

# Reduced-Order Modeling for Co-Simulation of Circuit and Electromagnetic Interactions

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## Abstract

Accurately modeling the interaction of circuits and electronic devices within an electromagnetic environment is critical to the analysis and design of their constituent systems. Creating a rational macromodel of a distributed circuit-field element is a reliable and effective method for describing these interactions in a manner which can be incorporated into a circuit simulator. Due to limitations present in rational modeling algorithms, further processing of these models is necessary to both confirm whether a model is passive, and to correct any non-passivity which is present. A class of iterative inverse algorithms is utilized to create a complete passive macromodel which can readily be incorporated within a simulator to analyze the behaviors caused by the interactions of the electromagnetic and circuit elements.

## 1 Introduction

Interconnects, transmission lines, microwave circuits, and antenna-to-antenna links are all distributed circuits with rich frequency-dependent characteristics, dispersive effects, and time-of-flight delays that cannot be adequately represented by lumped element circuits. In a simulator, the behavioral data of a structure must be replaced by an interpolative model of the frequency-domain data which, ideally, is valid in both the frequency domain and the time domain. This paper focuses on the development of a model for a transfer function of the response of a distributed element. All passive networks have a response that can be fitted to a Foster canonical transfer function model [1]. The general approach to the development of a transfer function model is to create a Laplace-domain rational function which approximates the behavioral data. The vector fitting (VF) method proposed in [2,3] applies an iterative fitting algorithm [4] utilizing partial fraction (PF) basis functions to generate the approximation. While the VF algorithm often reliably generates accurate models, unfortunately this technique can suffer from numerically unstable convergence when the number of poles in the fitted model is chosen greater than the number of poles present in the original circuit. Additional convergence problems arise when the distributed circuit has closely spaced or high-order multiple poles [5,6]. The problem is fitting an efficient and robust reduced-order model to data that possibly does not have a good pole-zero characteristic. The solution presented here develops a form of the Foster model that results in a robust stamp for a transient simulator. An enhanced fitting algorithm is also presented.

When generating a model of a distributed circuit, it is desirable to use a large number of fitted poles to ensure there are enough to accurately represent the measured behavior, however this can lead to convergence issues when using the VF method. The proposed method implements an iterative fitting algorithm which alternatively uses rational first-order (RFO) basis functions to characterize the circuit. This method provides a robust solution to the ill-conditioning caused by the selection of a high fitted-model pole order to fit a distributed circuit with unknown properties. The goal of the algorithm is to develop a rational first-order model [7] of a transfer function,

$$t_{\text{fit}}(s) = \sum_{k=1}^M \frac{q_k s - r_k}{s - p_k} + g + sc, \quad (1)$$

where  $t_{\text{fit}}(s)$  is the transfer function being fitted,  $q_k$  and  $r_k$  are the first and zeroth order coefficients respectively, the  $p_k$  are the poles,  $g$  is a constant coefficient term, and  $c$  is an optional frequency proportional coefficient. The rational first-order basis provides a solution to the modeling and transient simulation of distributed linear networks, as shown for the bandpass filter example in Fig. 1(a). The improved set of basis functions are proposed as a candidate for use in an iterative model fitting procedure. This procedure is shown to display convergence characteristics better than the partial fraction basis. The modeling procedure

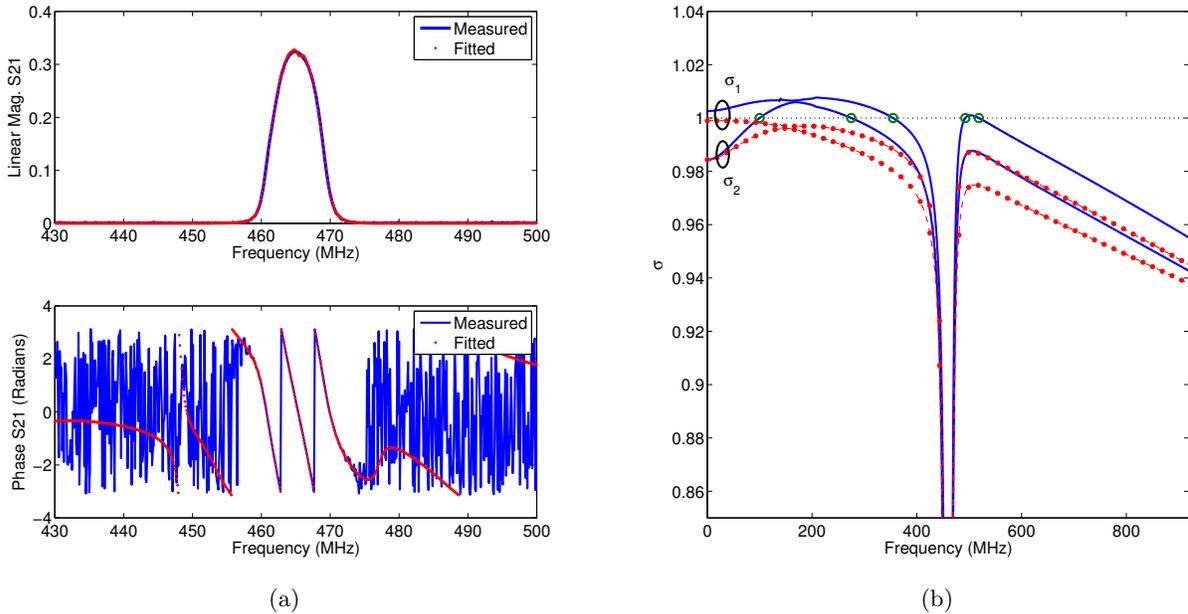


Figure 1: Seventh-order Chebyshev filter responses: (a)  $S_{21}$  measured and fitted model; and (b) singular value curves of the filter model numbered relative to their DC value, with passivity violations (solid lines) and boundary frequencies indicated (circles), and after passivity has been enforced (dash-dotted lines.)

fits a modified form of the Foster canonical rational transfer function model to the network, and has particular advantages for characterizing narrowband bandpass behaviors. The primary advantage of the proposed fitting procedure is its dependable convergence when fitting transfer functions that may not have precise pole-zero descriptions, or whose properties are not known a priori.

## 2 Passivation of Reduced-Order Macromodels

Reduced-order modeling of distributed structures for transient and steady-state circuit simulation transforms discrete frequency-domain network parameters to a set of rational functions. The models are ideally causal and passive with passivity being the most difficult property to assure, especially when the distributed structures incorporate propagation delay effects or the available network parameters have limited bandwidth. Small errors in the frequency-domain network parameters, or out-of-band assumptions, can yield models that result in unstable transient simulations. Here an inverse singular value method is developed that imposes the smallest perturbation required to simultaneously modify the residues, poles and coupling coefficients of the rational function-based model to achieve passivity. The process enables selection of the frequency ranges for which the model is required to be most accurate. The method is based on the observation that a macromodel is passive if the singular values of the scattering parameter matrix are less than unity at all frequencies.

Several methods have been proposed for developing reduced-order interpolative macromodels from network parameters [2,3,7–13]. While these methods generate a model that is causal and stable, they impose no constraint on the passivity of the macromodel. Utilizing an inverse method [14] is a highly effective alternative for enforcing passivity of a macromodel. This method incorporates the strengths of previous methods by enabling the simultaneous modification of the residues, poles, and direct coupling coefficients of the model. Utilizing a set of iteratively-scaled perturbations and projections, the model variables are efficiently adapted to form a passive macromodel. This method additionally enables the simultaneous modification of any desired subset of the variables to yield a passive macromodel. The method finds the smallest perturbations required to achieve passivity, thus ensuring that minimal error is introduced. The result is an efficiently-calculated stable passive macromodel usable in both time- and frequency-domain analyses.

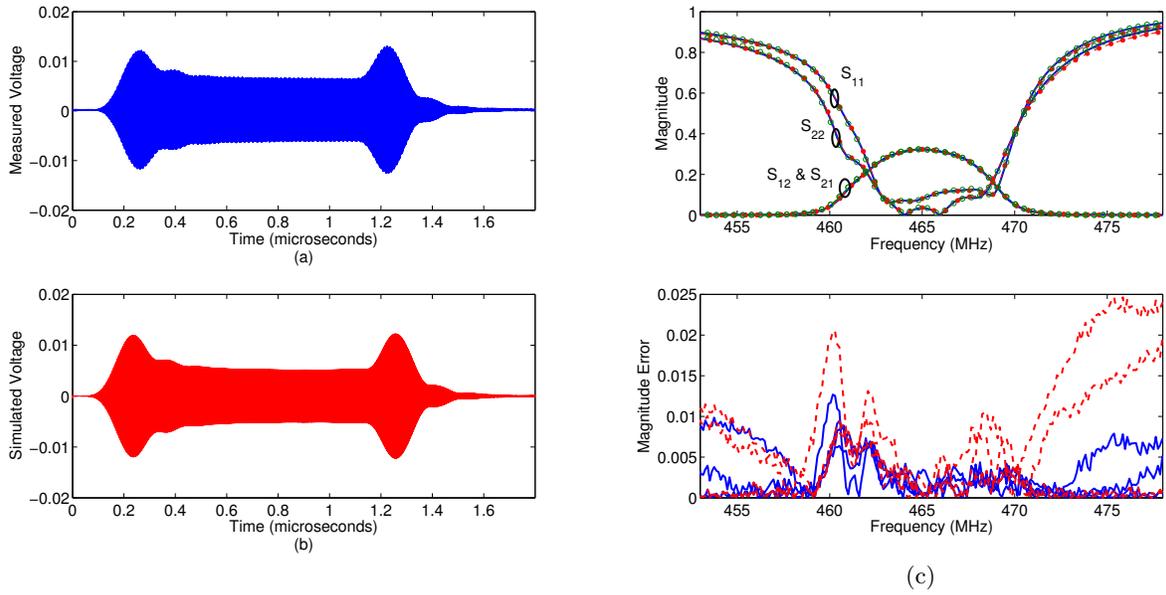


Figure 2: Chebyshev bandpass filter responses: (a) measured output with pulsed RF excitation: and (b) simulated output using the RFO model; and (c)  $S$ -parameter magnitudes of a Chebyshev bandpass filter: measured (circles), modeled prior to (solid lines) and after (dash-dotted lines) passivity is enforced are shown in the upper subplot (magnitude errors relative to the measured data for the fitted model prior to (solid lines) and after (dashed lines) passivity is enforced are shown in the lower subplot).

The passivity of a model can be readily established by first converting the rational model into a state-space model of the form

$$\mathbf{T}(s) = \mathbf{C}(s\mathbf{I}_{M_T} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}, \quad (2)$$

where the transfer function  $\mathbf{T}(s)$  can be either an  $S$ -parameter or  $Y$ -parameter description of a distributed circuit. For  $S$ -parameter descriptions, if at least one singular value of the  $S$ -parameters is greater than unity at any frequency, the model is non-passive. Thus, for passivity the condition must be true:

$$\sigma_u \{\mathbf{S}(s)\} < 1, \quad u = 1 \dots N, \quad \forall s. \quad (3)$$

For  $Y$ -parameter descriptions, if any eigenvalue of the Hermitian part of the admittance matrix is less than zero, this model is nonpassive. For passivity to be established for an admittance description of a circuit,

$$\lambda_u \{\mathbf{Y}'(s)\} > 0, \quad u = 1 \dots N, \quad \forall s, \quad (4)$$

where  $\mathbf{Y}'(s) = \frac{1}{2} [\mathbf{Y}(s) + \mathbf{Y}^*(s)]$  is the Hermitian part of the admittance matrix. Utilizing this information to determine passivity, a set of iterative inverse algorithms can be applied to manipulate the state-space parameters of the model which directly alters the singular values or eigenvalues to correct any passivity violation which is found in the model.

The iterative inverse singular value method for enforcing passivity of rational macromodels is applied to an  $S$ -parameter model of a bandpass filter example in Fig. 1(b). The perturbation-projection approach of the algorithm allows for the simultaneous modification of both linear coefficients and nonlinear poles, enabling a minimal modification to be applied to the model in order to achieve passivity, which thus preserves the accuracy of the macromodel, as seen in Fig. 2(c). The example shown in Figs. 2(a) and 2(b) displays both the measured output of the bandpass filter example from a pulsed RF input signal, and the corresponding simulated response of a model of the bandpass filter implemented in the modified nodal admittance matrix of the fREEDA<sup>TM</sup> simulator [15–17].

### 3 Conclusion

The general methodology for converting the measured response of a distributed circuit-field element has been presented. The data is first utilized to generate a rational model which accurately describes the behaviors of the element, but due to limitations in the algorithm is not guaranteed to be passive. Further model processing is applied which evaluates the passivity of the model, and utilizes a set of iterative inverse algorithms to enforce passivity to create a complete macromodel which is guaranteed to be passive.

### 4 References

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