Effect of Conductor Backing on Coupled Coplanar Waveguide with Finite Ground Planes

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Abstract

In this paper, quasi-static coupling characteristics of conductor-backed coupled coplanar waveguide (CB CCPW) with finite ground planes are presented. CAD-oriented closed-form expressions for calculating the even and odd mode quasistatic parameters of conductor-backed coupled coplanar structures are derived by using conformal mapping techniques (CMT). Numerical results of obtained analytic formulas are compared with those available in the literature for similar structures. According to the comparisons good agreement between the results are observed.

1. Introduction

Various transmission lines on elliptical and cylindrical dielectric materials are of great interest for a variety of applications, in particular for different non-conventional surfaces in aircraft, missiles, and mobile communication applications to feed printed and wrapped around antennas as expressed in [1-4]. Recently, considerable studies [1-10] have been devoted to compute the design parameters of these type transmission lines by using different methods. Analyses of mentioned structures by using most existing numerical approaches need more computation time and resources than CMT based on the quasi-TEM method [5]. Obtained fast and accurate closed form analytical expressions using CMT can be perfect alternative for using in CAD-oriented design tools.

Coplanar coupled transmission line structures are widely used for many applications including directional couplers, filters, and other devices in RF and microwave applications [11-13]. Conductor backing for CPW is frequently offered in order to improve the mechanical strength and the power-handling capability of coplanar structures whose substrate material is typically thin and fragile. In this work, CCPW proposed by Wen [14] as CB CCPW with finite substrate thickness and finite width lateral ground planes is designed and analyzed. To the author’s best knowledge, analysis of CB CPW with finite substrate thickness and finite lateral ground planes width was not realized. Via CMT the analysis of CCPW with finite substrate thickness and infinite lateral ground plane [11], CB CCPW with finite substrate thickness and infinite lateral ground plane configurations was investigated in [12-13]. In design step of practical applications, width of conductors used for the ground planes of the transmission line should be as small as possible as mentioned in [15]. This is because the conductors affect the line density seriously for the overall structure. Also, the analysis of elliptical and cylindrical CCPW without conductor backing was investigated in [8,10].

2. Analysis of CB CCPW with Finite Ground Planes

In Fig.1, configuration of CB CCPW with finite ground planes width to be investigated is shown. As seen in Fig.1, total width of the structure is $2d$, widths of metal strips are $(b-a)$, and spacing between both strips is $2a$. Side ground planes' widths are $(d-c)$ and spacing between each coupled strip and lateral ground plane is $(c-b)$. Thickness of dielectric substrate is $h$ and relative dielectric constant of the structure is $\varepsilon_r$. In order to be compatible through the analysis, dielectric substrate is assumed to be an isotropic and lossless medium. In the following analysis, all conductors are assumed to be infinitely thin and perfectly conducting. The boundaries between air and dielectric medium are supposed as magnetic wall. By means of calculating even and odd modes per unit length capacitances, the electrical parameters of the structure are obtained. Based on the partial capacitance approximation, the overall capacitances per unit length of the structure for each mode are considered to sum of the capacitance in free-space (upper air region) and dielectric layer. In analyses to be realized, $C_{C(d)}$ and $C_{C(a)}$ capacitances stands for the free-space capacitance and the dielectric layer capacitance, respectively. The analyses can be formulated as follows for even and odd modes.
2.1 Even-Mode

As demonstrated in Fig.1, for the even mode calculations a magnetic wall has been located along A-A' line. For calculation of the even mode capacitance, the partial capacitances $C_{0\text{ (even)}}$ and $C_{d\text{ (even)}}$ can be obtained using successive conformal transformation steps [15] as illustrated in Fig.2(a-c) and Fig.3(a-d), respectively. So air capacitance relations are given by

$$C_{\text{even}} = C_{0\text{ (even)}} + C_{d\text{ (even)}}$$  \hspace{1cm} (1)

Similarly, the capacitance of the dielectric layer is,

$$C_{d\text{ (even)}} = \varepsilon_0 \varepsilon_\text{r} \frac{K(k_{d\text{ (even)}})}{K(k'_{d\text{ (even)}})}$$  \hspace{1cm} (2)

where the related modulus can be expressed as

$$x_0 = \sqrt{\frac{\exp(\pi d / h) - 2}{\exp(\pi d / h) + 2}}, \quad 0 < \frac{d}{h} < 1$$  \hspace{1cm} (3)

where $x_a$, $x_b$ and $x_c$ can be found using the relations given below

$$F(\arcsin(x_c / x_h), x_h) = \frac{d}{d} K(x_h), \quad F(\arcsin(x_c / x_h), x_h) = \frac{b}{d} K(x_h), \quad F(\arcsin(x_c / x_h), x_h) = \frac{c}{d} K(x_h)$$  \hspace{1cm} (4)

with $F(\varphi,k)$ as the incomplete elliptic integral of the first kind in Jacobi’s notation. General parameters required for calculating the even mode dielectric layer capacitance are expressed below with following formulas

$$\frac{W_h}{H_h} = \frac{K(k_{d\text{ (even)}})}{K(k'_{d\text{ (even)}})} = \alpha_t, \quad \frac{W_d}{W_h} = \frac{F(\arcsin(x_d / h), x_d)}{K(k_{d\text{ (even)}})} = \beta_t, \quad \frac{Q_d}{W_d} = \frac{F(\arcsin(x_d / h), x_d)}{K(k_{d\text{ (even)}})} = \beta_t, \quad A = \frac{x_h^2 - x_c^2}{1 - x_h^2}, \quad B = \frac{x_a^2 - x_b^2}{x_d^2 - x_a^2}$$  \hspace{1cm} (5)

Fig. 1 Cross sectional view of CB CCPW with finite ground plane width.

Fig. 2 Conformal transformation steps to calculate the air capacitance for even mode (a-c) and odd mode (d-f)

Fig. 3 Conformal transformation steps to calculate the dielectric capacitance for even mode (a-d) and odd mode (a, b, c, f).
2.2 Odd-Mode

In order to obtain the capacitances for the odd mode, it is assumed that the symmetry line A-A’ in Fig. 1 is an electrical wall. The odd-mode capacitance considered as the sum of the capacitances of air and dielectric medium (C_{0,odd} and C_{d,odd}) can be evaluated using a sequence of conformal transformation steps shown in Fig. 2(d-f) and Fig. 3(a-b,e-f) similar to the even mode case. Then the capacitance equations for free space can be formulated as

\[ C_{\text{odd}} = C_{0,\text{odd}} + C_{d,\text{odd}}, \quad C_{0,\text{odd}} = \varepsilon_r \left( \frac{K(k_{0,\text{iu}})}{K(k_{0,\text{iu}})} + \frac{K(k_{0,\text{id}})}{K(k_{0,\text{id}})} \right), \quad C_{d,\text{odd}} = \varepsilon_r \left( \frac{K(k_{0,\text{id}})}{K(k_{0,\text{id}})} + \frac{K(k_{0,\text{id}})}{K(k_{0,\text{id}})} \right) \]  

where the related modulus are

\[ k_{\text{iu}} = \sqrt{\frac{c^2(b^2-a^2)}{b^2(c^2-a^2)}}, \quad k'_{\text{iu}} = \sqrt{1-k_{\text{iu}}^2}, \quad k_{\text{id}} = \sqrt{\frac{x_i^2(x_i^2-x_e^2)}{x_i^2(x_i^2-x_o^2)}}, \quad k'_{\text{id}} = \sqrt{1-k_{\text{id}}^2} \]  

(9)

The general parameters required for calculating the odd mode air capacitance and dielectric layer capacitance are expressed below using subscript i where i = a, d

\[ \frac{W_i}{H_i} = \frac{K(k_{0,i})}{K(k_{0,i})} = \alpha_i, \quad \frac{P_i}{W_i} = \frac{\varepsilon_i}{K(k_{0,i})} = \beta_i, \quad \frac{Q_i}{W_i} = \frac{\varepsilon_i}{K(k_{0,i})} = \beta_i \]  

relations of are given by

\[ A_i = 1 - \left( \frac{a}{c} \right)^2, \quad A_e = \frac{x_i^2-x_e^2}{x_i^2(1-x_e^2)}, \quad B_e = \frac{d^2(c^2-a^2)}{c^2(d^2-a^2)}, \quad B_e = \frac{x_i^2(x_i^2-x_e^2)}{x_i^2(x_i^2-x_o^2)} \]  

(11)

where xa, xb, xc, and xd are stated earlier in Equations 6(a-b) and 7(a-c). Lastly, in order to find k01 and k03 the following equations can be used

\[ \frac{\varepsilon_{\text{eff,even}}}{\varepsilon_{\text{eff,even}}} = \frac{C_{0,\text{even}} + C_{d,\text{even}}}{C_{0,\text{even}}} = \frac{1}{\varepsilon_{\text{eff,even}}} + C_{\text{coupling}} = \frac{Z_{\text{even}}}{Z_{\text{even}}} + \frac{Z_{\text{even}}}{Z_{\text{even}}} \]  

(13)

3. Numerical Results

Fig. 4 shows the even and odd mode quasistatic parameters (for both even and odd modes) of CB CCPW with finite ground planes (Fig. 1) as a function of the gap between the metal strips (a) and the half structure width (d). In Fig. 4, it can obviously be seen that variation of the gap between the metal strips and the half structure width changes significantly the characteristic impedance, the effective dielectric constant, and the coupling coefficient values in a wide range. As seen from Fig. 4, the numerical values of CB CCPW with finite ground planes approach to numerical values of the CB CCPW with infinite lateral ground planes with increasing the half structure width (d), and they overlap approximately the CB CCPW with infinite ground planes for d = 2000 µm.

![Fig. 4 (a) Variation of Z_0(even) and Z_0(odd) of CB CCPW with finite ground plane width versus (µm for defined dimensional parameters, (b) Variation of ε_{eff,even} and ε_{eff,odd} of CB CCPW with finite ground plane width versus (µm for defined dimensional parameters, (c) Variation of coupling coefficient of CB CCPW with finite ground plane width versus (µm for defined dimensional parameters.](image-url)
4. Conclusion

In this study, CB CCPW with a finite ground width configuration have been proposed and analysed using the CMT. The closed form analytical expressions have been obtained to calculate the effective dielectric constant, characteristic impedance (for even and odd mode), and coupling coefficient. The variations of quasi-static TEM parameters with respect to the dimensional parameters of CB CCPW proposed in this study has been investigated in Section 3. It is clearly seen that the work undertaken in this paper has been well supported with those available results for similar structures in the literature. Another crucial point of this study is the presentation of the fast and accurate analytical expressions for the above-mentioned transmission lines. In addition, the analytical expressions reported in this paper can be used in CAD programs for designing microwave-and millimetre-wave integrated circuits because of their highspeed-computation feature.

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6. References