Rigorous Analysis of the Parallel-Plate Waveguide by the Hybrid Mode Formulation: From the TEM mode to the Surface Plasmon Polariton

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Abstract

This paper presets an analysis of the parallel-plate waveguide, based on an hybrid mode formulation. The non-ideal metallic conductors of the waveguide are treated as a media characterized by an equivalent permittivity. The frequencies of interest in the presented analysis are at the terahertz band (from 300 GHz to 30 THz) and appropriate models are used for the correct characterization of metallic conductors at these frequencies. The behavior of the electromagnetic field of the fundamental mode is studied in a wide frequency range. At low frequencies (microwave regime) the fundamental mode is the well-known *TEM* mode; as frequency increases, the electromagnetic field changes significantly and a surface wave or Surface Plasmon Polariton (*SPP*) behavior is observed at the highest frequencies of the terahertz band. This paper shows an unified formulation that explains this transformation in the electromagnetic field behavior.

1. Introduction

The analysis of the ideal parallel-plate waveguide (with perfect conductor) is carried out in several references e.g. [1]. More elaborate studies can be found in [2] where a parallel-plate waveguide with different arbitrary surface impedance boundary conditions is analyzed. In [3] an equivalent circuit of a parallel-plate waveguide working at optical frequencies is proposed.

Beyond the theoretical characterization, the parallel-plate waveguide is often used in experiments and measurement systems. In the references [4, 5] several systems for terahertz time-domain spectroscopy are presented, losses and dispersion of the parallel-plate waveguide are obtained. It is worth noting the proposal of using the parallel-plate waveguide like a lens [6], based on the concept of equivalent refractive index of a propagating mode. Other possible practical applications of the parallel-plate waveguide at terahertz frequencies are presented in [7].

2. Rigorous electromagnetic field formulation

The parallel-plate waveguide to solve is shown in Fig. 1. It consists of two metallic indefinite plates separated a distance of $2h$. The metallic plates are considered infinite in x and z directions. The propagation direction of the electromagnetic field is assumed in the z coordinate. This waveguide presents a symmetry plane at $y = 0$. Therefore, all the modes are represented by an even or an odd function in the y coordinate. This is equivalent to a perfect electric wall (*PEW*) or a perfect magnetic wall (*PMW*) at the xz plane.

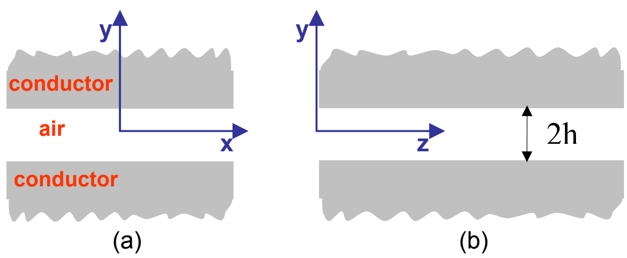

Fig. 1 Parallel-plate waveguide. (a) Cross-sectional view. (b) Longitudinal view.

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In order to take into account the behavior of the metallic conductors accurately, these are considered as another medium. The frequencies of interest in the presented analysis are at the terahertz band and thus the Drude model is used for the correct characterization of the metallic conductors [8]. Silver is assumed and the parameters for the Drude model are given by [9].

In that way, the waveguide to characterize is a non-homogeneous waveguide with two different regions. In the first region ($i=1,\ 0<y<h$) air is assumed and in the second region ($i=2,\ h<y<\infty$) the equivalent dielectric permittivity of the Drude model is used. Since the waveguide is non-homogeneous a hybrid mode formulation is required, i.e. each mode has, in principle, both longitudinal electric and magnetic components.
The longitudinal components are obtained solving the Helmtholtz equations and the transverse components are calculated from the longitudinal ones as:

$$\begin{bmatrix} \vec{E}_{ti} \\ \vec{H}_{ti} \end{bmatrix} = \frac{1}{\gamma_{ci}^2} \begin{bmatrix} \gamma \nabla_t & -j\omega\mu_i \hat{z}\times\nabla_t \\ j\omega\varepsilon_i \hat{z}\times\nabla_t & \gamma\nabla_t \end{bmatrix} \begin{bmatrix} E_{zi} \\ H_{zi} \end{bmatrix}, \quad i=1,2, \tag{1}$$

where $\gamma=\alpha+j\beta$ is the propagation constant and $\gamma_{ci}^2=-\omega^2\mu_i\varepsilon_i-\gamma^2$. The subindex t in the operators refers to the transverse coordinates.

The continuity of the tangential components of the electromagnetic field at the interface $y=h$ is finally imposed in order to obtain the dispersion or characteristic equation of the waveguide. In this case all the modes obtained are *TM* or *TE*, although, the formulation has started from the general case, assuming hybrid modes. The dispersion equations obtained are summarized in Table I.

Table I: Dispersion or characteristic equations.

	PEW symmetry plane	*PMW* symmetry plane
TM modes	$\frac{\varepsilon_1}{\gamma_{c1}}\cosh(\gamma_{c1}h)+\frac{\varepsilon_2}{\gamma_{c2}}\sinh(\gamma_{c1}h)=0$	$\frac{\varepsilon_1}{\gamma_{c1}}\sinh(\gamma_{c1}h)+\frac{\varepsilon_2}{\gamma_{c2}}\cosh(\gamma_{c1}h)=0$
TE modes	$\frac{\mu_2}{\gamma_{c2}}\cosh(\gamma_{c1}h)+\frac{\mu_1}{\gamma_{c1}}\sinh(\gamma_{c1}h)=0$	$\frac{\mu_2}{\gamma_{c2}}\sinh(\gamma_{c1}h)+\frac{\mu_1}{\gamma_{c1}}\cosh(\gamma_{c1}h)=0$

3. Results

The propagation constant of each mode is obtained from the dispersion equations by means of root finding in the complex plane. It is highlighted that the propagation constants of a non-homogeneous waveguide can present an arbitrary behavior with frequency and the root finding must be done for each frequency.

The fundamental mode is a *TM* mode (with *PEW* symmetry plane). The longitudinal component of the electric field of this fundamental mode tends to zero when the metallic conductor tends to be a perfect conductor. At low frequencies the metallic conductors behave as good conductors, thus the fundamental mode is accurately represented by the classical solution, the *TEM* mode. When frequency increases the behavior of metallic conductors change significantly. It can be proved, that in the asymptotic limit (high frequency and electrically large height) the solution of the characteristic equation is:

$$\gamma = j\omega\sqrt{\mu_0\varepsilon_0}\sqrt{\frac{\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}}}. \tag{2}$$

This is the propagation constant of a surface wave or *SPP* propagating over a metal-dielectric interface [10, 11].

In Fig. 2 the real part of the exact propagation constant (attenuation) is compared with the classical approach obtained by the power loss method [1], and with the surface wave, equation (2). As can be seen, the exact solution is quite similar to the power loss one at frequencies until some terahertzs, where a significant change in the behavior of the real part of the propagation constant is observed. At highest frequencies the exact solution fits perfectly with the *SPP* solution.

Only the real part of the propagation constant has been shown, for the imaginary part of the propagation constant, all the solutions (ideal, exact an surface wave) are quite similar in all the analyzed frequency range.

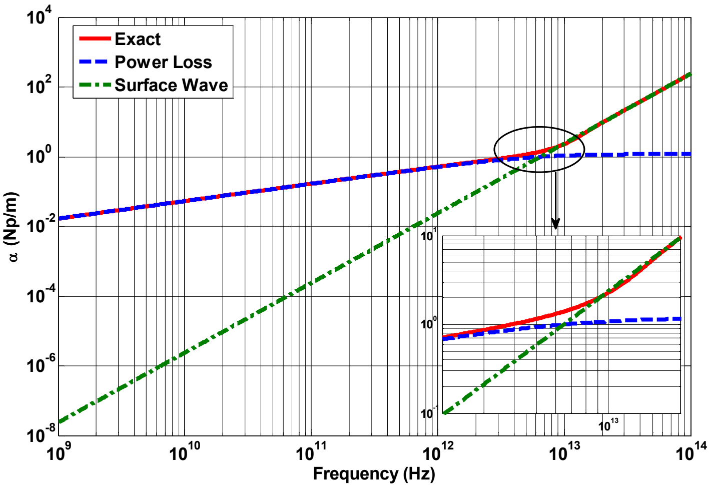

Fig. 2 Real part of the propagation constant (α) for the fundamental mode of a parallel-plate waveguide with *2* mm of height. Comparison of the exact, power loss and surface wave solution.

In Fig. 3, the electric field ($\hat{y}$ component) of the fundamental mode at different frequencies is shown. The change of the electric field with frequency clearly explains the aforementioned behavior of the fundamental mode. At low frequencies the fundamental mode is the classical *TEM* and at the highest frequency the *SPP* behavior of the electromagnetic field is observed.

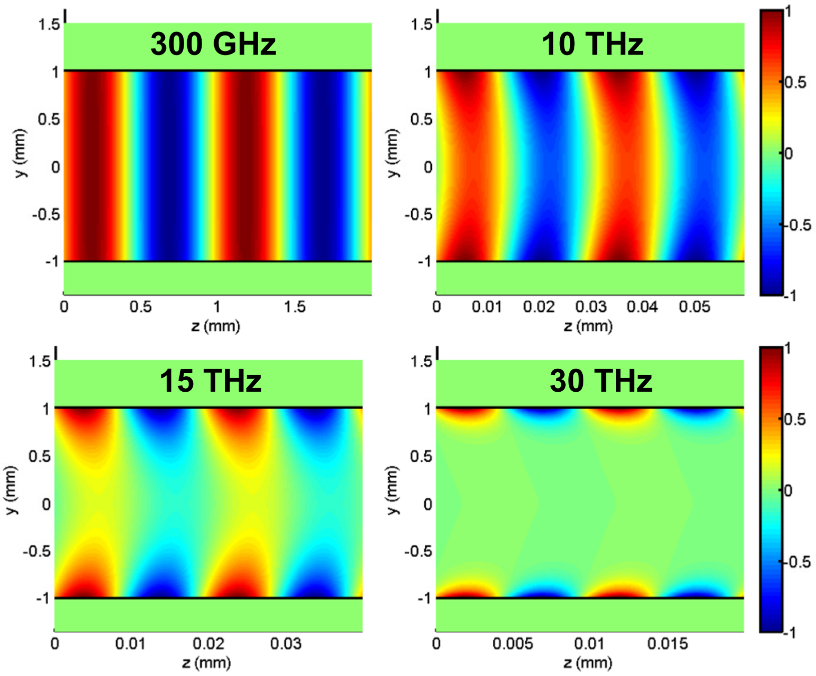

Fig. 3 Electric field ($\hat{y}$ component) for the fundamental mode of a parallel-plate waveguide with *2 mm* of height.

4. Conclusion

The parallel-plate waveguide has been analyzed by means of an hybrid mode formulation. The analysis has been carried out at terahertz frequencies. At low frequencies the fundamental mode is perfectly characterized by the *TEM* mode of the ideal waveguide (with perfect conductors), but when the frequency increases the electromagnetic field distribution changes. At the highest frequencies of the terahertz band a surface wave or SPP behavior of the electromagnetic field is obtained. The evolution of the fundamental mode from the *TEM* mode to the *SPP* has been obtained.

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