Material Invariant Formulation of Plane Wave Scattering by a Moving Dielectric Half-Space

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Abstract

In virtue of the axiomatic structure of a recently introduced Material Invariant Electromagnetic Field Theory, we introduce the general formulation of a scattering problem for arbitrarily moving material media and provide the solution for the canonical case of TM plane wave scattering by a dielectric half-space in uniform rectilinear motion.

1. Introduction

This work is complementary to the recent paper [1] where the present author has provided the mathematical tools and the postulations which lead us to a material invariant (or frame indifferent) classical electromagnetic field theory. Based on the new field equations of moving media introduced in [1] in what follows we introduce the general formulation of a scattering problem for moving objects and provide the solution for the special case of plane wave incidence on a dielectric half space in uniform rectilinear motion. Throughout the investigation we assume the reader to be familiar with the notation, symbols and all definitions introduced in [1] since they cannot be repeated due to size limitation.

2. General Formulation

We consider the scenario in Fig.1 where, according to an E-observer, the incident electromagnetic wave with fields $\left(\vec{E}_{inc}(\vec{r};t),\vec{H}_{inc}(\vec{r};t)\right)$ and sources $\left(\rho_{Tx}(\vec{r};t),\vec{J}_{Tx}(\vec{r};t)\right)$ generated by a stationary transmitter in medium I is impinging on an object occupying a region D and in arbitrary relative motion with velocity $\vec{\mathrm{v}}(\vec{r};t)$

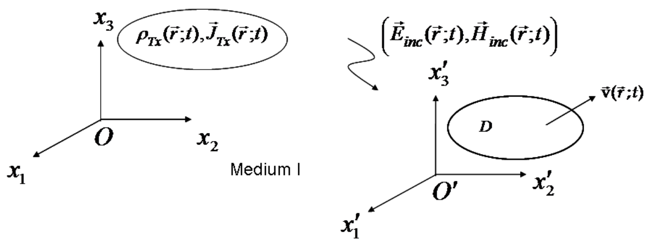

Fig.1. An illustration of the problem

2.1 The Incoming Wave

In E-frame $Ox_1x_2x_3$ the incident fields (with argument $(\vec{r};t)$) satisfy the Maxwell equations of stationary media

$$\mathrm{curl}\,\vec{E}_{inc}+\frac{\partial}{\partial t}\vec{B}_{inc}=\vec{0}\ \ ,\ \mathrm{curl}\,\vec{H}_{inc}-\frac{\partial}{\partial t}\vec{D}_{inc}=\vec{J}_{Tx}\ \ ,\ \mathrm{div}\,\vec{D}_{inc}=\rho_{Tx}\ \ ,\ \mathrm{div}\,\vec{B}_{inc}=0\,. \tag{2.1}$$

Let us assume the medium I simple and lossless with constitutive parameters (ε,μ). Then the incident fields in E-frame satisfy the stationary wave and Helmholtz equations

$$\left(lap-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{pmatrix}\vec{E}_{inc}\\ \vec{H}_{inc}\end{pmatrix}=\begin{pmatrix}(1/\varepsilon)\,grad\,\rho_{Tx}\\ \vec{0}\end{pmatrix}\ \ ,\ \ \left(lap+k^2\right)\begin{pmatrix}\vec{E}_{inc}\\ \vec{H}_{inc}\end{pmatrix}=\begin{pmatrix}(1/\varepsilon)\,grad\,\rho_{Tx}\\ \vec{0}\end{pmatrix} \tag{2.2}$$

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where $c = 1/\sqrt{\mu\varepsilon}$ and $k = \omega\sqrt{\mu\varepsilon} = 2\pi/\lambda$ with time dependence taken as $\exp(-i\omega t)$. For an observer in L-frame $Ox_1'x_2'x_3'$ the object is stationary and the surrounding medium I is in relative motion with a velocity $\vec{v}'(\vec{r}';t)$. Accordingly, in L-frame the incident fields (with argument $(\vec{r}';t)$) satisfy the material invariant field and wave equations

$$\mathrm{curl}'\vec{E}'_{inc} + \frac{\Diamond'}{\Diamond' t}\vec{B}'_{inc} = \vec{0} \quad , \quad \mathrm{curl}'\vec{H}'_{inc} - \frac{\Diamond'}{\Diamond' t}\vec{D}'_{inc} = \vec{J}'_{Tx} \quad , \quad \mathrm{div}'\vec{D}'_{inc} = \rho'_{Tx} \quad , \quad \mathrm{div}'\vec{B}'_{inc} = 0 \tag{2.3}$$

$$\left(lap' - \frac{1}{c^2}\frac{\Diamond'^2}{\Diamond' t^2}\right)\begin{pmatrix}\vec{E}'_{inc}\\ \vec{H}'_{inc}\end{pmatrix} = \begin{pmatrix}(1/\varepsilon)\,grad'\rho'_{Tx}\\ \vec{0}\end{pmatrix} \quad , \quad \left(lap' + k^2\right)\begin{pmatrix}\vec{E}'_{inc}\\ \vec{H}'_{inc}\end{pmatrix} = \begin{pmatrix}(1/\varepsilon)\,grad'\rho'_{Tx}\\ \vec{0}\end{pmatrix} \tag{2.4}$$

Here the comoving time derivative of a vector $\vec{A}'_{inc}$ is given by

$$\frac{\Diamond'}{\Diamond' t}\vec{A}' = \frac{D'}{D't}\vec{A}' - \vec{A}'\cdot grad'\vec{v}' + \vec{A}'(div'\vec{v}') = \frac{\partial}{\partial t}\vec{A}' + \vec{v}'\cdot grad'\vec{A}' - \vec{A}'\cdot grad'\vec{v}' + \vec{A}'(div'\vec{v}')\,. \tag{2.5}$$

2.2 The Scattered Wave

Let us express the total field in space in E- and L-frames respectively as

$$\left(\vec{E}_{tot},\vec{H}_{tot}\right) = \begin{cases}\left(\vec{E}_{inc},\vec{H}_{inc}\right) + \left(\vec{E}_{sc},\vec{H}_{sc}\right), \text{in medium I}\\ \left(\vec{E}_D,\vec{H}_D\right), \text{in region D}\end{cases} \quad \text{and} \quad \left(\vec{E}'_{tot},\vec{H}'_{tot}\right) = \begin{cases}\left(\vec{E}'_{inc},\vec{H}'_{inc}\right) + \left(\vec{E}'_{sc},\vec{H}'_{sc}\right), \text{in medium I}\\ \left(\vec{E}'_D,\vec{H}'_D\right), \text{in region D}\end{cases}.$$

In L-frame of the scattered field, i.e. according to an observer traveling with the scattered field, the ambient source-free medium I is in motion with linear velocity $-\vec{v}'(\vec{r}';t)$. Accordingly, the scattered fields in medium I satisfy the material invariant field and wave equations

$$\mathrm{curl}'\vec{E}'_{sc} + \frac{\bar{\Diamond}'}{\bar{\Diamond}' t}\vec{B}'_{sc} = \vec{0} \quad , \quad \mathrm{curl}'\vec{H}'_{sc} - \frac{\bar{\Diamond}'}{\bar{\Diamond}' t}\vec{D}'_{sc} = \vec{0} \quad , \quad \mathrm{div}'\vec{D}'_{sc} = 0 \quad , \quad \mathrm{div}'\vec{B}'_{sc} = 0 \tag{2.6}$$

$$\left(lap' - \frac{1}{c^2}\frac{\bar{\Diamond}'^2}{\bar{\Diamond}' t^2}\right)\begin{pmatrix}\vec{E}'_{sc}\\ \vec{H}'_{sc}\end{pmatrix} = \vec{0} \quad , \quad \left(lap' + k^2\right)\begin{pmatrix}\vec{E}'_{sc}\\ \vec{H}'_{sc}\end{pmatrix} = \vec{0} \tag{2.7}$$

where the accompanying comoving time derivative of a vector $\vec{A}'_{sc}$ is defined as

$$\frac{\bar{\Diamond}'}{\bar{\Diamond}' t}\vec{A}'_{sc} = \frac{D'}{D't}\vec{A}'_{sc} + \vec{A}'_{sc}\cdot grad'\vec{v}' - \vec{A}'_{sc}(div'\vec{v}') = \frac{\partial}{\partial t}\vec{A}'_{sc} - \vec{v}'\cdot grad'\vec{A}'_{sc} + \vec{A}'_{sc}\cdot grad'\vec{v}' - \vec{A}'_{sc}(div'\vec{v}')\,. \tag{2.8}$$

2.3 Total Field Inside the Moving Object

In L-frame of the region D with fields $\left(\vec{E}'_D,\vec{H}'_D\right)$ and sources $\left(\rho'_D,\vec{J}'_D\right)$, the region is stationary since the ambient medium I is observed as source-free. Therefore in region D the field equations of stationary media

$$\mathrm{curl}'\vec{E}'_D + \frac{\partial}{\partial t}\vec{B}'_D = \vec{0} \;,\; \mathrm{curl}'\vec{H}'_D - \frac{\partial}{\partial t}\vec{D}'_D = \vec{J}'_D \;,\; \mathrm{div}'\vec{D}'_D = \rho'_D \;,\; \mathrm{div}'\vec{B}'_D = 0 \tag{2.9}$$

are satisfied. When the region D simple with constitutive parameters $(\varepsilon_D,\mu_D,\sigma_D)$, (2.9) yield the stationary wave equations

$$\left(lap' - \frac{1}{{c_D}^2}\frac{\partial^2}{\partial t^2} - \sigma_D\mu_D\frac{\partial}{\partial t}\right)\begin{pmatrix}\vec{E}'_D\\ \vec{H}'_D\end{pmatrix} = \begin{pmatrix}(1/\varepsilon_D)\,grad'\rho'_D\\ \vec{0}\end{pmatrix} \tag{2.10}$$

with $c_D = 1/\sqrt{\mu_D\varepsilon_D}$. For E-observer the field and wave equations (2.9), (2.10) read

$$\mathrm{curl}\,\vec{E}_D + \frac{\Diamond}{\Diamond t}\vec{B}_D = \vec{0} \;,\; \mathrm{curl}\,\vec{H}_D - \frac{\Diamond}{\Diamond t}\vec{D}_D = \vec{J}_D \;,\; \mathrm{div}\,\vec{D}_D = \rho_D \;,\; \mathrm{div}\,\vec{B}_D = 0 \tag{2.11}$$

$$\left(lap - \frac{1}{{c_D}^2}\frac{\Diamond^2}{\Diamond t^2} - \sigma_D\mu_D\frac{\Diamond}{\Diamond t}\right)\begin{pmatrix}\vec{E}_D\\ \vec{H}_D\end{pmatrix} = \begin{pmatrix}(1/\varepsilon_D)\,grad\,\rho_D\\ \vec{0}\end{pmatrix} \tag{2.12}$$

where the accompanying comoving time derivative of a vector $\vec{A}_D$ is defined as

$$\frac{\Diamond}{\Diamond t}\vec{A}_D = \frac{D}{Dt}\vec{A}_D - \vec{A}_D \cdot grad\,\vec{\mathrm{v}} + \vec{A}_D\left(div\vec{\mathrm{v}}\right) = \frac{\partial}{\partial t}\vec{A}_D + \vec{\mathrm{v}}\cdot grad\vec{A}_D - \vec{A}_D\cdot grad\,\vec{\mathrm{v}} + \vec{A}_D\left(div\vec{\mathrm{v}}\right). \tag{2.13}$$

2.4 Boundary Relations on the Moving Object

On the enclosure $S = \partial D$ of the moving medium, which is assumed a simple interface that may support surface charges and currents $\left(\rho_S'(\vec{r}_S';t), \vec{J}_S(\vec{r}_S';t)\right)$, the distributional form of stationary field (Maxwell) equations in L- frame read

$$\hat{n}'\times\left[\vec{E}_{inc}'(\vec{r}_S';t)+\vec{E}_{sc}'(\vec{r}_S';t)\right] = \hat{n}'\times\vec{E}_D'(\vec{r}_S';t)\ ,\quad \hat{n}'\times\left[\vec{H}_{inc}'(\vec{r}_S';t)+\vec{H}_{sc}'(\vec{r}_S';t)\right]-\hat{n}'\times\vec{H}_D'(\vec{r}_S';t) = \vec{J}_S'(\vec{r}_S';t) \tag{2.14a,b}$$

$$\hat{n}'\cdot\left[\vec{D}_{inc}'(\vec{r}_S';t)+\vec{D}_{sc}'(\vec{r}_S';t)\right]-\hat{n}'\cdot\vec{D}_D'(\vec{r}_S';t) = \rho_S'(\vec{r}_S';t)\ ,\quad \hat{n}'\cdot\left[\vec{B}_{inc}'(\vec{r}_S';t)+\vec{B}_{sc}'(\vec{r}_S';t)\right] = \hat{n}'\cdot\vec{B}_D'(\vec{r}_S';t) \tag{2.14c,d}$$

Along with constitutive relations and radiation, edge, tip, periodicity, etc. type conditions complementing the boundary relations, the associated boundary value problem can be solved uniquely to yield the L-fields $\left(\vec{E}_{sc}', \vec{H}_{sc}'\right)$ and $\left(\vec{E}_D', \vec{H}_D'\right)$, whose images also yield the E-fields $\left(\vec{E}_{sc}, \vec{H}_{sc}\right)$ and $\left(\vec{E}_D, \vec{H}_D\right)$.

3. TM Plane Wave Scattering by a Moving Dielectric Half Space

Let us consider an incoming TM plane wave propagating along $\hat{n}_{inc} = (\cos\alpha, \sin\alpha)$ direction in (x_1, x_2) plane with phase velocity c and fields represented by

$$\vec{H}_{inc}(\vec{r};t) = \hat{x}_3 f(\hat{n}_{inc}\cdot\vec{r}-ct) = \hat{x}_3 f(x_1\cos\alpha + x_2\sin\alpha - ct) = \hat{x}_3 g(k\hat{n}_{inc}\cdot\vec{r}-\omega_{inc}t)\ ,\ \vec{E}_{inc}(\vec{r};t) = Z\vec{H}_{inc}(\vec{r};t)\times\hat{n}_{inc} \tag{3.1}$$

with angular frequency $\omega_{inc} = 2\pi f_{inc}$, phase velocity $\vec{\mathrm{v}}_{Pinc} = \hat{x}_1 c = \hat{x}_1\,\omega_{inc}/k = \hat{x}_1\lambda f_{inc}$ and $Z = \sqrt{\mu/\varepsilon}$. For E-observer we assume the half-space $x_1 > 0$ in uniform rectilinear motion with velocity $\vec{\mathrm{v}} = \pm G\hat{x}_1, G > 0$ while it is moving with linear velocity $\vec{\mathrm{v}}' = \mp G\hat{x}_1'$ for L-observer. Incorporating the coordinate transformations $x_1 = x_1' \pm Gt$, $x_2 = x_2'$, $x_3 = x_3'$, the incoming fields in L-frame can be given as

$$\vec{H}_{inc}'(\vec{r}';t) = \hat{x}_3' f(\hat{n}_{inc}'\cdot\vec{r}'-c_{inc}'t) = \hat{x}_3' g(k\hat{n}_{inc}'\cdot\vec{r}'-\omega_{inc}'t)\ ,\quad \vec{E}_{inc}'(\vec{r}';t) = Z\vec{H}_{inc}'(\vec{r}';t)\times\hat{n}_{inc}' \tag{3.2}$$

with $\hat{n}_{inc}'\cdot\vec{r}' = x_1'\cos\alpha + x_2'\sin\alpha$, $c_{inc}' = c\mp G\cos\alpha = c(1\mp\beta\cos\alpha)$, $\omega_{inc}' = \omega_{inc}(1\mp\beta\cos\alpha)$, $f_{inc}' = f_{inc}(1\mp\beta\cos\alpha)$.
We observe $\beta < \cos\alpha$ as a physical limit on G for the realization of scattering phenomenon. The scattered field and the total field in region D can be given by

$$\vec{H}_{sc}'(\vec{r}';t) = \hat{x}_3' R_{TM} f(\hat{n}_{sc}'\cdot\vec{r}'-c_{sc}'t) = \hat{x}_3' R_{TM}\, g(k\hat{n}_{sc}'\cdot\vec{r}'-\omega_{sc}'t),\ \vec{E}_{sc}'(\vec{r}';t) = Z\vec{H}_{sc}'(\vec{r}';t)\times\hat{n}_{sc}' \tag{3.3}$$

$$\vec{H}_D'(\vec{r}';t) = \hat{x}_3' T_{TM} f(\hat{n}_D'\cdot\vec{r}'-c_D't) = \hat{x}_3' T_{TM}\, g(k_D\hat{n}_D'\cdot\vec{r}'-\omega_D't),\ \vec{E}_D'(\vec{r}';t) = Z_D\vec{H}_D'(\vec{r}';t)\times\hat{n}_D \tag{3.4}$$

with $\hat{n}_{sc}'\cdot\vec{r}' = -x_1'\cos\alpha_{sc} + x_2'\sin\alpha_{sc}$, $\hat{n}_D'\cdot\vec{r}' = x_1'\cos\alpha_D + x_2'\sin\alpha_D$, $Z_D = \sqrt{\mu_D/\varepsilon_D}$. The unknowns c_{sc}', α_{sc}, c_D', α_D, R_{TM}, T_{TM} are solved from the boundary value problem

$$\begin{cases} \left(lap' - \dfrac{1}{c^2}\dfrac{\Diamond'^2}{\Diamond' t^2}\right)\vec{H}_{sc}'\left(x_1', x_2', x_3'; t\right) = \vec{0}, \text{in medium I} \\ \left(lap' - \dfrac{1}{c_D^2}\dfrac{\partial^2}{\partial t^2}\right)\vec{H}_D'\left(x_1', x_2', x_3'; t\right) = \vec{0}, \text{in region D} \\ \vec{H}_{inc}'(0, x_2', x_3'; t) + \vec{H}_{sc}'(0, x_2', x_3'; t) = \vec{H}_D'(0, x_2', x_3'; t),\ \ \forall x_2', x_3', t \\ Z\left[\vec{H}_{inc}'(0, x_2', x_3'; t)\times\hat{n}_{inc}' + \vec{H}_{sc}'(0, x_2', x_3'; t)\times\hat{n}_{sc}'\right] = Z_D\vec{H}_D'(0, x_2', x_3'; t)\times\hat{n}_D',\ \ \forall x_2', x_3', t \\ \text{Radiation Conditions as } x_1' \to \pm\infty \end{cases} \tag{3.5a-e}$$

The wave equation (3.5a) in medium I, namely,

$$\left(lap' - \frac{1}{c^2}\frac{\Diamond'^2}{\Diamond' t^2}\right)\vec{H}_{sc}' = \hat{x}_3'\left(lap' - \frac{1}{c^2}\left(\frac{\partial}{\partial t}\pm G\frac{\partial}{\partial x_1'}\right)^2\right)f(\hat{n}_{sc}'\cdot\vec{r}'-c_{sc}'t) = \hat{x}_3'\left(1-\frac{1}{c^2}\left(-c_{sc}'\mp G\cos\alpha_{sc}\right)^2\right)f = \vec{0}$$

require $c'_{sc} = c(1 \mp \beta\cos\alpha_{sc})$, $\omega'_{sc} = \omega_{inc}(1 \mp \beta\cos\alpha_{sc})$, while (3.5b) in region D require $c'_D = c_D$ and $\omega'_D = \omega_{inc}$. Boundary conditions (3.5c,d) require

$$f(x'_2 \sin\alpha - c'_{inc}t) + R_{TM} f(x'_2 \sin\alpha_{sc} - c'_{sc}t) = T_{TM} f(x'_2 \sin\alpha_D - c'_D t), \quad \forall x'_2, t \tag{3.6a}$$

$$Z\left[\cos\alpha\, f(x'_2 \sin\alpha - c'_{inc}t) - \cos\alpha_{sc}\, R_{TM} f(x'_2 \sin\alpha_{sc} - c'_{sc}t)\right] = \cos\alpha_D\, Z_D T_{TM} f(x'_2 \sin\alpha_D - c'_D t), \quad \forall x'_2, t \tag{3.6b}$$

From (3.6) one uniquely obtains I) $dx'_2/dt = c'_{inc}/\sin\alpha = c'_{sc}/\sin\alpha_{sc} = c'_D/\sin\alpha_D$, which requires $\alpha_{sc} = \alpha$, $c'_{sc} = c'_{inc}$, $\omega'_{sc} = \omega'_{inc}$ and the extended Snell relation $\sin\alpha = n(1 \mp \beta\cos\alpha)\sin\alpha_D$ with the refractivity $n = c/c_D = \sqrt{\varepsilon_D \mu_D}\big/\sqrt{\varepsilon\mu}$; II) the reflection and transmission coefficients $R_{TM} = \dfrac{Z\cos\alpha - Z_D\cos\alpha_D}{Z\cos\alpha + Z_D\cos\alpha_D}$, $T_{TM} = \dfrac{2Z\cos\alpha}{Z\cos\alpha + Z_D\cos\alpha_D}$.

The Brewster angle α_B for which one has zero reflection coefficient ($R_{TM} = 0$) is calculated from the equation $Z\cos\alpha = Z_D\cos\alpha_D$, which, upon substituting the extended Snell relation, shapes into the fourth order transcendental equation $Z_D^2(n^2 - \sin^2\alpha_B) = Z^2 n^2 \cos^2\alpha_B (1 \mp \beta\cos\alpha_B)^2$. A change of variables $\xi = \cos\alpha_B \in [0,1)$ provides a compact closed form representation

$$n^2 - 1 + \xi^2 - \left(Z/Z_D\right)^2 n^2 \xi^2 (1 \mp \beta\xi)^2 = 0. \tag{3.7}$$

For $\forall n, \beta$ there is always one and only one root of the polynomial (3.7) that falls into the described range of ξ. For the special case $\mu_D = \mu$ one has $Z/Z_D = n$ and (3.7) simplifies as

$$n^4\beta^2\xi^4 \mp 2\beta n^4 \xi^3 + (n^4 - 1)\xi^2 + 1 - n^2 = 0. \tag{3.8}$$

A first order approximation in β requires $n^4\beta^2\xi^4 << 2\beta n^4\xi^3$, namely $\beta\xi << 2$, which can also be considered as roughly equivalent to $\beta < 0.2$. Under this condition (3.8) reduces to the cubic polynomial

$$\mp 2\beta n^4 \xi^3 + (n^4 - 1)\xi^2 + 1 - n^2 = 0. \tag{3.9}$$

(3.8) and (3.9) can always be solved uniquely for the described physical range of ξ by Cardano's analytical formulas for third and fourth order polynomials. The limiting case $\beta = 0$ yields the classical result $\xi = 1\big/\sqrt{n^2 + 1}$.

The total reflection mechanism $R_{TM} = 1$ is observed for $\alpha_D = \pi/2$ and the angle of total reflection α_{TR} is calculated from the relation $\sin\alpha_{TR} \pm n\beta\cos\alpha_{TR} = n$, which, by a change of variables $\varsigma = \sin\alpha_{TR} \in [0,1)$, can be shaped into the quadratic equation

$$\varsigma^2(1 + n^2\beta^2) - 2n\varsigma + n^2(1 - \beta^2) = 0. \tag{3.10}$$

For a physical solution the discriminant of (3.10) requires to be positive: $\Delta = 4n^2\beta^2[1 - n^2(1 - \beta^2)] \geq 0$,or equivalently,

$$n \leq 1\big/\sqrt{(1 - \beta^2)}. \tag{3.11}$$

Since $\beta \in [0,1)$, (3.11) can be satisfied for $n > 1$ when $\beta \neq 0$ as well. Under the condition (3.11) the angles of total reflection for $\vec{v} = +G\hat{x}_1$ and $\vec{v} = -G\hat{x}_1$ are found respectively, as

$$\alpha_{TR} = \sin^{-1}\left[\frac{n}{(1 + n^2\beta^2)}\left[1 - \beta\sqrt{1 - n^2(1 - \beta^2)}\right]\right] \quad , \quad \alpha_{TR} = \sin^{-1}\left[\frac{n}{(1 + n^2\beta^2)}\left[1 + \beta\sqrt{1 - n^2(1 - \beta^2)}\right]\right]. \tag{3.12}$$

Finally, the images of $\left(\vec{E}'_{sc}, \vec{H}'_{sc}\right)$ and $\left(\vec{E}'_D, \vec{H}'_D\right)$ in E-frame read

$$\vec{H}_{sc}(\vec{r};t) = \hat{x}_3 R_{TM} f(\hat{n}_{sc}\cdot\vec{r} - c_{sc}t) = \hat{x}_3 R_{TM}\, g(k\hat{n}_{sc}\cdot\vec{r} - \omega_{sc}t) \ , \ \vec{E}_{sc}(\vec{r};t) = Z\vec{H}_{sc}(\vec{r};t)\times\hat{n}_{sc} \tag{3.13}$$

$$\vec{H}_D(\vec{r};t) = \hat{x}_3 T_{TM} f(\hat{n}_D\cdot\vec{r} - c_{tr}t) = \hat{x}_3 T_{TM}\, g(k_D\hat{n}_D\cdot\vec{r} - \omega_{tr}t) \ , \ \vec{E}_D(\vec{r};t) = Z_D\vec{H}_D(\vec{r};t)\times\hat{n}_D \tag{3.14}$$

with $\hat{n}_{sc}\cdot\vec{r} = -x_1\cos\alpha + x_2\sin\alpha$, $\hat{n}_D\cdot\vec{r} = x_1\cos\alpha_D + x_2\sin\alpha_D$ and $c_{sc} = c(1 \mp \beta\cos\alpha)$, $c_{tr} = c_D(1 \pm \beta n\cos\alpha_D)$, $\omega_{sc} = \omega_{inc}(1 \mp 2\beta\cos\alpha)$, $\omega_{tr} = \omega_{inc}(1 \pm \beta n\cos\alpha_D)$.

5. References

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