Properties of Inflective Nano Wires

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Abstract

The wave phenomena around inflection points are studied from a new point of view in order to coordinate inflective shapes analytically. The inflective coordinate systems are used to compute the exact solutions. The differential equations obtained using these recently introduced coordinate systems are solved by extending the usual separation method. These extensions bring out new, special functions and series, and the achieved results give the exact meaning of physics related with inflection points. In addition, the electromagnetic wave is calculated around inflection point. The approach is extended to the nano-scale.

1. Introduction

The inflection points still rise some critical problems in electromagnetic phenomena. There are some results related to these problems, to name some: diffraction from S- shaped discontinuities [1], diffraction from the edged concave-convex boundaries [2], and smooth targets with inflection points [3-4]. These works use some approximations and focus on the uniformization of physical optics. The first paper requires a radius of curvature that is relatively large at every point on the surface but gives more compact solution than the solution given in the second paper. The second paper describes an asymptotic, high frequency solution for the scattering from a concave-convex shaped boundary with an edge. This second paper requires that the associated reflected ray caustic with transition regions of two points of inflection do not come close together. The authors' PO based uniform analysis fails near grazing angles of incidence on the reflecting boundaries with points of inflection. The third and fourth papers have used PO approximations. Therefore, it is clear that the influence of inflection points on electromagnetic field still has some open questions. A recent paper aimed to open the analytical insights behind the inflection points when the solutions of the basic field equations are asked [5]. This last paper describes inflective circular cylindrical coordinates and derives the essential analytical expressions for the solution of problems with inflection points. The evaluation is due to the new defined inflective series obtained by applying the extracted separation method, which is defined here. The extracted separation method comes with some modifications on extended separation method in [5]-[6].

In view of the above comments, this paper aims to extend the analytical perspectives of [5]-[7] for the wave phenomena around inflection points in nano-scale.

2. Weighted inflective circular cylindrical coordinate system

The most simple inflection point can be arranged by two semi-cylinders that are placed side by side as in Fig.1a. Here the curves $\mathrm{L}_{\mathrm{i}}^{\mathrm{ra}}$ and $\mathrm{L}_{\mathrm{i}}^{\mathrm{la}}$ are the upper and lower halves of cylinders with radii a. When the points on such curves are coordinated by repeating the curve $\mathrm{L}_{\mathrm{i}}^{\mathrm{a}} \overset{\Delta}{\equiv} \mathrm{L}_{\mathrm{i}}^{\mathrm{la}} \cup \mathrm{L}_{\mathrm{i}}^{\mathrm{ra}}$ without changing the inflection point O(0,0) as in Fig. 1b, it is seen that the trajectory obtained is more versatile than using the common cylindrical coordinate lines. However, if we use the weighted inflective circular cylindrical coordinates x^1 and x^2 of point P then the separations of the differential equations arising from field calculations become possible. These coordinates are defined as

$$x^1 \equiv q\rho/\cos\phi, \qquad x^2 \equiv p\,\phi \tag{2.1}$$

in [5] where p and q are finite real numbers other than zero. The distance ρ and angle ϕ are usual circular cylindrical coordinates of P (see Fig. 1c). We call weight parameters p and q.

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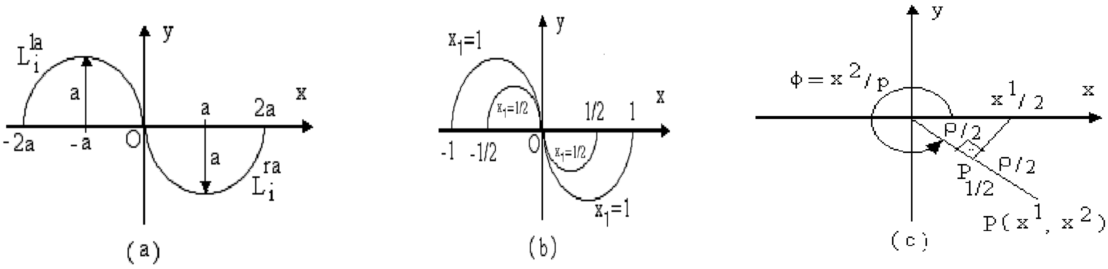

Figure 1. Inflective circular cylindrical coordinates system: (a) Simplest inflection point. (b) More versatile trajectory. (c) Relation with circular coordinates of P.

To address the essential mathematical analysis we deal with the scalar wave and consider the solution of Helmholtz's equation.

3. The Waves in Inflective Coordinates

Let u be a scalar, and time independent wave function then a wave propagation problem can be solved by extending the usual separation method in inflective coordinates. If u is written as $u(x^1, x^2)=X_1(x^1)X_2(x^2)$, then Helmholtz's equation gives

$$\frac{X_1''}{(x^1)^2 X_1} + \frac{1}{X_2}\cos^4(\tfrac{x^2}{p})\sin^2(\tfrac{x^2}{p})\{X_2'' \frac{p^2}{\sin^2(\frac{2x^2}{p})\cos^2(\frac{2x^2}{p})} - X_2' \frac{8p\cos(\frac{x^2}{p})}{\sin^2(\frac{2x^2}{p})\sin(\frac{4x^2}{p})}[\cos^4 \tfrac{x^2}{p} +$$

$$+ (\sin^2 \tfrac{x^2}{p})(1 - 3\cos^2 \tfrac{x^2}{p} + \cos \tfrac{x^2}{p})]\} +$$

$$+ \frac{X_1'}{X_1} x^1 \{\frac{X_2'}{X_2} \frac{2p\cos^2(\frac{x^2}{p})\sin^2(\frac{x^2}{p})}{\sin(\frac{2x^2}{p})\cos(\frac{2x^2}{p})} - [1 + 3\cos^2(\tfrac{x^2}{p})]\} +$$

$$+ (x^1)^2 \frac{k^2}{q^2}\cos^4(\tfrac{x^2}{p})\sin^2(\tfrac{x^2}{p}) = 0 \qquad (3.1)$$

Here k is the wave number of the medium. The first and second terms of the sum in (3.1) are only the functions of x^1 and x^2, respectively. So we may write

$$X_1'' - (x^1)^3 X_1' + (\kappa_1 x^1/q)^2 X_1 = 0 \qquad (3.2a)$$

$$X_2'' - X_2' \frac{1}{p} \mathrm{seng}(x^2/p) + X_2(2\kappa_2/q)^2 \frac{1}{p^2}[s_c(x^2/p)]^2 = 0 \qquad (3.2b)$$

and

$$\frac{X_1'}{X_1} x^1 [\frac{X_2'}{X_2} \frac{p\cos^2(\frac{x^2}{p})\sin(\frac{x^2}{p})}{\mathrm{COS}(\frac{x^2}{p})\cos(\frac{2x^2}{p})} - 3\cos^2(\tfrac{x^2}{p})] = -(x^1)^2 \frac{k^2}{q^2}\cos^4(\tfrac{x^2}{p})\sin^2(\tfrac{x^2}{p}) - \frac{\kappa_{12}{}^2}{q^2} \qquad (3.2c)$$

since p is arbitrary by the algorithm of extended separation method [5]. Here κ_1, κ_2, and κ_{12} are suitable constants such that

$$(\tfrac{\kappa_1}{q})^2 + (\tfrac{\kappa_2}{q})^2 + (\tfrac{\kappa_{12}}{q})^2 = 0 \qquad (3.3)$$

is satisfied. The function $\mathrm{seng}(x^2/p)$ is defined as

$$\mathrm{seng}(x^2/p) \overset{\Delta}{\equiv} s_n(x^2/p)s_d(x^2/p) \tag{3.4a}$$

$$s_n(x^2/p) \overset{\Delta}{\equiv} \cos(2x^2/p)/\sin(x^2/p) \tag{3.4b}$$

$$s_d(x^2/p) \overset{\Delta}{\equiv} 2\left\{\cos^4(\tfrac{x^2}{p}) + \sin^2(\tfrac{x^2}{p})\left[\cos(\tfrac{x^2}{p})\left(1 - \cos(\tfrac{x^2}{p})\right) - \cos(2\tfrac{x^2}{p})\right]\right\} \tag{3.4c}$$

$$s_c(x^2/p) \overset{\Delta}{\equiv} \cos(2x^2/p)/\cos(x^2/p) \tag{3.4d}$$

Assumption: If κ_1, κ_2, κ_{12}, and κ_{21} are suitable constants then we select following equalities since q is arbitrary when $x^2 \neq np\pi$ if $n\in \mathbb{Z} \overset{\Delta}{\equiv} \{0, \pm1, \pm2, \pm3,\dots\}$ and Im k=0, with respect to p and q:

$$\text{i.} \quad x^1X_1'+[(\kappa_1 x^1)^2+ \kappa_{12}^2]X_1 = 0 \tag{3.5a}$$

$$\text{ii.} \quad \frac{X_2'}{X_2}\frac{p\cos^2(\frac{x^2}{p})\sin(\frac{x^2}{p})}{\cos(\frac{x^2}{p})\cos(\frac{2x^2}{p})} - 3\cos^2(\tfrac{x^2}{p}) = \lambda_2^2\cos^4(\tfrac{x^2}{p})\sin^2(\tfrac{x^2}{p}) - \lambda_{21}^2 \tag{3.5b}$$

In the case of Im k≠0 we have to put $-\kappa_2^2$ instead of κ_2^2 in (3.5b). We have

$$\kappa_1\neq0, \quad \kappa_2\neq0, \quad \kappa_1\kappa_2=\pm k/q, \quad \kappa_{12}\equiv0, \quad \kappa_{21}\equiv0, \quad \kappa_{12}=0 \tag{3.6}$$

since equation (3.1) is supplied for $\forall x_1$ and $\forall x_2$, identically; therefore, we obtain

$$X_1'+ x^1\kappa_1^2X_1 = 0 \tag{3.7a}$$

$$X_2' - X_2\frac{\lambda_2^2\cos^4(\frac{x^2}{p})\sin^2(\frac{x^2}{p})+3\cos^2(\frac{x^2}{p})}{p\sin(\frac{x^2}{p})\cos(\frac{x^2}{p})}\cos(\tfrac{2x^2}{p}) = 0 \tag{3.7b}$$

from equation (3.2c) and assumption. Here we placed

$$-\lambda_1^2\lambda_2^2 + \frac{k^2}{q^2} = 0 \tag{3.8}$$

The equation pairs (3.2a)-(3.2b) and (3.7a)-(3.7b) are called extended equations and extracted equations, respectively. The extracted equations cannot be used to solve X_1 and X_2 as basic equations. First, extended equations must be solved. The extracted equations should be used for the solutions of (3.2a) and (3.2b) as constrained equations. The parameters κ_1 and κ_2 are separation constants. The parameter κ_{12} is extended separation constant whereas the parameters κ_1 and κ_2 are extra-separation constants. The parameter κ_{12} is extracted separation constant. The equations (3.3) and (3.8) are called "extended separation equation" and "extra separation equation", respectively. The equations (3.6) are called "extraction conditions" [8].

4. Conclusion

The separation of wave equation in inflective circular cylindrical coordinates is given. The wave phenomenon is studied near inflection points. The analytical essentials are derived for the solution of Helmholtz's equation when inflective boundaries are included. The evaluation is obtained by the extracted separation method. The results are given by inflective wave series. Nano-scale manipulations are studied.

5. References

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