# Reflection and Transmission of Full-Vector X-Waves Normally Incident on Dielectric Half Spaces


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## Abstract

The reflection and transmission of full-vector X-Waves incident normally on a planar interface between two lossless dielectric half-spaces are investigated. Full-vector X-Waves are obtained by superimposing transverse electric and magnetic polarization components, which are derived from the scalar X-Wave solution. The analysis of transmission and reflection is carried out via a straightforward but yet effective method: First, the X-Wave is decomposed into vector Bessel beams via the Bessel-Fourier transform. Then, the reflection and transmission coefficients of the beams are obtained in the spectral domain. Finally, the transmitted and reflected X-Waves are obtained via the inverse Bessel-Fourier transform carried out on the X-wave spectrum weighted with the corresponding coefficient.


## 1. Introduction

X-Waves were first introduced as wide-band Localized Wave (LW) solutions to the scalar wave equation in [1]. Since they are free from backward-propagating components, they are more appealing for practical realization than the other types of LWs, and thus have been extensively studied in the literature (see [2] for a historical review). For constructing electromagnetic counterparts of scalar X-Waves, single polarization (mostly transverse-electric (TE)-polarization) type of localized solutions to the vector wave equation were considered (see, [3-9] and references therein). Full-vector X-Waves obtained by superimposing localized TE and transverse magnetic (TM) polarization components were first introduced in [10] and their unusual propagation characteristics were revealed. In this work, we study the reflection and transmission characteristics of the full-vector X-Waves incident normally on a planar interface between to lossless dielectric half-spaces.

The reflection and transmission of obliquely incident TE LWs were first investigated in [4]. The study considered only a two-dimensional representation of LWs with backward-propagating components and concluded that an obliquely incident TE LW will disintegrate upon passing through a lossless dielectric interface. These results were later shown to hold true for X-Waves as well [5], yet X-Waves maintain their localization upon normal incidence on lossless and non-dispersive dielectric interfaces. In [6], a pulsed plane-wave decomposition approach was used to investigate reflection and transmission of X-waves normally incident on lossy and dispersive dielectric interfaces. In this work, a straightforward but yet effective method is used to study reflection and transmission of X-waves normally incident on a planar interface between two lossless dielectric half spaces. The full-vector X-Wave is represented in terms of weighted vector Bessel beams [11] (i.e., Bessel-Fourier transform), which is a more natural decomposition than the plane-wave-based transformation given the cylindrical nature of LWs in general. Note that all Bessel beams that constitute the X-Wave are normally incident on the interface, thus the pertinent reflection and transmission coefficients can be easily obtained. The expressions of the reflected and transmitted vector X-Waves are obtained via the inverse Bessel-Fourier transform applied to the product of the reflection and transmission coefficients and the X-Wave's spectrum.


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## 2. Full-Vector X-Waves

In this section, expressions of the full-vector X-Waves are derived from the scalar solution of the wave equation. In principle, X-Wave solutions to the scalar wave equation are obtained by integrating Bessel beam expressions weighted by an appropriate spectrum that preserves the necessary undistorted propagation condition $\omega = Vk_z$ and the dispersion relation $k_\rho^2 + k_z^2 = k^2$ over the spectral variables $k_\rho$, $k_z$ and $k = \omega / c$. Here, $V$ is the velocity of the X-Wave, $\omega$ is the angular frequency, $k$ is the magnitude of the wave vector $\mathbf{k}$ with the components $(k_\rho, k_z)$ in the transverse and longitudinal directions, respectively, and $c$ is the speed of light in free-space. The 'standard' X-Wave spectrum [2] is given by

$$\tilde{\Psi}_n(k_\rho, k_z, \omega) = 2^n \left(\frac{\omega}{V}\right)^m e^{-a\omega} \delta\left(k_z - \frac{\omega}{V}\right) \delta\left(k_\rho^2 - \left[\frac{\omega^2}{c^2} - k_z^2\right]\right), \tag{1}$$

where *m* is the order of the X-Wave and *a* is an arbitrary positive real constant. Taking the Bessel-Fourier transform of (1) yields $\Psi_n(\rho, \phi, z, t)$, an X-wave propagating in *z*-direction, viz.

$$\Psi_n(\rho, \phi, z, t) = \int_{-\infty}^{\infty} d\omega \int_0^{\infty} dk_z \int_0^{\infty} dk_\rho \tilde{\Psi}_n(k_\rho, k_z, \omega) J_n(k_\rho \rho) e^{in\phi} e^{i(k_z z - \omega t)}, \tag{2}$$

where $J_n(z)$ is the ordinary Bessel function of first kind and order *n*. The generalized *m*-th order scalar X-Wave is obtained by carrying out the integral (2) analytically using formula (6.621.1) in [10], as

$$\Psi_n(\rho, \phi, \zeta) = e^{in\phi} \frac{(\gamma\rho)^n}{\tau^{1+m+n}} \frac{(m+n)!}{n!} {}_2F_1\left(\frac{1+m+n}{2}, \frac{2+m+n}{2}; 1+n; -\eta\right), \tag{3}$$

where $\gamma = \sqrt{(V/c)^2 - 1}$, $\tau = aV - i\zeta$, $\zeta = z - Vt$, $\eta = (\gamma\rho/\tau)^2$, and ${}_2F_1(a, b; c; z)$ is the Gauss hypergeometric function [12].

X-Wave solution to the vector wave equation can be derived using single-component Hertz vector potentials as

$$\begin{aligned} \mathbf{E} &= \nabla(\nabla \cdot \Pi_e) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Pi_e - \mu_0 \nabla \times \left(\frac{\partial}{\partial t}\Pi_h\right), \\ \mathbf{H} &= \epsilon_0 \nabla \times \left(\frac{\partial}{\partial t}\Pi_e\right) + \nabla(\nabla \cdot \Pi_h) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Pi_h, \end{aligned} \tag{4}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field vectors, $\Pi_e = A_e \Psi_n(\rho, \phi, \zeta)\hat{\mathbf{z}}$ and $\Pi_h = A_h \Psi_n(\rho, \phi, \zeta)\hat{\mathbf{z}}$ are the electric and magnetic Hertz vector potentials, $A_e$ and $A_h$ are arbitrary complex amplitudes, $\hat{\mathbf{z}}$ is the unit vector in the *z*-direction, and $\mu_0$ and $\epsilon_0$ are the free-space permeability and permittivity. The expressions (4) thus describe the generalized electromagnetic full-vector X-Wave propagating in the *z*-direction in free-space. In previous studies on vector X-Waves, only the real part of the Hertz vector potential (with a real-valued amplitude) is used to derive the vector X-Wave expressions, as it exhibits a symmetric localization in space and time around the centroid. In this work, we keep the complex vector potentials (with complex-valued amplitudes) to weight the contributions of the real and the imaginary parts in the scalar solution, and finally retain only the real part of the electric and magnetic fields.

## 3. Reflection and Transmission at a Planar Interface

Consider a vector X-Wave propagating in *z*-direction and incident normally on a planar interface at $z = 0$ between the two half-spaces $M_0$ and $M_1$. Here $M_0$, $-\infty < z \le 0^-$, represents the free-space, while $M_1$, $0^+ \le z < \infty$, represents a lossless dielectric medium with relative permittivity $\epsilon_1$ and relative permeability $\mu_1$. Components of the wave vector in $M_1, \mathbf{k}_1$, are $(k_{\rho 1}, k_{z1})$. The analysis is carried out by treating each polarization of the vector wave separately. To obtain the reflected and transmitted wave expressions, we expand the X-Wave in terms of vector Bessel beams, obtain the reflection and transmission coefficients of the vector Bessel beams, then integrate the spectral function (1) weighted with these coefficients over $\omega$. For a vector Bessel beam, enforcing the continuity of the tangential electric and magnetic fields at the interface $(z=0)$ yields

$$\gamma_e = \frac{\mu_1 k_z - \mu_0 k_{z1}}{\mu_1 k_z + \mu_0 k_{z1}},\ \gamma_h = \frac{\epsilon_1 k_z - \epsilon_0 k_{z1}}{\epsilon_1 k_z + \epsilon_0 k_{z1}},\ \tau_e = \frac{2\mu_0 k_z}{\mu_1 k_z + \mu_0 k_{z1}},\ \tau_h = \frac{2\epsilon_0 k_z}{\epsilon_1 k_z + \epsilon_0 k_{z1}}, \tag{5}$$

where $\gamma_e$ and $\gamma_h$ are the reflection coefficients of the TE and TM polarizations, respectively, and $\tau_e$ and $\tau_h$ are the corresponding transmission coefficients. The Dirac-delta functions in (1) dictate the values of the wave vector components in $M_0$ to be $k_\rho = k\sqrt{1-q^2}$ and $k_z = kq$, with $V = c/q$. As a result of the boundary condition at the interface, the transverse wave vector component is preserved in the reflected and transmitted fields and the longitudinal wave vector component in $M_1$ is directly prescribed as $k_{z1} = k\sqrt{n_1^2-(1-q^2)}$, with $n_1 = \sqrt{(\epsilon_1\mu_1)/(\epsilon_0\mu_0)}$. Figure 1 shows a plot of the azimuthal electric field component of a full-vector X-Wave as it passes through a dielectric interface with $\mu_1 = \mu_0$ and $\epsilon_1 = 2\epsilon_0$ at the time instances $t = \{-T, 0, T\}$ ,with $T = 2.5\times10^{-15}s$. The X-Wave is of zero-order (*m* = 0), has an azimuthal dependence *n* = 1, with the parameter $a = 2\times10^{-16}s$ and a peak velocity $V = 1.01c$. The figure also reveals the new characteristic of change in the centroid spot-size and longitudinal length.

It should be noted here that this simple but effective way of computing reflected and transmitted X-waves could easily be adopted for X-waves in planar layered media. Results of these studies will be presented at the conference.

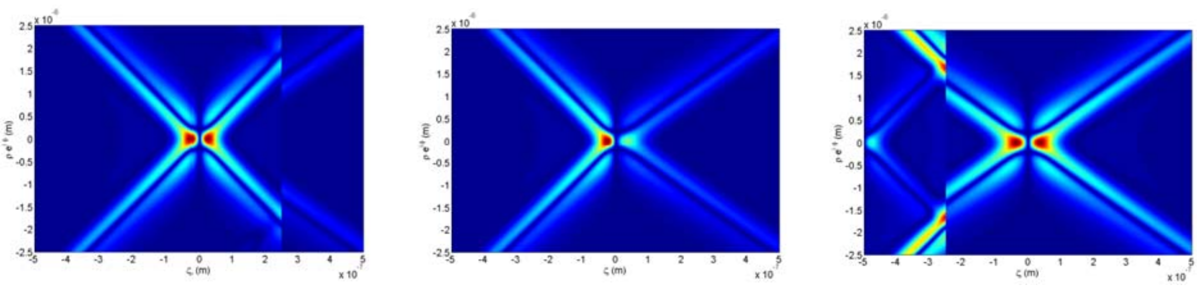

**Figure 1** Reflection and transmission of a full-vector X-Wave as it passes through a dielectric interface at time instances $t = \{-T, 0, T\}$ with $T = 2.5\times10^{-16}s$ showing the change in its spot-size and longitudinal length.

## 4. Conclusion

A straightforward method for analyzing the reflection and transmission of full-vector X-Waves normally incident on an interface between two lossless dielectric half spaces is described. The method is based on the decomposition of X-waves in terms of Bessel beams, which describes a more natural decomposition with direct physical interpretations than a plane-wave based transformation. This method can easily be extended for analyzing scattering behavior of vector LWs, such as oblique incidence, reflection, and transmission at interfaces between lossy and dispersive media.

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