Scattering of a Gaussian Beam from the End-Face of a Waveguide System

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Abstract

The scattering of a Gaussian beam from the end-face of a waveguide system composed of a large number of cores is treated by the volume integral equation for the electric field and the first term of a perturbation solution for TE and TM wave incidence is analytically derived. The far scattered field does not almost depend on the polarization of an incident wave. Some numerical examples are shown.

1. Introduction

An image fiber is composed of a large number of cores embedded in a single cladding, which is used to transmit directly an optical image. By illuminating the end-face of an image fiber with a laser beam a diffraction pattern reflecting the arrangement of cores can be simply observed. We can see experimentally that the diffraction pattern does not almost depend on the polarization of the laser beam. In this paper we deal with the problem from a theoretical point of view.

The reflection and transmission of a guided mode by the cut-end of a dielectric slab waveguide and the coupling of a beam wave to a dielectric slab waveguide have been treated by the boundary integral equation[1],[2]. In their papers the approximate form of the Green's function in a waveguide region has been used and the integral equation has been solved numerically. The analytical treatment of the boundary integral equation is very difficult.

The scattering of an electromagnetic wave from a dielectric body has been treated by the volume integral equation for the electric field[3]. The scattered field is the field radiated from the electric polarization induced in a dielectric body by the incident field and the physical image is very clear. The solution of the volume integral equation can be easily expanded into a perturbation series and each term of the series can be derived analytically. The analytical representation gives a deep understanding of the scattering properties of a dielectric body.

The scattering of a plane wave from the end-face of a waveguide system composed of a large number of cores has been treated by the volume integral equation for the electric field and it has been shown that the far scattered field does not almost depend on the polarization of an incident wave[4].

In this paper the scattering of a Gaussian beam from the end-face of a two-dimensional waveguide system is treated by the volume integral equation and the first order term of a perturbation solution for TE and TM wave incidence is derived analytically. The dependence of the scattering pattern on the polarization of an incident wave is clarified.

2. Formulation of the problem

We consider the scattering of a Gaussian beam from the end-face of a two-dimensional waveguide system composed of a large number of cores and a single cladding as shown in Fig.1. The waveguide system is a model of an image fiber. The plane of incidence is xz-plane and the polarization of an incident beam is arbitrary. We assume that a beam source is located at $z_1 = L$. For the total electric field $\boldsymbol{E}$ we can write

$$\nabla \times \nabla \times \boldsymbol{E} - k_0^2 n^2(\boldsymbol{r})\boldsymbol{E} = 0 \tag{1}$$

Here k_0 is the wave number in the air region and $n(\boldsymbol{r})$ is the refractive index distribution for the whole region where $\boldsymbol{r}$ is a position vector in the xz-plane. The total field $\boldsymbol{E}$ is divided as follows:

$$\boldsymbol{E} = \begin{array}{ll} \boldsymbol{E}^i + \boldsymbol{E}^r + \boldsymbol{E}^s, & z > 0 \\ \boldsymbol{E}^t + \boldsymbol{E}^s, & z < 0 \end{array} \tag{2}$$

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where $\boldsymbol{E}^i$ is the incident field and $\boldsymbol{E}^r$ and $\boldsymbol{E}^t$ are the reflected and transmitted fields by a plane surface between the air and the cladding, respectively. $\boldsymbol{E}^s$ is the scattered field. By substituting Eq.(2) into Eq.(1) and using the dyadic Green's function we obtain

$$\boldsymbol{E}^s(\boldsymbol{r}) = k_0^2 \delta n^2 \sum_m \int_{S_m} \overline{\boldsymbol{G}}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{E}^t(\boldsymbol{r}') d\boldsymbol{r}' + k_0^2 \delta n^2 \sum_m \int_{S_m} \overline{\boldsymbol{G}}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{E}^s(\boldsymbol{r}') d\boldsymbol{r}' \tag{3}$$

where S_m is a volume region occupied by the m-th core and

$$\delta n^2 = n_1^2 - n_2^2 \tag{4}$$

Here n_1 and n_2 are the refractive indices of core and cladding, respectively. $\overline{\boldsymbol{G}}$ is the dyadic Green's function for a two-layered medium composed of the air and the cladding,

$$\overline{\boldsymbol{G}} = \boldsymbol{G}^{(x)} \overline{x} + \boldsymbol{G}^{(y)} \overline{y} + \boldsymbol{G}^{(z)} \overline{z} \tag{5}$$

$\overline{i}$ $(i = x, y, z)$ is the unit vector and $\boldsymbol{G}^{(i)}$ is the vector Green's function with a source pointed in the i-direction. We can easily derive the vector Green's functions in a spectrum domain[5]. $\boldsymbol{G}^{(i)}$ needed to calculate the scattered field in the air region is listed in Appendix. In this paper only the first term of the right-hand side of Eq.(3) is calculated, which is the radiated field from the electric polarization induced in the cores by the transmitted field.

3. Far Scattered Field

3.1 Case of TE Wave Incidence

The incident beam field is expressed as

$$\boldsymbol{E}^i = \frac{\overline{y}}{2\pi} \int_{-\infty}^{\infty} A(\xi) e^{-j\zeta L + jk_0\{x\cos(\vartheta_i+\vartheta_1)+z\sin(\vartheta_i+\vartheta_1)\}} d\xi \tag{6}$$

where

$$A(\xi) = \sqrt{2\pi} w_0 e^{-\frac{(w_0\xi)^2}{2}}, \qquad \zeta = \sqrt{k_0^2 - \xi^2} \tag{7}$$

w_0 is a spot size at the beam waist, ϑ_i is the angle of incidence of the Gaussian beam and $\vartheta_1 = \tan^{-1}(\xi/\zeta)$. Then the transmitted beam field is

$$\boldsymbol{E}^t = \frac{\overline{y}}{2\pi} \int_{-\infty}^{\infty} A(\xi) T_\perp(\vartheta_i + \vartheta_1) e^{-j\zeta L + jk_0 n_2\{x\cos(\vartheta_t+\vartheta_2)+z\sin(\vartheta_t+\vartheta_2)\}} d\xi \tag{8}$$

where

$$n_2 \cos(\vartheta_t + \vartheta_2) = \cos(\vartheta_i + \vartheta_1) \tag{9}$$

ϑ_t is the angle of transmission of the Gaussian beam. $T_\perp$ is the transmission coefficient,

$$T_\perp(\vartheta_i) = 2\sin\vartheta_i / (\sin\vartheta_i + n_2 \sin\vartheta_t) \tag{10}$$

By substituting Eq.(8) into Eq.(3) we have

$$\begin{aligned} \boldsymbol{E}^s(\boldsymbol{r}) = \; & \tfrac{k_0^2 \delta n^2}{j4\pi^2} \textstyle\sum_m \int_{S_m} \int \int_{-\infty}^{\infty} \frac{A(\xi) T_\perp(\vartheta_i+\vartheta_1)}{q_0+q_2} \\ & \times e^{-j\zeta L - jp(x-x') - jq_0 z + jq_2 z' + jk_0 n_2\{x'\cos(\vartheta_t+\vartheta_2)+z'\sin(\vartheta_t+\vartheta_2)\}} d\xi dp dx' dz' \end{aligned} \tag{11}$$

where

$$q_0 = \sqrt{k_0^2 - p^2}, \qquad q_2 = \sqrt{k_0^2 n_2^2 - p^2} \tag{12}$$

The far scattered field is

$$\boldsymbol{E}^s(\boldsymbol{r}) \sim \sqrt{\frac{2\pi}{k_0 r}} e^{-jk_0 r + j\frac{\pi}{4}} F_\perp(\vartheta) \overline{y}, \quad r \to \infty \tag{13}$$

where $F_\perp$ is the scattering amplitude,

$$F_\perp(\vartheta) = -\frac{k_0^2 \delta n^2}{2\pi} \frac{T_\perp(\vartheta_i)}{q_0 + q_2} \frac{k_0 \sin\vartheta}{q_2 + k_0 n_2 \sin\vartheta_t} \sum_m \tilde{\phi}_{a_m}(k_0 \cos\vartheta + k_0 \cos\vartheta_i) g_m \tag{14}$$

g_m is the amplitude of the incident beam at $x = x_m$ and $z = 0$,

$$g_m = \frac{w_0}{w} \exp\left(-\frac{1}{2} \frac{(x_m \sin\vartheta_i)^2}{w^2} - jk_0 L\right) \tag{15}$$

where

$$w = \sqrt{w_0^2 - j\frac{L - x_m \cos\vartheta_i}{k_0}} \tag{16}$$

$\tilde{\phi}_{a_m}$ is the Fourier transform of the m-th core region,

$$\tilde{\phi}_{a_m} = 2\frac{\sin pa_m}{p} e^{jpx_m} \tag{17}$$

3.2 Case of TM Wave Incidence

The incident beam field is expressed as

$$\boldsymbol{E}^i = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\xi) \{-\overline{x} \sin(\vartheta_i + \vartheta_1) + \overline{z} \cos(\vartheta_i + \vartheta_1)\} e^{-j\zeta L + jk_0\{x \cos(\vartheta_i + \vartheta_1) + z \sin(\vartheta_i + \vartheta_1)\}} d\xi \tag{18}$$

Then the transmitted field is

$$\boldsymbol{E}^t = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\xi) T_\parallel(\vartheta_i + \vartheta_1) \{-\overline{x} \sin(\vartheta_t + \vartheta_2) + \overline{z} \cos(\vartheta_t + \vartheta_2)\} e^{-j\zeta L + jk_0 n_2\{x \cos(\vartheta_t + \vartheta_2) + z \sin(\vartheta_t + \vartheta_2)\}} d\xi \tag{19}$$

where $T_\parallel$ is the transmission coefficient

$$T_\parallel(\vartheta_i) = 2n_2 \sin\vartheta_i / (n_2 \sin\vartheta_i + \sin\vartheta_t) \tag{20}$$

The far scattered field is

$$\boldsymbol{E}^s(\boldsymbol{r}) \sim \sqrt{\frac{2\pi}{k_0 r}} e^{-jk_0 r + j\frac{\pi}{4}} F_\parallel(\vartheta) \overline{\vartheta}, \quad r \to \infty \tag{21}$$

where

$$F_\parallel(\vartheta) = -\frac{k_0^2 \delta n^2}{2\pi} \frac{q_2 \sin\vartheta_t + p \cos\vartheta_t}{k_0 n_2 (n_2^2 q_0 + q_2)} \frac{k_0 \sin\vartheta T_\parallel(\vartheta_i)}{q_2 + k_0 n_2 \sin\vartheta_t} \sum_m \tilde{\phi}_{a_m}(k_0 \cos\vartheta + k_0 \cos\vartheta_i) g_m \tag{22}$$

By putting $g_m = 1$ we can obtain the results for TE and TM plane wave incidence from Eqs.(14) and (22), respectively.

4. Numerical Examples

The scattering pattern by a single core for TE and TM wave incidence is shown in Fig.2. For numerical calculations the wavelength λ, the core radius a and the refractive indices of the core and the cladding n_1 and n_2 are chosen as $\lambda = 0.633\mu m$, $a = 2.5\mu m$, $n_1 = 1.457$ and $n_2 = 1.437$. The angle of incidence ϑ_i is chosen as the Brewster angle $\vartheta_i = 34.83(deg)$. We assume that the spot size of the beam is $w_0 = 10\mu m$ and the beam waist is positioned at $L = 0$. Although the values of the scattering patterns differ from each other there is no essential difference of the angle dependence.

The scattering pattern by the end-face of an ordered waveguide system composed of identical cores of equal spacing for TM wave incidence is shown in Fig.3. The spacing between cores is $8\mu m$ and the number of cores is 21. The other parameters are the same as those for Fig.2. We can see peaks at scattering angle $\vartheta \sim 137.8°,\ 145.2°,\ 154.2°$, which agree with diffraction angles calculated by the grating formula. The scattering pattern for TM plane wave incidence is shown in Fig.4. The parameters are the same as those for Fig.3. The diffraction beams are sharp compared to those in Fig.3 and we can see ripples between the major diffraction beams.

5. Conclusion

The scattering of a Gaussian beam from the end-face of a two-dimensional waveguide system has been treated by the volume integral equation for the electric field and the first term of the perturbation solution for TE and TM wave incidence has been analytically derived. The far scattered field can be expressed as the product of the plane wave scattering and the Gaussian beam correction and does not almost depend on the polarization of an incident wave.

6. References

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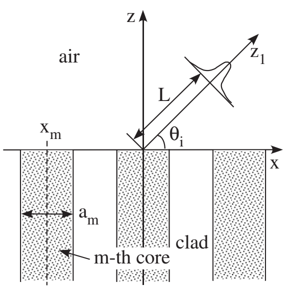

Fig.1 Waveguide system and an incident beam.

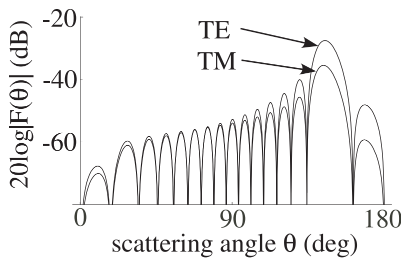

Fig.2 Scattering pattern by a single core for TE and TM incidence.

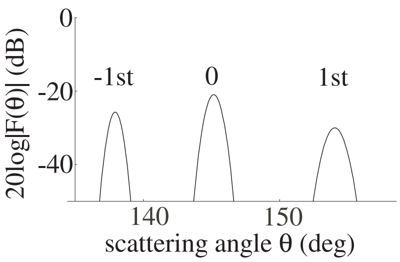

Fig.3 Scattering pattern of a Gaussian beam.

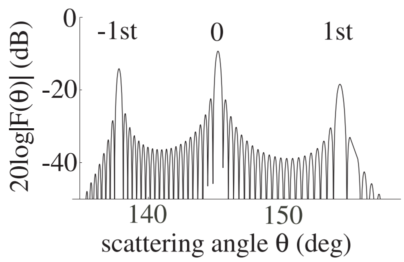

Fig.4 Scattering pattern of a plane wave.

7. Appendix: Vector Green's Functions

$$\boldsymbol{G}^{(x)} = \frac{1}{j2\pi}\int_{-\infty}^{\infty} \frac{q_2(q_0\overline{x} - p\overline{z})}{k_0^2(n_2^2 q_0 + q_2)} e^{-jp(x-x')-jq_0 z + jq_2 z'} dp, \quad z > 0 \tag{23}$$

$$\boldsymbol{G}^{(y)} = \frac{1}{j2\pi}\int_{-\infty}^{\infty} \frac{\overline{y}}{q_0 + q_2} e^{-jp(x-x')-jq_0 z + jq_2 z'} dp, \quad z > 0 \tag{24}$$

$$\boldsymbol{G}^{(z)} = \frac{1}{j2\pi}\int_{-\infty}^{\infty} \frac{p(-q_0\overline{x} + p\overline{z})}{k_0^2(n_2^2 q_0 + q_2)} e^{-jp(x-x')-jq_0 z + jq_2 z'} dp, \quad z > 0 \tag{25}$$