Aperture Field Estimation in Waveguide Circuits for Non-Sinusoidal, Periodic Excitation

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Abstract

In this paper an attempt has been made to analyze waveguide circuits for non-sinusoidal, periodic excitations. The excitation functions, taken as example, are trapezoidal and clipped sine wave in nature and the waveguide circuit, that has been considered, is two interacting thick inductive irises. The excitation functions have been expressed in terms of Fourier series. The circuit has been analyzed using Multiple Cavity Modeling Technique and finally the normalized electric field distribution at the apertures have been calculated and plotted.

1. Introduction

Waveguide and waveguide based components are used since World War II and they are still continued to be in use. These waveguide based elements have got wide applications in ground based, air borne and ship borne radars in various frequency bands ranging from 1 GHz to 1000 GHz. Modern radar system employs a number of waveguide components and circuit elements. Some of them are the irises, septum, windows, filters etc.

During post World War II, Lewin [1] and Harrington [2] gave an extensive material on the waveguides. Till then huge works have been carried out on different waveguide based circuits, like, diaphragms, steps, bends, filters, power divides, couplers etc, using variational technique, Method of Moment (MoM), Finite Difference Time Domain (FDTD) method etc. Vengadarajan [3] and Das [4] provided a brief literature survey on few of this circuits. However, all of these analyses assume a dominant mode excitation in a time harmonic form. A little attention was paid on the analysis of waveguide circuits under non-sinusoidal, periodic excitations like pulses, square wave, trapezoidal wave, clipped sine wave, triangular wave, saw tooth wave etc. Out of these the first four have huge technical significance in high power applications like radar. This is because in Continuous Wave Doppler radar when the gain and / or input of the amplifier are very high then the output may be clipped off resulting in a clipped sinusoidal wave. Depending on the clipping level it may be approximated as a trapezoidal or even a square wave. On the other hand pulses are used in Moving Target Indicator and Pulsed Doppler radar for its operation.

The results available in the literatures show that the value of network parameters of waveguide circuits changes with frequencies and their harmonics. This is due to changes in the electrical lengths of different circuit parameters and possible existence of higher order modes. Now a pulse / clipped sine / trapezoidal / square wave can be represented in terms of Fourier series and the circuit will behave differently for different Fourier components. So a circuit, say filter, which has been designed assuming a time harmonic excitation at fundamental frequency, will no more behave as expected when excited by a non-sinusoidal periodic wave.

Though the literature survey shows that few works have been carried out on transients and pulse propagation in waveguides [5], till the authors are concerned no literature is available on the frequency response of a waveguide based circuits under pulse, clipped sine wave and trapezoidal or square wave excitation.

In this paper, a methodology for the analysis of waveguide based passive circuits under non-sinusoidal periodic excitation has been presented. Interacting thick inductive diaphragms inside a waveguide has been taken as example for the waveguide circuit whereas clipped sinusoidal and trapezoidal wave form have been considered as excitation function. The cross sectional view of the proposed circuit is shown in figure 1(a). This particular circuit is being considered because when $L_1 = a$ or $L_2 = a$ the circuit reduces to a thick inductive diaphragm and when $L_1 = L_2 = a$ the circuit reduces to a waveguide. The circuit also forms a first order resonant iris band pass filter.

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2. Theory

The proposed structure has been analyzed using Multiple Cavity Modeling Technique (MCMT) [3-4, 6]. The details of different regions and fictitious magnetic currents (equivalent source to electric field) at the apertures are shown in figure 1(b). The structure has five regions, namely, two waveguide regions & three cavity regions and four interfacing apertures between them. The electric field distributed on apertures (and hence the fictitious magnetic currents) are unknown and to be determined. The ξ component of electric field at the ith aperture can be approximated in terms of unknown complex basis coefficients ($E_{pq}^{i,\xi}$) and known piecewise triangular basis function $\left(T\left(\xi\right)\right)$ as [3 - 4]

$$E_{\xi}^{i} = \sum_{p=1}^{M}\sum_{q=1}^{M} E_{pq}^{i,\xi} T_p\left(x\right) T_q\left(y\right) \quad \text{where } \xi = x/y \tag{1}$$

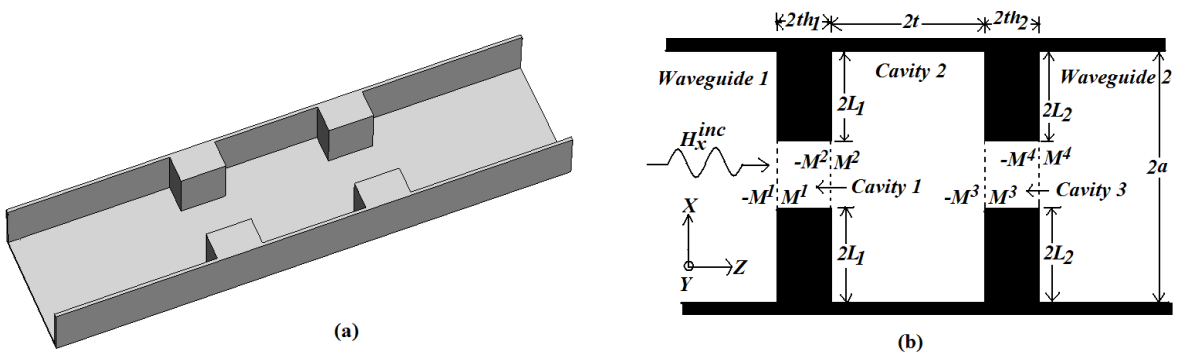

Figure 1. (a) Cross sectional view of two interacting thick inductive diaphragm inside a rectangular waveguide and (b) Details of regions and fictitious magnetic currents at the aperture of a thick interacting inductive diaphragm

The boundary conditions (continuity of tangential component of magnetic field across the dielectric – dielectric boundary) at the apertures can be obtained using superposition principle and can be given by

$$H_x^{wvg1}\left(M_x^1\right) + H_x^{cav1}\left(M_x^1\right) + H_x^{wvg1}\left(M_y^1\right) + H_x^{cav1}\left(M_y^1\right) - H_x^{cav1}\left(M_x^2\right) - H_x^{cav1}\left(M_y^2\right) = 2H_x^{inc} \tag{2}$$

$$H_y^{wvg1}\left(M_x^1\right) + H_y^{cav1}\left(M_x^1\right) + H_y^{wvg1}\left(M_y^1\right) + H_y^{cav1}\left(M_y^1\right) - H_y^{cav1}\left(M_x^2\right) - H_y^{cav1}\left(M_y^2\right) = 0 \tag{3}$$

$$\begin{aligned} &-H_{\xi}^{cav(i)}\left(M_x^i\right) - H_{\xi}^{cav(i)}\left(M_y^i\right) + H_{\xi}^{cav(i)}\left(M_x^{i+1}\right) + H_{\xi}^{cav(i+1)}\left(M_x^{i+1}\right) + \\ &H_{\xi}^{cav(i)}\left(M_y^{i+1}\right) + H_{\xi}^{cav(i+1)}\left(M_y^{i+1}\right) - H_{\xi}^{cav(i+1)}\left(M_x^{i+2}\right) - H_{\xi}^{cav(i+1)}\left(M_y^{i+2}\right) = 0 \end{aligned} \quad \text{where i = 1, 2} \tag{4}$$

$$-H_{\xi}^{cav3}\left(M_x^3\right) - H_{\xi}^{cav3}\left(M_y^3\right) + H_{\xi}^{cav3}\left(M_x^4\right) + H_{\xi}^{wvg2}\left(M_x^4\right) + H_{\xi}^{cav3}\left(M_y^4\right) + H_{\xi}^{wvg1}\left(M_y^4\right) = 0 \tag{5}$$

The magnetic field scattered inside the cavity region due to the fictitious magnetic currents at the apertures can be determined by using cavity Green's function of the electric vector potential. The cavity Green's function has been derived by solving the Helmholtz equation for the electric vector potential for unit magnetic current source [3]. The scattered magnetic fields in the waveguide region due to the presence of the fictitious transverse magnetic current densities are solved by rigorous mode matching technique [2 - 3]. The incident magnetic fields are assumed to be X-directed and can be obtained by solving wave equations [7]. The amplitude A_{mn} and B_{mn} in the expressions of the incident field [7] can be obtained from the Fourier coefficients of the excitation function. If τ be the rise time and T be the time period then the Fourier coefficients for trapezoidal and clipped sine wave respectively can be given by (for both the cases a_0 and a_n is zero)

$$b_{n,Trapezoidal} = \left\{4\sin\left(n\omega\tau\right)\left[1-\cos\left(n\omega T/2\right)\right]\right\} \Big/ \left(n^2\omega^2 T\tau\right) \tag{6}$$

$$b_{n,Clipped\ Sinusoidal} = \frac{2}{T}\left\{\frac{A\tau}{2} sin\,c\left[\omega\tau\left(n-1\right)\right]\left\{1+2\,cos\left[\frac{\omega T}{2}\left(n-1\right)\right]+cos\left[\omega T\left(n-1\right)\right]\right\}-\frac{A\tau}{2} sin\,c\left[\omega\tau\left(n+1\right)\right]\times \right.$$
$$\left.\left\{1+2\,cos\left[\frac{\omega T}{2}\left(n+1\right)\right]+cos\left[\omega T\left(n+1\right)\right]\right\}+\frac{2}{n\omega}cos\left(n\omega\tau\right)\left[1-cos\left(\frac{n\omega T}{2}\right)\right]\right\} \tag{7}$$

Solving the boundary conditions, provided in equation [2-5] using Galerkin's specialization of Method of Moment the aperture electric field distribution can be obtained.

3. Results and Discussion

The normalized electric field distribution at the input and output apertures of thick interacting identical inductive irises are shown in figures [2-3] respectively for trapezoidal and clipped sinusoidal excitations. VSWR data obtained from literature [8], measurement [8] and aperture electric field distribution, obtained using present method, also have been plotted as a function of frequency in figure 4 and compared to validate the analysis. The results are in good agreement.

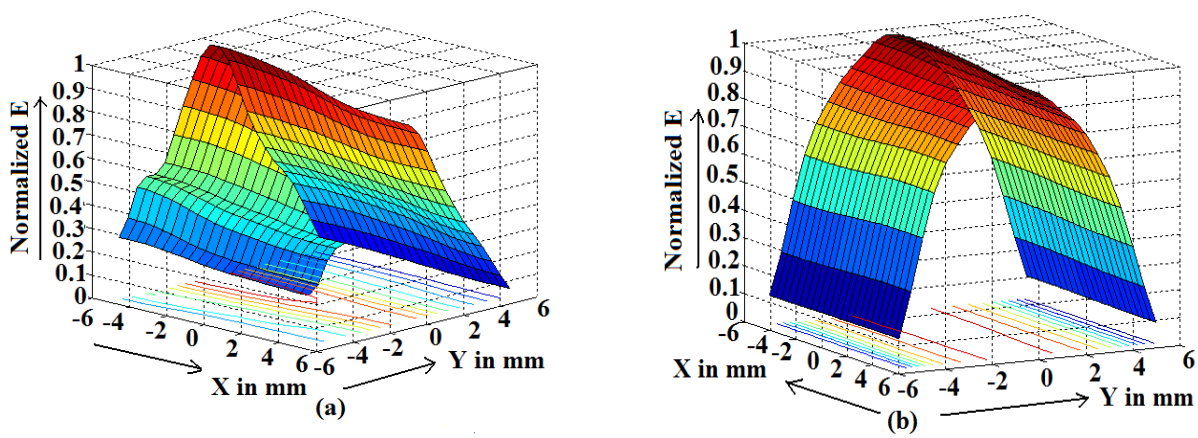

Figure 2. Normalized electric field distribution in (a) aperture 1 and (b) aperture 4 for trapezoidal wave excitation

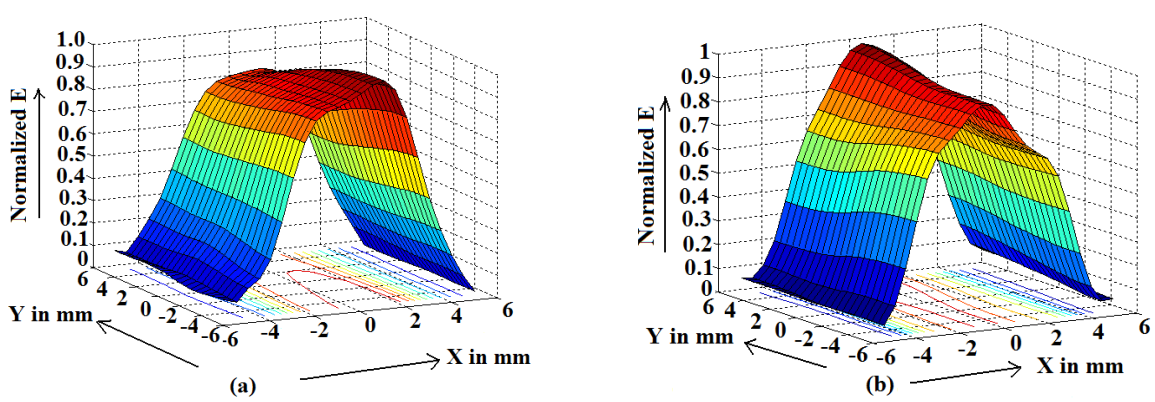

Figure 3. Normalized electric field distribution in (a) aperture 1 and (b) aperture 4 for clipped sine wave excitation

The figures [2-3] show that the aperture electric field distributions are not purely sinusoidal. This is due to the presence of the higher order modes at the apertures. In the result, only the electric field distributions at apertures 1 and 4 have been shown instead of different network parameters. This is because once the aperture distribution is known the other network parameters can be calculated. The electric field distribution in the rest of the apertures can also be plotted similarly.

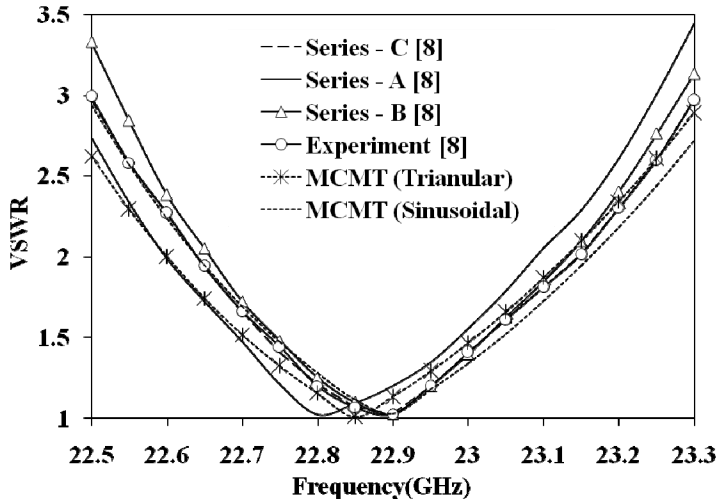

Figure 4. Comparison of the VSWR data obtained from literature [8], measurement [8] and aperture electric field distribution, obtained using present method

4. Conclusion

The paper presents a methodology to find the electric field distribution at the apertures of thick identical inductive irises. The methodology is general and can be extended to other waveguide circuits excited by any deterministic and periodic waveform that can be extended in Fourier series.

5. Acknowledgment

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6. References

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