Detecting Electromagnetic Cloaks using Backward-Propagating Waves

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Abstract

A novel approach for detecting transformation-optics invisibility cloaks is proposed. The detection method takes advantage of the unusual backward-propagation characteristics of recently reported beams and pulses to induce electromagnetic scattering from the cloak. Even though waves with backward-propagating energy flux cannot penetrate the cloaking shell and interact with the cloaked objects (i.e., they do not make the cloaked object visible), they provide a mechanism for detecting the presence of cloaks.

1. Introduction

Designing and constructing invisibility cloaks is becoming an important practical application of transformation optics [1]. This became more evident with the successful experimental realizations of cloaks for large objects [2] and under-the-carpet cloaking [3]. Transformation optics suggests controlling the field distribution inside the cloak shell by changing the cloak material properties anisotropically. Such changes in material (tensoral) properties transform the fields from the electromagnetic (original) space into a physical (transformed) space. For an invisibility cloak to work properly, it must shield the object located inside the cloak from interacting with the incident electromagnetic fields without disturbing the fields outside the cloak. This is achieved by "squeezing" the electromagnetic space into the cloak shell; accordingly, the material of cloak shell has to be anisotropic and inhomogeneous with identical electric permittivity and magnetic permeability.

To design a transformation-optics invisibility cloak, one needs to find a Jacobian matrix $\mathbf{J}$ and (anistropic and inhomogeneous) material parameters that allow the "physical construction" of such Jacobian to transform the fields from the electromagnetic space (designated by the coordinates $\mathbf{r}'$) into the physical cloaking space (designated by the coordinates $\mathbf{r}$). The field transformation from the electromagnetic space into the physical space is thus written as

$$\mathbf{E}(\mathbf{r}) = \mathbf{J}(\mathbf{r},\mathbf{r}')\mathbf{E}'(\mathbf{r}'), \quad \mathbf{H}(\mathbf{r}) = \mathbf{J}(\mathbf{r},\mathbf{r}')\mathbf{H}'(\mathbf{r}'), \tag{0}$$

where $\mathbf{E}$ and $\mathbf{E}'$ are the electric fields in the physical space and the electromagnetic space, respectively; and $\mathbf{H}$ and $\mathbf{H}'$ are the corresponding magnetic fields. The boundary conditions at the outer and inner interfaces of the spherical cloak are enforced by the coordinate transformation. At the outer boundary Ω_2, the matching of the tangential fields must yield "no-scattering," i.e., an impedance matching with the ambient medium must be enforced:

$$\hat{\mathbf{n}}_2 \times \mathbf{E}(\Omega_2) = \hat{\mathbf{n}}_2 \times \mathbf{E}'(\Omega_2), \quad \hat{\mathbf{n}}_2 \times \mathbf{H}(\Omega_2) = \hat{\mathbf{n}}_2 \times \mathbf{H}'(\Omega_2), \tag{1}$$

where $\hat{\mathbf{n}}_2$ is the unit normal vector to the surface Ω_2. No field components are allowed to penetrate Ω_1; this requirement can be formulated as zero energy flux passing through Ω_1:

$$\hat{\mathbf{n}}_1 \cdot \left[\mathbf{E}(\Omega_1) \times \mathbf{H}(\Omega_1)\right] = 0, \tag{2}$$

with $\hat{\mathbf{n}}_1$ is the unit normal vector to the surface Ω_1 Following several algebraic manipulations from [4], the boundary conditions (2) and (3) are used to determine $\mathbf{J}(\mathbf{r},\mathbf{r}')$, such that

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$$\mathbf{J}(\mathbf{r},\mathbf{r}') = \frac{\mathbf{E}\otimes\mathbf{H}' - \mathbf{H}\otimes\mathbf{E}'}{\mathbf{S}'^2}\mathbf{S}'^{\times} + \mathbf{a}\otimes\mathbf{S}' \tag{3}$$

where $\mathbf{a}$ is an arbitrary vector, $\mathbf{S}' = \mathbf{E}'\times\mathbf{H}'$, and $\mathbf{S}'^{\times}$ is a three-dimensional anti-symmetric tensor dual to the vector $\mathbf{S}'$. As $\mathbf{J}$ should agree with Maxwell's equations in order to construct a valid transformation from the electromagnetic to the physical space, the following the identity condition must hold true

$$\nabla\times\mathbf{J}(\mathbf{r},\mathbf{r}') = 0. \tag{4}$$

An example of such a transformation Jacobian matrix can be found in [1], where the Jacobian is designed to map the electromagnetic space to the physical space of a spherical shell of inner radius R_1 and outer radius R_2. Since the transformation enforces (3), any material confined within the region $r < R_1$ is left completely isolated from the electromagnetic fields (see Figure 1 left; no field can penetrate the $r = R_1$ surface). Additionally, since the transformation enforces the impedance relation (2), the cloaking shell does not disturb the fields external to its outer boundary, Ω_2 (see Figure 1 left; fields are not disturbed outside the $r = R_2$ surface).

Recently, it was shown in [5] and [6] that vector Bessel beams [5] and vector X-Waves [6] could support negative energy flux confined to a small volume around their axis of propagation. It is conjured in [5] that if the Bessel beam is apertured such that the entire field incident on the aperture has a net negative energy flux, a reflection would occur from the open aperture with the reflected portion having its energy flux vector and longitudinal wave vector component in opposite directions. In this work, such waves are named backward-propagating waves.

Fields of a backward-propagating wave, when incident on a transformation-optics cloak having a Jacobian matrix $\mathbf{J}$, designed to match forward-propagating waves (see (4)), immediately violate the identity condition (5). The boundary condition (3) would still hold true; the internal part of the cloak would still be completely shielded from electromagnetic fields (see Figure 1 right; no field can penetrate the $r = R_1$ surface). On the other hand, the boundary condition (2) would not yield an impedance match and part of the incident wave field will reflect back from the outer surface of the cloak disturbing the external fields (see Figure 1 right; fields are disturbed outside the $r = R_2$ surface). This phenomenon suggests that a transformation-optics invisibility cloak can be detected using backward-propagating waves.

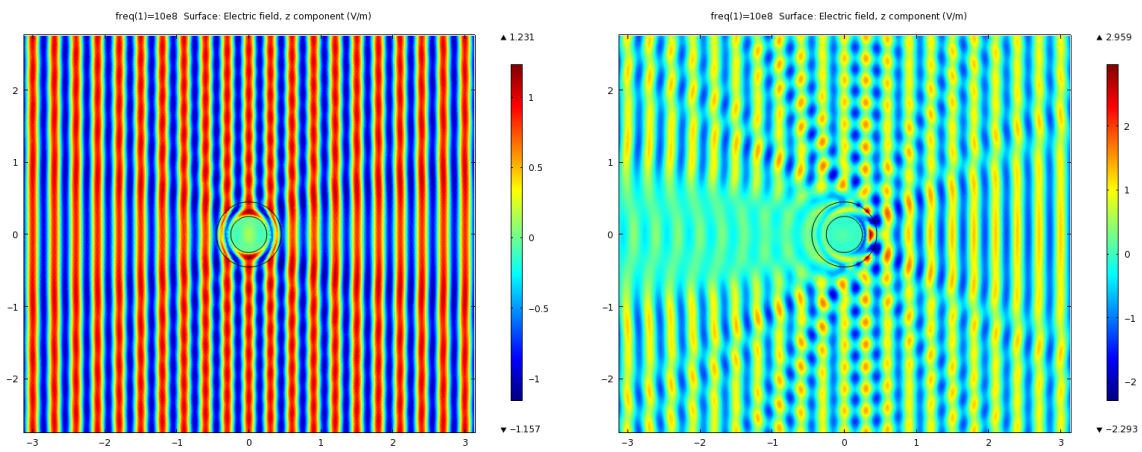

Figure 1 A spherical cloak illuminated by a forward-propagating field (left) and a backward-propagating field (right), showing that cloak detection is possible in the case of the backward-propagating field illumination.

2. References

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