Metadispersion in Anisotropic and Bianisotropic Media

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Abstract

In this work an innovative method for the characterization of dispersion in metamaterials is presented. The attention is focused on structures realized by stacks of planar periodic surfaces. The analysis procedure subdivides the artificial material into its constituent planes, performing a full wave analysis of the individual planar periodic structures, and uses analytic formulas from Bloch theory to determine the dispersion characteristics of the overall artificial medium. Physical properties of the equivalent admittance matrix modeling the single planar sheets are exploited to analytically describe the dispersion properties of the metamaterial in the whole first Brillouin zone, starting from a limited number of full-wave simulations.

1. Introduction

In the last decade the electromagnetic engineering community has focused its attention on the analysis and modeling of metamaterials [1]. Among the various parameters characterizing a metamaterial, the dispersion diagram provides a clear physical picture for understanding the wave propagation and constitutes a useful basis for the definition of practical design criteria. Dispersion diagrams can be obtained through transmission measurements of a metamaterial sample or 3D simulations of the metamaterial unit cell. However, these approaches are not sufficient to provide a clear insight into the relationship between the geometrical characteristics of the metamaterial and the corresponding physical behavior. The large number of measurements or simulations required for the determination of the whole dispersion diagram is another drawback of these approaches.

In this work, we present a general approach for the efficient analysis of the dispersion properties of artificial materials consisting of stacks of planar periodic structures. This class includes many metamaterials of practical interest, since artificial materials are typically fabricated by stacking planar arrays of patches printed on dielectric substrates [2].

2. Derivation of the Dispersion Characteristics

The analysis procedure starts from the decomposition of the artificial material into its constituent planar structures. The analysis of the individual planar structures is then performed using an equivalent transmission line model. The assumption is made that the distance between adjacent planar structures is sufficiently large to render the interactions among higher order Floquet modes negligible. As a consequence, the equivalent network for the planar periodic structure must only account for the dominant Floquet mode, and consists of two transmission lines associated with the transverse electric (TE) and the transverse magnetic (TM) polarizations loaded by a shunt load represented through an equivalent admittance matrix [3]. The three-dimensional metamaterial is therefore modeled through a couple of periodically loaded transmission lines (Fig. 1a). When the direction of incidence is on a principal plane of the planar periodic structure the admittance matrix is diagonal and the two equivalent transmission lines are decoupled. In this case, the conventional Bloch theory [4] can be applied to study the propagation in the metamaterial. A similar approach has been employed in [5] to study the propagation in the wire medium. However, out of the principal planes the application of the conventional Bloch theory would require the solution of a 4x4 eigenvalue equation. In this work, a novel and simpler approach is proposed for the analysis of the periodically loaded coupled lines. It is based on a projection of the normalized modal fields onto the eigenvector basis of the normalized single layer admittance matrix and leads to the definition of a new equivalent transmission line network in which the two constituent transmission lines are decoupled (Fig. 1b) and are associated with the actual modes supported by the 3D periodic structure.

For any given transverse phasing, two modes supported by the metamaterial are identified, which are in general hybrid (*i.e.* neither TE nor TM) and exhibit different longitudinal propagation constants. This means that the metamaterial is in general a birefringent medium, *i.e.* an incident wave impinging on it from free space can give rise

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to two refracted waves with different ray directions and ray velocities. Furthermore, the Bloch analysis also provides the characteristic impedance of the equivalent transmission lines (*i.e.* the ratio between the transverse electric and magnetic field at the boundary of the unit cell), which can be used to set equivalent boundary conditions that accurately describe the scattering at from a finite metamaterial slab.

The final result of the analysis is the complete dispersion relationship for the artificial medium; in particular, for any given combination of frequency and transverse phasing, a closed-form expression of the corresponding longitudinal propagation constants is provided as a function of the entries of the equivalent admittance matrix of the constituent planar periodic structure. In turn, these latter depend on frequency and transverse phasing.

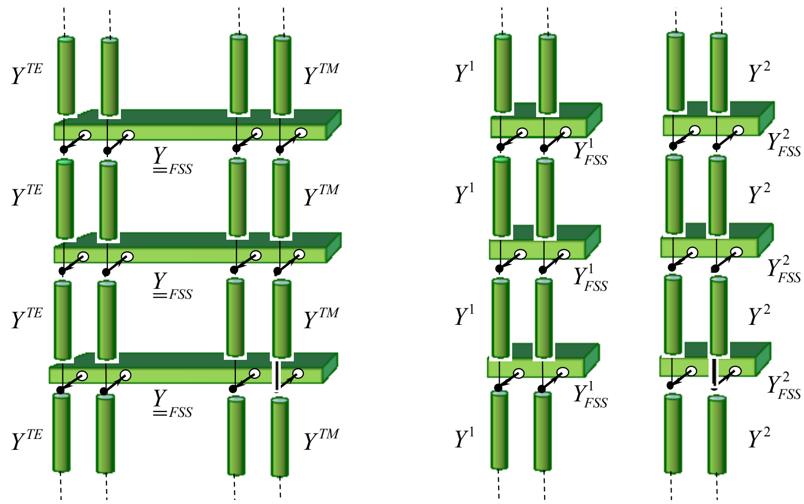


Fig. 1 Equivalent transmission line model for a generic stack of planar periodic structures: a): classical model with coupled TE and TM equivalent transmission lines; b): novel model with two decoupled equivalent transmission lines

A key point for the efficiency of the dispersion analysis is the possibility to completely reconstruct the dependence of the equivalent admittance matrix entries on frequency and transverse phasing starting from a limited number of full wave simulations. In this work, this is done by using a pole-residue matching technique similar to the one described in [6]. The pole-residue matching technique, which is briefly summarized in the following section, exploits the Foster's properties of lossless two-port networks to derive an analytical description of the frequency dependence of the entries of the equivalent admittance matrix based on the identification of poles and residues. These parameters exhibit a weak dependence on the transverse phasing, thus, allowing one to apply a numerically efficient interpolation algorithm to obtain the overall response of the periodic sheet from a limited number of full-wave simulations.

3. Analytical Reconstruction of the Dispersion Surfaces

In the absence of losses the equivalent admittance matrix $\underline{\underline{Y}}_{FSS}$ of the single layer is antihermitian. Furthermore, it can be demonstrated that for any given transverse phasing all the entries, seen as a function of frequency, have the same poles, and the diagonal entries respect the Foster's reactance theorem, *i.e.* the rate of change of their imaginary part with frequency is positive [4]. From this theorem it follows that for the diagonal entries the poles and zeros lie on the real ω axis and are *simple* and *alternate;* a zero (patch type planar periodic structure) or a pole (slot type planar periodic structure) must be at ω=0; poles and zeros are symmetrically displaced with respect to the origin and the residues are real and positive. An important consequence of these properties is that, in the frequency range of interest, each element of the admittance matrix can be well approximated by the following pole-residue representation:

$$y_{ij}(\omega) = \sum_{n=1}^{N} \frac{r_n}{\omega - \omega_p^{(n)}} + j\omega C, \tag{1}$$

where $\omega_p^{(n)}$, r_n and C indicate the n-th pole, the corresponding residue and the quasi-static equivalent capacitance, respectively. The identification of poles and residues is efficiently performed by using the algorithm described in [7]. The number N of poles to be included in the representation in (1) depends on the frequency range of interest. As a practical rule, all the poles within the frequency range of interest plus the closest one outside the same range must be included in the representation. Since poles and residues exhibit a weak dependence on the transverse phasing, their behavior in the whole irreducible Brillouin zone can be determined through a polynomial fitting of the values obtained from a limited number of full wave simulations.

The final outcome of the procedure is an analytic representation of the equivalent admittance matrix $\underline{\underline{Y}}_{FSS}$, which allows one to readily obtain the value corresponding to an arbitrary combination of frequency and transverse phasing without performing further full wave simulations.

4. Numerical Results

The proposed technique has been applied to the analysis of the dispersion relation in a square Split Ring Resonator metamaterial. The unit cell, whose geometry is depicted in Fig. 2, consists of a pair of concentric square metallic rings with slits etched in opposite sides [8].

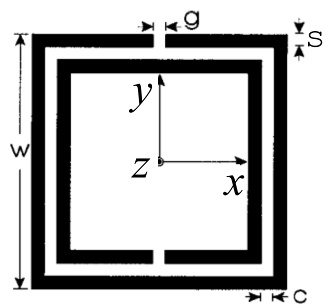
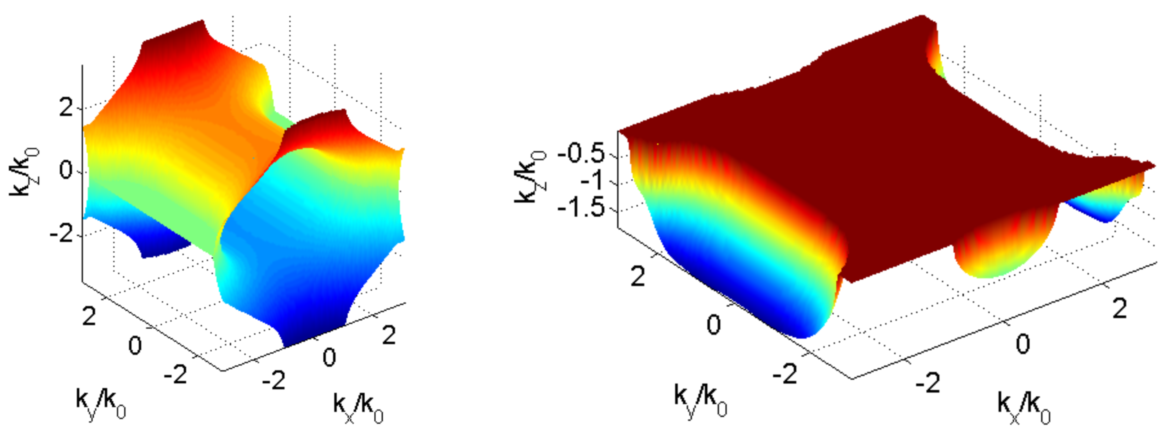


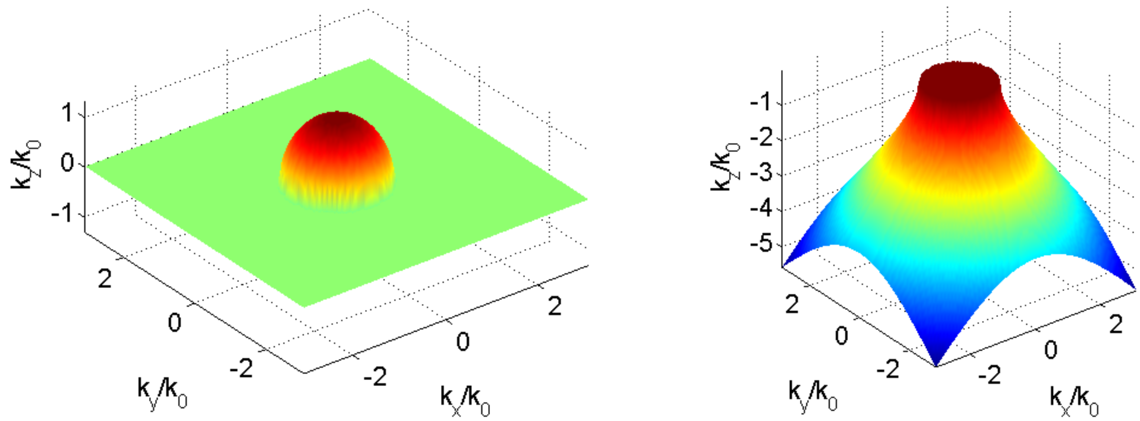
Fig. 2 Unit cell for the SRR metamaterial. Geometrical parameters: d_x= d_x=d=10mm, w=0.8d, s=0.08d, g=c=0.04d.

Figures 3 and 4 show two samples of the dispersion surfaces obtained for the infinite periodic medium in the case of a cubic cell, with periods along the three directions equal to d_x=d_y=d_z=1 cm. The surfaces show the values of the real and imaginary part of the normalized longitudinal propagation constant k_z/k_0 for the two modes supported by the infinite medium at the frequency of 4.37GHz (close to the split ring resonance) as a function of the normalized transverse components of the propagation constant, k_x/k_0 and k_y/k_0, where k_0 is the free space propagation constant. Real values and imaginary values of k_z indicate propagation or attenuation along z, respectively.



(a) (b)

Fig. 3 Longitudinal propagation constant of the first mode as a function of k_x and k_y at the frequency of 4.37GHz: (a) real part, (b) imaginary part.



(a) (b)

Fig. 4 Longitudinal propagation constant of the second mode as a function of k_x and k_y at the frequency of 4.37GHz: (a) real part, (b) imaginary part.

Comparing the surfaces relevant to the two modes it can be seen that for the second mode the propagation is only slightly influenced by the periodic structure, and the dispersion surface is similar to the one of free space. On the other hand, the dispersion surface relevant to the first mode is significantly different from the one of free space since this mode strongly interacts with the periodic structure.

5. Conclusions

A novel method for the complete analysis of dispersion in a metamaterial consisting of a stack of planar periodic structures has been presented. The proposed approach is systematic and computationally efficient, since it only requires the full wave analysis of a single periodic sheet for a limited number of transverse phasings.

6. Acknowledgment

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7. References

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