Perturbation Analysis of Planar Periodic Leaky-Wave Antennas Fed by Cylindrical Surface-Waves

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Abstract

An accurate but simple perturbation analysis of planar cylindrical leaky-wave antennas (LWAs) is presented. Initially a printed LWA, realized by a grounded dielectric slab (GDS) and fed by an array of slots in the ground plane for surface wave (SW) excitation, is considered. Cylindrical leaky wave (LW) radiation by the $n = -1$ spatial harmonic and perturbation of the TM_0 SW mode of the slab is achieved by the addition of a printed metallic grating. Attenuation and phase constants for the cylindrical LW mode are derived along with the pointing angle and beamwidth as a function of frequency. Results are also in good agreement with full-wave simulations and measurements for some recently designed and fabricated structures.

1. Introduction

Leaky-wave antennas (LWAs) have sustained much interest and attention in the electromagnetics community. In particular, printed 2-D LWAs are desirable for their compatibility with other planar devices, low cost, and conical or pencil beam patterns that scan as a function of frequency. Applications include radar systems and communications. Generally, a 2-D LWA consists of a planar structure that can support cylindrical leaky waves (LWs), i.e., cylindrical waves with a complex wavenumber which leak energy into free space during their radial propagation [1]-[5]; the interface between air and the planar guiding surface defines the 2-D antenna aperture.

Such planar LWAs can be realized by a grounded dielectric slab (GDS) covered with circular metallic strip gratings defining an effective partially reflecting surface (PRS) as shown in Figs. 1 and 2. Practical feeding techniques include unidirectional and bi-directional surface wave (SW) launching from directive and non-directive slot sources embedded within the ground plane of the GDS [3]-[5] as respectively depicted in Figs. 2(a) and (b). By using these source configurations, one-sided and/or two-sided beam patterns can be realized in the far-field. Physically, the guided TM_0 SW mode is perturbed by the addition of the metallic grating and radiation is achieved by a fast $n = -1$ spatial harmonic. Using these excitation techniques, radial SWs and thus LWs can exist on such planar guiding surfaces respectively defining the antenna feeding network and radiation mechanism.

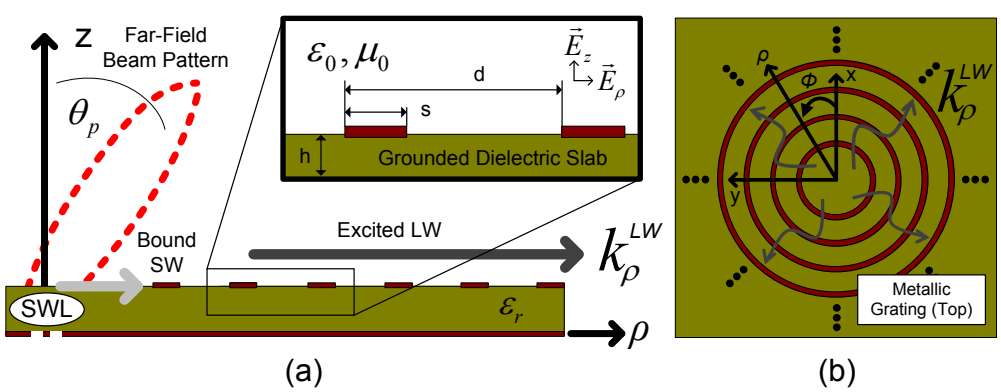

Figure 1: Antenna structure considered in this work. A slotted magnetic dipole acts as the SW source generating a radial TM_0 SW mode that is bound to GDS. The addition of periodic metallic gratings (defining an effective PRS) can excite cylindrical LWs.

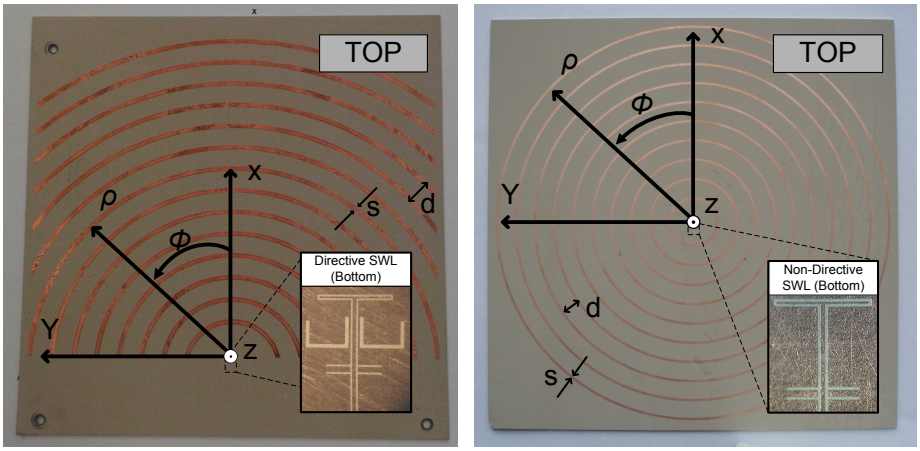

Figure 2: To excite the dominant TM_0 SW mode on the GDS, 50-Ω coplanar waveguide transmission lines can be utilized to feed the directive and non-directive SWLs. Such half wavelength slots in the ground plane have field distributions which can inductively couple to the dominant TM_0 SW mode. Substrate properties ($\epsilon_r = 10.2$, $h = 1.27$ mm and $\tan\delta = 0.0023$) are typically chosen such that more than 80% of the input power is coupled into the dominant TM_0 SW mode of the slab [7]. Moreover, for the operating frequency range of these LWA designs, substrate permittivities and thicknesses are typically chosen to support only one SW mode. (a): LWA with a directive SWL source for one-sided beam patterns in the far-field [4], and (b): bull-eye design with a non-directive SWL for bi-directional beam scanning [5]. In both cases SW propagation is normal to the gratings as considered in the presented perturbation analysis for cylindrical waves.

To further characterize such LW radiation this work provides a perturbation analysis of these planar LWA structures in an effort to characterize the complex radial wavenumber, k_ρ^{LW}. Initially, cylindrical SW fields generated from the slot sources are considered. Then the phase propagation constant of the radiating spatial harmonic, β_{-1}, and the modal attenuation constant, α, are determined from the excited TM_0 SW mode. Once β_{-1} and α are known the main beam direction, θ_p ($\approx \sin^{-1}\{\beta_{-1}/k_0)\}$, where $k_0 = 2\pi/\lambda_0$ is the free space wavenumber) can be calculated along with the beamwidth ($\approx 2\alpha/k_0\cos\theta_p$) [1]. In particular, the phase constant is fundamental when predicting the pointing angle, whereas the attenuation constant is a measure of power leaked per unit length and defines the effective aperture of the antenna [8].

2. Perturbation Analysis of the Planar Guiding Structure

For the investigated GDS, the cylindrical TM_0 SW fields are $[H_\phi, E_\rho, E_z]$ where,

$$H_\phi = \begin{cases} k_\rho \cos(k_{z1}z) H_1^{(2)}(k_\rho\rho) & 0 \le z \le h \\ k_\rho \cos(k_{z1}h) e^{-\alpha_0(z-h)} H_1^{(2)}(k_\rho\rho) & z \ge h \end{cases}, \tag{1}$$

$k_\rho = \beta_0$, $k_{z1} = (\epsilon_r k_0^2 - \beta_0^2)^{1/2}$, and $\alpha_0 = (\beta_0^2 - k_0^2)^{1/2}$. $H_1^{(2)}$ is the first order Hankel function of the second kind defining an outward traveling cylindrical wave, while β_0 and α_0 define TM_0 SW propagation and attenuation in the $\hat{\rho}$ and $\hat{z}$ directions, respectively. In addition, E_ρ and E_z can be obtained from H_ϕ by Maxwell`s Equations [9] and an $e^{+j\omega t}$ time variation is assumed throughout. For simplicity, perturbation of any TE fields excited by the magnetic dipole source are assumed negligible and azimuthal symmetry is chosen for the excited cylindrical fields.

Small perturbations of the fundamental SW mode are considered as the cylindrical waves are incident and normal [6] to the metal strips (with period 'd' and width '$s << d$') along the radial aperture. We may then assume that the SW fields remain unperturbed within the dielectric. Namely the tangential electric field below the interface, $z = h^-$, is equal to $E_\rho(h)$. However, the same field at $z = h^+$ on the metal strips must vanish. To support such discontinuities, we impose an infinite array of magnetic line sources (with period d) at the positions of the metal strips with a magnetic current density $\vec{m}_\phi = \hat{\phi}E_{\rho 0}$ (in [V/m]). As a periodic function $\vec{m}_\phi$ can be expressed as a Fourier series:

$$\vec{m}_\phi = -\hat{\rho}E_{\rho 0}\left\{ \frac{s}{d} + \sum \frac{s}{d}\,\mathrm{sinc}\left(\frac{n\pi s}{d}\right) e^{-j(\beta_n - \beta_0)\rho} \right\} \times \hat{z}, \tag{2}$$

where $E_{\rho 0} = (\alpha_0 k_\rho / j\omega\epsilon_0)\cos(k_{z1}h)H_1^{(2)}(k_\rho \rho)$ and $\beta_n = \beta_0 + 2\pi n/d$. The summation in Eq. (2) is for all positive and negative integer values of n (except $n = 0$) to account for the excitation of an infinite number of spatial harmonics as m_ϕ is periodic in the $\hat{\rho}$ direction. The $n = 0$ term is also intentionally removed from the summation to isolate the perturbation of the fundamental TM_0 SW mode. Now the radial directed electric field in the air region is given by the original $E_{\rho 0}$ field and the derived perturbation,

$$E_\rho(\rho, z) = E_{\rho 0}\left(1 - \frac{s}{d}\right)e^{-u_0(z-h)} - E_{\rho 0}\frac{s}{d}\sum \text{sinc}\left(\frac{n\pi s}{d}\right)e^{-j(\beta_n - \beta_0)\rho}e^{-u_n(z-h)}, \tag{3}$$

where $u_0 = (\beta_0^2 - k_0^2)^{1/2}$ and $u_n = (\beta_n^2 - k_0^2)^{1/2}$. It should be noted that as s tends to zero the metal strips vanish and the E_ρ field derived in Eq. (3) approaches the original $E_{\rho 0}$ field at $z = h^+$.

Moreover, the lost SW power along the aperture at $z = h$ (or the radiated LW power due to the perturbations) can be derived as the radiation per unit length 'l':

$$P_{Rad}\,/\,l = -2\pi\rho\, m_\phi(\rho, h)\, H_\phi^*(\rho, h)\,\frac{s}{d} = \frac{4jk_\rho\alpha_0^2\cos^2(k_{z1}h)}{\omega\epsilon_0}\frac{s}{d}\left\{\left(1 - \frac{s}{d}\right)/u_0^* - \frac{s}{d}\sum \text{sinc}\left(\frac{n\pi s}{d}\right)/u_n^*\right\} \tag{4}$$

with

$$H_\phi(\rho, h) = j\omega\epsilon_0 E_{\rho 0}\left\{\left(1 - \frac{s}{d}\right)/u_0 - \frac{s}{d}\sum \text{sinc}\left(\frac{n\pi s}{d}\right)/u_n\right\}. \tag{5}$$

This H_ϕ term can be derived from Eq. (3) through Maxwell's Equations. Furthermore, the radiated power of unit length '$P_{Rad}\,/\,l$' can be described as the outward-directed power leaked along the interface and is independent of the radial distance from the source [9]. In addition, if dielectric losses are neglected, the unperturbed SW power flowing along the aperture, P_{SW}, is purely real and can be obtained from [7] by analyzing an analogous 2-D guiding structure with no strips. Furthermore, the LW attenuation constant can now be obtained:

$$\alpha = \frac{\text{Re}\,\{P_{Rad}\,/\,l\}}{2P_{SW}}, \quad \text{since} \quad \frac{\partial P_{SW}}{\partial \rho} = -2\alpha P_{SW} = -P_{Rad}\,/\,l, \tag{6}$$

as the perturbed SW power along the air-dielectric interface can be described by $P_0 e^{-2\alpha\rho}$ (where P_0 is defined as the unperturbed SW power at the origin). Thus, the complex wavenumber describing cylindrical LW propagation along the radial aperture can now be stated for the $n = -1$ spatial harmonic,

$$k_\rho^{LW} = \beta_{-1} - j\alpha = \beta_0 - 2\pi/d - j\alpha. \tag{7}$$

3. Analysis and Experimental Verification

Figure 3 compares calculated pointing angles and beamwidths to recent measurements of a finite, one-sided LWA structure using a directive SWL source as illustrated Fig. 2(a). Agreement can be observed. In addition, deviations in the measurements and calculations are a likely result of substrate variations, dielectric losses (which are ignored in the presented perturbation analysis), and fabrication tolerances due to microfabrication. It should be noted that similar downward frequency shifts between measurements and calculations were previously reported by the authors for other dielectric based LWA designs fed by SWLs [4]-[6]. To further investigate these effects a parametric analysis was completed by computing θ_p and the beamwidth for $\epsilon_r = 10.2, 11.2$, and 12.2. Results suggest that for this LWA structure, the relative dielectric constant was ≈ 11.5. Figure 4 compares calculated LW phase and attenuation constants to HFSS full-wave simulations of a finite LWA structure. General agreement can be observed for the infinite case and this finite structure (8 periods) suggesting respectable predictions for k_ρ^{LW}.

4. Conclusion

A simple perturbation analysis of a planar periodic LWA antenna structure was presented. Specifically, by examining the cylindrical SW fields excited by a slot source embedded in the ground plane of a GDS and the perturbed fields along the aperture, due to an added grating configuration, the radiated power can be established. By comparing this radiated power to the SW power, the LW attenuation constant can be determined along with the far-field beamwidth. The presented analysis is verified by comparing numerical calculations to measurements and HFSS full-wave simulations of a finite, one-sided LWA structure driven by a directive SWL source. Good agreement can be observed suggesting a respectable derivation of the cylindrical LW propagation constant.

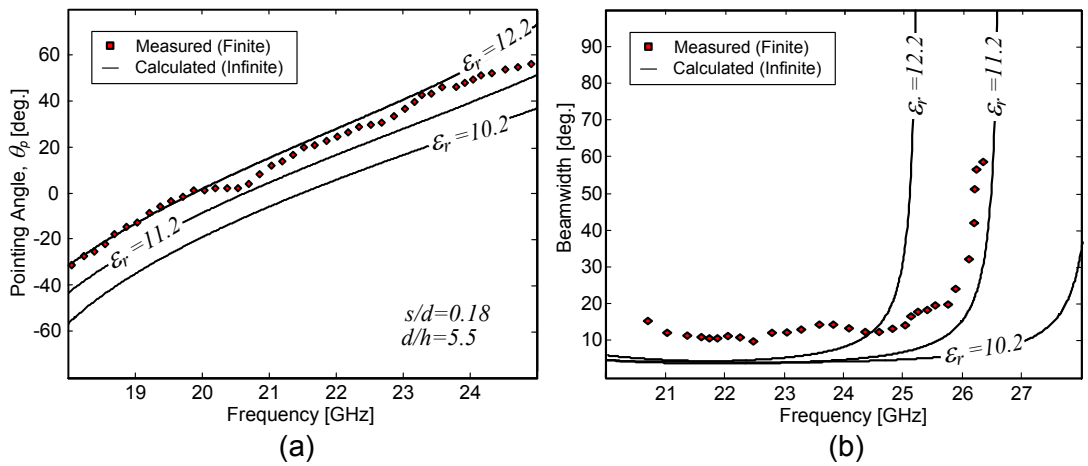

Figure 3: Pointing angle and beamwidth (in the x-z plane) for a one-sided LWA structure (as in Fig. 2(a)) with 10 circular strip gratings.

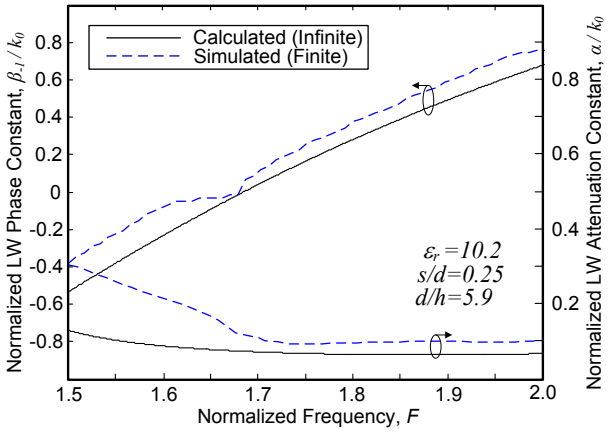

Figure 4: Calculated LW phase, β_{-1}, and attenuation constant, α, versus the normalized frequency, F $(= k_0 h\sqrt{\epsilon_r - 1})$. Results are also compared to full-wave simulations (along the $\hat{x}$-axis) of a finite LWA structure as in Fig. 2(a). Plotted wavenumbers normalized by k_0.

5. References

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