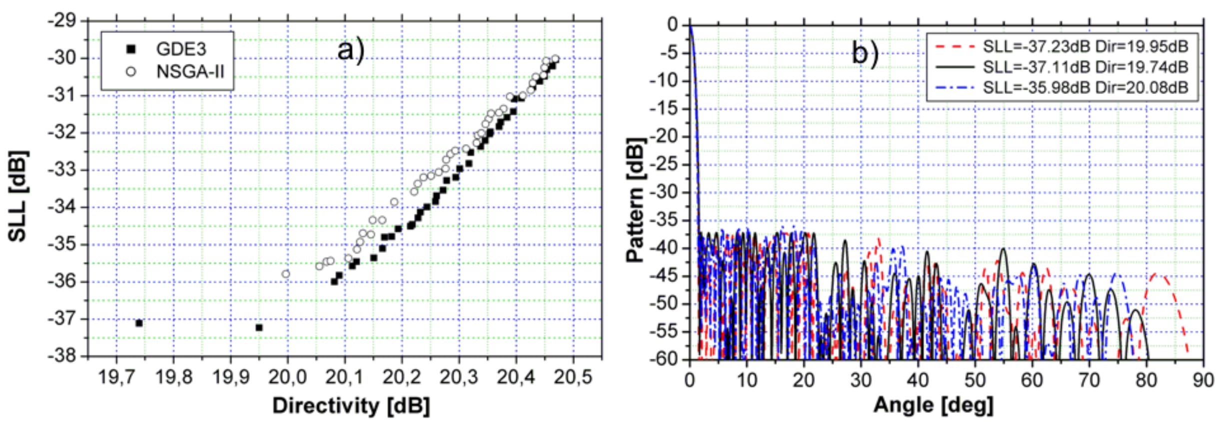

Fig. 2 Nonuniform subarraying (N=128, Q=16, N_{min}=4). a) Pareto fronts found by GDE3 and NSGA-II algorithms, b) Optimized power patterns found by the GDE3 algorithm.

4. Conclusions

The multi-objective algorithms GDE3 and NSGA-II have been employed in the synthesis of uniform and nonuniform subarrayed linear arrays. Sidelobe level and directivity have been simultaneously optimized, providing a set of non-dominated solutions (Pareto front) that satisfy specific design constraints; from the Pareto front the antenna array designer can select the best solution that fits a certain design case or application. Numerical results show that, besides the possibility of selection from a set of optimal solutions, the proposed multi-objective approach can give subarrayed arrays with better characteristics than those reported in the literature. GDE3 produces similar or slightly better results than NGSA-II, for the same population size and number of generations.

In our future work we intend to examine the design of subarrayed linear arrays, taking also into account phase control and beam steering. Additionally, possible modifications of the GDE3 algortithm will be examined, in order to produce arrays with even better characteristics.

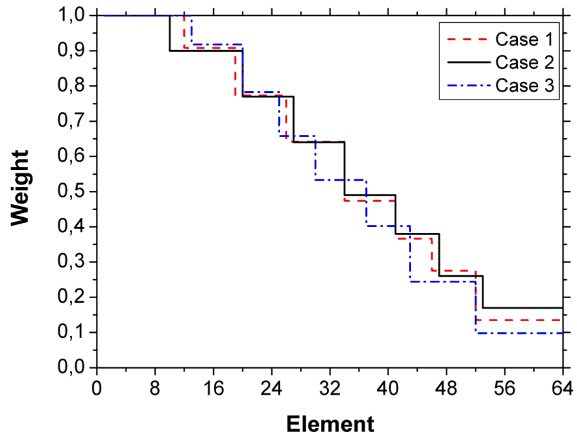

Fig. 3 Nonuniform subarraying (N=128, Q=16, N_{min}=4). Number of elements per subarray and weights plot.

5. References

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