Reflection and Transmission at Isotropic-Biaxial Interface

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Abstract

The problem of wave reflection and transmission at the interface between an isotropic (air) medium and biaxially anisotropic medium is solved and analyzed. We observe behavior that differs from the typical behavior at an isotropic-isotropic interface. It is seen that rotating the permittivity tensor of the biaxial medium with respect to the reference coordinate system significantly affects the reflection and transmission behavior.

1 Introduction

In this paper, we are interested in studying the reflection and transmission behavior of a wave incident from an isotropic region (air) upon a biaxially anisotropic region and vice versa. Reflection and transmission behaviors from biaxial half space interfaces have been studied by several others. Landry [1, 2] details the biaxial-biaxial boundary and considers the isotropic-biaxial boundary as a special case. However, Landry's analysis of this special case is limited. Stamnes and Sithambaranathan [3] provide derivations for reflection and transmission coefficients but no numerical results. Grzegorczyk et al. [4, 5] studied reflection and refraction but considered primarily left-handed metamaterials. We are interested in the general treatment of reflection and transmission for an isotropic-biaxial half-space and layered problem. The biaxial layer is considered to be both unrotated and rotated with respect to the laboratory coordinate system.

The defining property of electrically biaxial media is the permittivity tensor. Isotropic materials have a single permittivity. Uniaxially anisotropic materials have two different permittivity values with the same permittivity along two dimensions and a different permittivity along the third dimension. Biaxially anisotropic materials have three unique values in the permittivity tensor. In this work we have chosen to study the unrotated biaxial permittivity tensor is given by

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \tag{1}$$

Equation (1) represents a biaxial medium whose optic axes are aligned with the Cartesian coordinate system. If, however, the biaxial medium was arbitrarily oriented with respect to the coordinate system, the permittivity tensor would not be as simple. We can obtain the tensor for an arbitrarily oriented biaxial medium by applying rotations to the tensor in equation (1). In our formulation we use the same rotations used by Mudaliar and Lee [6]. Let us assume the permittivity tensor shown in (1) lies in the double-primed coordinate system. We want to transform the tensor to the system coordinates x, y, z. We begin by performing a counter-clockwise transformation about the x'' axis by an angle ψ_1 as shown in Figure 1(a), moving the tensor to the primed coordinate system. Then, we transform this tensor about the z' axis by an angle ψ_2 as shown in Figure 1(b) resulting in a full permittivity matrix in the unprimed system.

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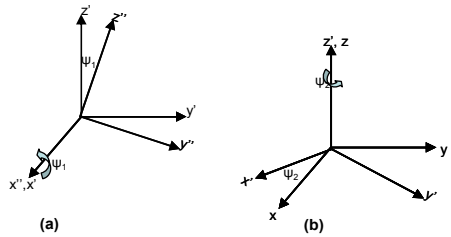

Figure 1: Rotation diagrams

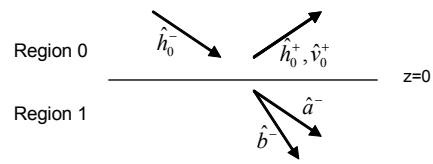

Figure 2: h-wave incident upon isotropic-biaxial interface

Waves in biaxially anisotropic materials exhibit birefringence. For a single wave incident upon the biaxial medium, two distinct waves will be generated with four propagation vectors. We call these waves the *a-wave* and *b-wave*. In general the *a-wave* propagates with the smaller propagation constant and the *b-wave* propagates with the larger propagation constant.

2 Analysis

We want to analyze the half-space problem of an isotropic (air) medium and biaxial medium. We define the isotropic region as region 0 and the anisotropic region as region 1. Normal to the boundary is the z-axis. We will focus on the case where a wave is downward incident from region 0 to region 1. To derive the half-space reflection and transmission coefficients, we formulate the fields in each region of interest and then apply the boundary conditions. Note that this derivation follows Pettis' work [7, Appendix G].

2.1 Horizontally polarized wave incident upon isotropic-biaxial interface

A horizontally polarized wave (*h-wave*) downward incident on the isotropic-biaxial interface (region 0 – region 1), will give rise to two reflected waves (one horizontally polarized and one vertically polarized in the isotropic region) and two transmitted waves (an *a-wave* and a *b-wave* in the biaxial medium). This behavior is depicted in Figure 2.

We must formulate the fields in each region to solve for the half-space coefficients. Based on Figure 2 we can write the electric fields in each region as

$$\overline{E}_0(\overline{r}) = \hat{h}_0^- e^{i\overline{\kappa}_0\cdot\overline{r}} + \hat{h}_0^+ R_{hh}^{01} e^{i\overline{k}_0\cdot\overline{r}} + \hat{v}_0^+ R_{hv}^{01} e^{i\overline{k}_0\cdot\overline{r}} \tag{2}$$

$$\overline{E}_1(\overline{r}) = \hat{a}^- X_{ha}^{01} e^{i\overline{\kappa}_a\cdot\overline{r}} + \hat{b}^- X_{hb}^{01} e^{i\overline{\kappa}_b\cdot\overline{r}} \tag{3}$$

where $\hat{h}$, $\hat{v}$, $\hat{a}$, and $\hat{b}$ are the electric field unit vectors of the *h*-wave, *v*-wave, *a*-wave and *b*-wave, respectively and the + and – superscripts represent upward and downward propagation. Given these fields, we can evaluate the unknown reflection and transmission coefficients by applying the boundary conditions at the interface. For each half-space problem, we put the interface at z=0 and assume there are no sources along the interface. Given this configuration, the boundary conditions are given by

$$\hat{z}\times\overline{E}_0(\overline{r}) = \hat{z}\times\overline{E}_1(\overline{r}), \quad \text{at z = 0} \tag{4}$$

$$\hat{z}\times\overline{H}_0(\overline{r}) = \hat{z}\times\overline{H}_1(\overline{r}) \longrightarrow \hat{z}\times\nabla\times\overline{E}_0(\overline{r}) = \hat{z}\times\nabla\times\overline{E}_1(\overline{r}), \quad \text{at z = 0} \tag{5}$$

Applying the boundary conditions to the formulated fields, we arrive at a system of four equations. We can write the four equations in matrix form as

$$\begin{bmatrix} -h_{0y}^{+} & -v_{0y}^{+} & a_{y}^{-} & b_{y}^{-} \\ -h_{0x}^{+} & -v_{0x}^{+} & a_{x}^{-} & b_{x}^{-} \\ -\left(k_x h_{0z}^{+} - k_{0z} h_{0x}^{+}\right) & -\left(k_x v_{0z}^{+} - k_{0z} v_{0x}^{+}\right) & \left(k_x a_z^{-} - k_z^{ad} a_x^{-}\right) & \left(k_x b_z^{-} - k_z^{bd} b_x^{-}\right) \\ -\left(k_y h_{0z}^{+} - k_{0z} h_{0y}^{+}\right) & -\left(k_y v_{0z}^{+} - k_{0z} v_{0y}^{+}\right) & \left(k_y a_z^{-} - k_z^{ad} a_y^{-}\right) & \left(k_y b_z^{-} - k_z^{bd} b_y^{-}\right) \end{bmatrix} \begin{bmatrix} R_{hh}^{01} \\ R_{hv}^{01} \\ X_{ha}^{01} \\ X_{hb}^{01} \end{bmatrix} = \begin{bmatrix} h_{0y}^{-} \\ h_{0x}^{-} \\ \left(k_x h_{0z}^{-} + k_{0z} h_{0x}^{-}\right) \\ \left(k_y h_{0z}^{-} + k_{0z} h_{0y}^{-}\right) \end{bmatrix} \tag{6}$$

The x and y subscripts on quantities in equation (6) are used to indicate the appropriate component of the given field vector. The matrix equation in (6) can be solved numerically to determine the half-space reflection and transmission coefficients. Other problems (wave incident from the biaxial medium upon uniaxial medium and the two layer problem), can be solved in a similar manner.

3 Results

In this analysis we are choosing the x-z plane as the plane of incidence, making k_y zero. The angle of incidence in the prescribed plane is given by θ. The reflection and transmission coefficients will be calculated and plotted as function of θ. We shall assume values of $\varepsilon_x = 2$, $\varepsilon_y = 5$, and $\varepsilon_z = 8$ for illustration here.

First, we consider the co-polarized half space reflection coefficients for the isotropic-biaxial interface. The isotropic medium is air and the biaxial medium is unrotated with the permittivity tensor shown in equation (1). The reflection (R) and transmission (X) coefficients are plotted against angle of incidence in Figure 3. The figure shows that at smaller angles, the vertically polarized wave is reflected more strongly than the horizontally polarized wave. For angles greater than approximately 40˚, this behavior is reversed and the horizontally polarized wave is reflected more strongly. This is in contrast with the typical behavior at an isotropic-isotropic half space boundary where the horizontally polarized wave is reflected more strongly for all incident angles. We can also observe the Brewster angle effect. At an incident angle just above 60˚, the vertically polarized wave has zero reflection and only the horizontally polarized wave is reflected.

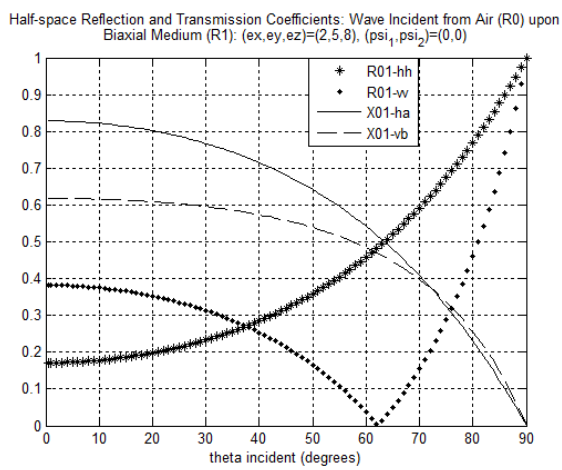

Figure 3: Half-space co-polarized reflection and transmission coefficients for incident wave from isotropic medium to unrotated biaxial medium (ε_x, ε_y, ε_z) = (2, 5, 8).

Now, we'd like to consider the same phenomena when the permittivity tensor of region 1 is rotated with respect to the reference coordinate system. The behavior observed in Figure 3 changes as we rotate. First, we rotate by ψ_1. As ψ_1 increases from 0˚, R_{hh} is not significantly changed while R_{vv} increases. However, when we increase ψ_2 we see more significant results. As ψ_2 increases from 0˚ R_{hh} increases and R_{vv} decreases. When ψ_2 reaches 45˚ R_{hh} and R_{vv} are equal at zero incidence angle and diverge for ψ_2 greater than 45˚. This behavior is shown in Figure 4.

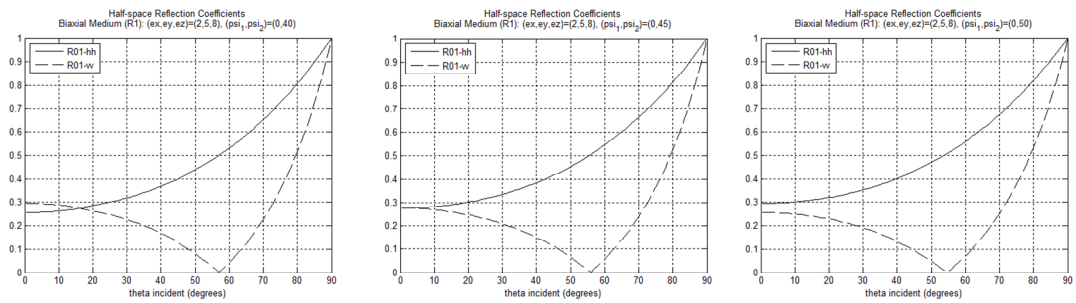

Figure 4: Half-space co-polarized reflection coefficients: ψ_2 varied, ψ_1=0 (ε_x, ε_y, ε_z) = (2, 5, 8).

The half-space reflection behavior changes when we move the angle of incidence to the *y-z* plane. In this plane, when the biaxial medium is unrotated, R_{hh} is greater than R_{vv} for all incident angles. As ψ_2 increases from 0˚ R_{hh} decreases and R_{vv} increases. When ψ_2 reaches 45˚ R_{hh} and R_{vv} are equal at zero incident angle and for ψ_2 greater than 45˚, R_{hh} is less than R_{vv} at low incident angles. Again changing ψ_1 results in less overall change, but in this plane, this ψ_1 has a greater impact on R_{hh} than R_{vv}.

Additional notes regarding the behavior at this interface. First, the cross-polarized reflection coefficients (R_{hv} and R_{vh}) are zero when the biaxial medium is unrotated and non-zero (but still small) when the medium is rotated. Next, we observed the half space transmission coefficients. When the horizontally polarized wave is incident, the energy is coupled to the a-wave, not the b-wave as X_{hb} is approximately zero. Similarly, the vertically polarized wave transmits into the b-wave with X_{va} approximately zero. The X_{ha} and X_{vb} behave like co-polarized transmission coefficients (see Figure 3) while X_{hb} and X_{va} behave like cross-polarized transmission coefficients for the unrotated case. These pairings breakdown when the biaxial medium is rotated. In this manner, the a-wave is acting like a horizontally polarized wave and the b-wave is acting like a vertically polarized wave for the given medium parameters. Lastly, to verify our results, we showed that power is conserved across the boundary.

4 Summary

In this paper we have presented the solution and numerical results of the half-space reflection and transmission problem of a wave incident from air upon a biaxially anisotropic region. We observed the unique behavior of the reflection coefficients as we rotate the biaxial permittivity tensor. Additionally, we have studied the half-space problem of a wave incident from the biaxial medium to the isotropic medium and the two layer problem of a biaxially anisotropic layer positioned between two isotropic layers. Brewster angle and critical angle effects have also been analyzed. Additional details of this and other analyses will be presented at the conference and in future work.

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