A Predictor-Corrector Scheme for Solving the Volterra Integral Equation

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Abstract

The occurrence of late time instabilities is a common problem of almost all time marching methods developed for solving time domain integral equations. Implicit marching algorithms are now considered stable with various efforts that have been developed for removing low and high frequency instabilities. On the other hand, literature on stabilizing explicit schemes, which might be considered more efficient since they do not require a matrix inversion at each time step, is practically non-existent. In this work, a stable but still explicit predictor-corrector scheme is proposed for solving the Volterra integral equation and its efficacy is verified numerically.

1. Introduction

In this work, we consider the Volterra integral equation technique [1] for characterizing transient wave interactions on inhomogeneous arbitrarily-shaped dielectric bodies. The technique updates the present-time total electric fields using the present-time incident electric field and the explicitly discretized retarded scattered field [2]. Contrary to implicit marching schemes, the Volterra integral equation technique does not require the inversion of a "right-now" interaction matrix at every time step; hence, it is more efficient albeit less stable. To obtain stable solutions, particular consideration is usually given to the numerical approximation of the Green's tensor operator. In addition, the accuracy of the solution highly depends on the characteristics of the interpolating functions utilized for evaluating the retarded field values that fall in between the surfaces of the spheres of integration from the already calculated nodal values on the uniform time mesh. However, it is usually observed that while a particular combination of these schemes, for discretizing the Green's tensor and evaluating the temporal interpolations, can produce accurate results; it typically can also have an adverse effect on the late time stability of the solution.

A predictor-corrector scheme for solving the Volterra integral equation, which builds upon the discretization technique presented in [2], is proposed here as a stable alternative to the fully-explicit scheme. To update present electric fields, the explicit scheme is initially executed for the causal values up to and including previous time step; thus giving the predictor step. At the corrector step, electric field values from the predictor step are employed to evaluate the corrected present field values; with the explicit scheme now replaced by an implicit one. It is noted; the latter step still does not involve a matrix inversion. Algorithmically, the proposed scheme is very similar to the traditional predictor-corrector methods developed for solving ordinary differential equations [3]. Numerical results, which compare the new predictor-corrector scheme's accuracy and stability to those of the former fully-explicit formulation, will be presented at the conference.

2. References

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978-1-4244-6051-9/11/$26.00 ©2011 IEEE