SEM-based Analysis of Antenna Radiation Properties

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Abstract

A novel analytical singularity-expansion-method-based approach to the modeling of transient antenna radiation is presented. Any electromagnetic field prediction model can be used to derive a minimal spherical harmonic expansion of the equivalent currents excited along a Huygens surface enclosing the antenna under analysis. In this way, the time-domain electromagnetic field radiated in the Fraunhofer region is evaluated analytically in terms of a newly introduced class of incomplete modified Bessel functions as the superposition of non-uniform spherical wave contributions which account for the natural resonant processes occurring in the structure.

1 Introduction

The answer to the growing demand for high-performance ultra-wideband (UWB) systems for wireless communications and radar applications is one of the present challenges for the industrial and scientific community. In this context, accurate electromagnetic field prediction models, useful to analyze the time-domain radiation properties of complex antennas [1], are highly desirable. Unfortunately, except for a few classes of radiators, such as aperture and impulse radiating antennas, analytical models for the representation of the transient electromagnetic field distribution in Fraunhofer region are not available in the scientific literature. Therefore, in most cases intensive numerical tools are to be used to compute the far-field parameters of interest. However, such an approach does not provide an integral physical insight into the mechanisms which are responsible for the electromagnetic behavior of the structure, and requires large computational times and storage of a large amount of data. To overcome this limitation, a suitable semi-analytical formulation based on the singularity expansion method (SEM) is described in this paper. By using the proposed procedure, the radiated electromagnetic field is presented directly in the time domain as the superposition of propagating non-uniform spherical waves related to the complex resonant processes occurring in the structure under analysis. To this end, any time-domain integral-equation or finite-difference technique can be adopted to carry out the full-wave analysis within a volume surrounding the antenna, and to determine on-the-fly in step with the numerical simulation a spherical harmonic expansion of the equivalent electric and magnetic currents excited on a suitable Huygens surface enclosing the radiating structure. Then, a pole/residue representation of the currents is derived by means of a dedicated time-domain vector fitting procedure. In this way, closed-form expressions of the time-domain effective height and antenna gain can be obtained in terms of a new class of incomplete modified spherical Bessel functions [1]. Furthermore, the suggested approach allows for a significant reduction of the computational resources.

2 Theoretical Formulation

Let us consider a general antenna operating in free-space and driven through a transmission line by a matched voltage generator $v_g(t)$ having internal resistance R_g. Denoting by S_h a spherical surface of radius R_h enclosing the structure, the time-domain electric field radiated in the Fraunhofer region can be readily obtained as the vector slant-stack transform (SST) of the equivalent electric and magnetic current densities $\boldsymbol{\mathcal{I}}_S(\vartheta,\varphi,t) = -\eta_0 \boldsymbol{\mathcal{J}}_S(\vartheta,\varphi,t) + \hat{\boldsymbol{r}} \times \boldsymbol{\mathcal{M}}_S(\vartheta,\varphi,t)$ excited along S_h, namely:

$$\boldsymbol{\mathcal{E}}(r,\vartheta,\varphi,t) \simeq SST\left\{\boldsymbol{\mathcal{I}}_S(\vartheta,\varphi,t)\right\} = \frac{1}{4\pi r c_0} \iint_{S_h} \dot{\boldsymbol{\mathcal{I}}}_S(\vartheta',\varphi',\tau + t_h\cos\gamma)\, dS', \tag{1}$$

where $\cos\gamma = \sin\vartheta \sin\vartheta' \cos(\varphi-\varphi') + \cos\vartheta\cos\vartheta'$, $\tau = t - r/c_0$ is the spherical-wave delayed time, and $t_h = R_h/c_0$, with c_0 being the speed of light. As usual, the quantities $\{\vartheta,\varphi\}$ and $\{\vartheta',\varphi'\}$ in (1) denote the polar angles relevant to the observation and source point, respectively, r is the radial distance, and η_0 is

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the free-space wave impedance. As known from theory, the magnetic field can be determined by using the plane-wave-like expression:

$$\boldsymbol{\mathcal{H}}\left(r,\vartheta,\varphi,t\right)\simeq\frac{1}{\eta_0}\hat{\boldsymbol{r}}\times\boldsymbol{\mathcal{E}}(r,\vartheta,\varphi,t)\,. \tag{2}$$

At any time, can be conveniently approximated as a finite superposition of spherical harmonics:

$$\boldsymbol{\mathcal{I}}_S(\vartheta,\varphi,t)\simeq\sum_{n=0}^{N}\sum_{m=-n}^{n}\boldsymbol{\psi}_{n,m}(t)\,Y_n^m(\vartheta,\varphi)\,. \tag{3}$$

Therefore, provided that the considered antenna is excited by a finite-duration pulse, the following modified *SEM*-based representation of the vector current expansion coefficients in (3) can be adopted:

$$\boldsymbol{\psi}_{n,m}(t)\simeq\sum_{k=1}^{K}\boldsymbol{\rho}_{n,m,k}e^{s_{n,m,k}t}u(t)\,, \tag{4}$$

with $u(t)$ being the usual Heaviside unit-step distribution, and where $s_{n,m,k}$, $\boldsymbol{\rho}_{n,m,k}$ denote the complex poles and vector residues, respectively, of the exponential terms accounting for the natural resonant processes occurring in the antenna. Hence, substituting equations (3) and (4) into the equation (1) yields:

$$\boldsymbol{\mathcal{E}}(r,\vartheta,\varphi,t)\simeq\frac{t_hR_h}{4\pi r}\sum_{n=0}^{N}\sum_{m=-n}^{n}\sum_{k=1}^{K}s_{n,m,k}e^{s_{n,m,k}\tau}\boldsymbol{\rho}_{n,m,k}\iint_{\Omega_h}e^{s_{n,m,k}t_h\cos\gamma}Y_n^m(\vartheta',\varphi')\,d\Omega', \tag{5}$$

where $d\Omega'=\sin\vartheta' d\vartheta' d\varphi'$ is the infinitesimal solid angle, and $\Omega_h=\Omega_h\left(\vartheta,\varphi,\tau/t_h\right)=\{(\vartheta',\varphi'):\cos\gamma>-\tau/t_h\}$ denotes the angular domain of the equivalent currents on S_h contributing, at the normalized time τ/t_h, to the radiated electromagnetic field excited at the observation point $\{r,\vartheta,\varphi\}$. By making judicious use of the Laplace integral representation of $u(t)$, and exploiting some advanced properties of the surface harmonics, the radiation integrals in (5) can be evaluated analytically, after some mathematical manipulations, as:

$$\iint_{\Omega_h}e^{s_{n,m,k}t_h\cos\gamma}Y_n^m(\vartheta',\varphi')\,d\Omega'=4\pi i_n\!\left(s_{n,m,k}t_h,\min\!\left\{1,\frac{\tau}{t_h}\right\}\right)u\!\left(1+\frac{\tau}{t_h}\right)Y_n^m(\vartheta,\varphi)\,, \tag{6}$$

having introduced the incomplete modified spherical Bessel function of order n [1]:

$$i_n(\xi,w)=\frac{1}{2}\int_{-w}^{1}e^{\xi z}P_n(z)\,dz, \tag{7}$$

$P_n(z)$ denoting the Legendre polynomial of order n. By adopting the sum formula relevant to $P_n(z)$, the closed-form expression of $i_n(\xi,w)$ is derived as follows:

$$i_n(\xi,w)=i_n(\xi)-e^{\xi}\sum_{q=1}^{n+1}\binom{n}{q-1}\binom{-n-1}{q-1}\frac{\Gamma(q,\xi\,(1+w)\,,2\xi)}{(2\xi)^q}, \tag{8}$$

with $\Gamma(a,z_1,z_2)=\int_{z_1}^{z_2}\zeta^{a-1}e^{-\zeta}d\zeta$ being the generalized incomplete Gamma function, and $i_n(\xi)$ the canonical modified spherical Bessel function of order n. The incomplete functions described by equation (7) are particularly useful in the analysis of complex wave phenomena occurring in truncated spherical structures. In the context considered here, they clearly account for the early transient when only the portion of the surface S_h corresponding to the solid angle Ω_h gives contribution to the radiated electromagnetic field. In the limit $\tau\to t_h$ the incomplete Bessel function-related term appearing in (6) approaches to a time-independent quantity, meaning that the observation point starts to collect wave contributions from the whole Huygens sphere. Upon combining equation (6) with equation (5), and setting for shortness:

$$\bar{\imath}_n(\xi,w)=i_n(\xi,\min\{1,w\})\,u(1+w)\qquad(w\in\mathbb{R}), \tag{9}$$

the time-domain electric field radiated by the antenna in the Fraunhofer region is obtained as the following superposition of propagating non-uniform spherical waves attenuating along with the radial distance and time according to the real part of the complex poles $s_{n,m,k}$:

$$\boldsymbol{\mathcal{E}}(r,\vartheta,\varphi,t) \simeq \frac{t_h R_h}{r} \sum_{n=0}^{N} \sum_{m=-n}^{n} \sum_{k=1}^{K} s_{n,m,k} e^{s_{n,m,k}\tau} \bar{\imath}_n\left(s_{n,m,k} t_h, \frac{\tau}{t_h}\right) Y_n^m(\vartheta,\varphi)\, \boldsymbol{\rho}_{n,m,k}. \tag{10}$$

Finally, applying (10) and the shifting property of the unilateral Laplace transform $\mathcal{L}\{\cdot\}$, the complex frequency-domain equivalent representation immediately reads:

$$\boldsymbol{E}(r,\vartheta,\varphi,p) = \mathcal{L}\left\{\boldsymbol{\mathcal{E}}(r,\vartheta,\varphi,t)\right\}(p) \simeq t_h R_h \frac{e^{-jk_0 r}}{r} \sum_{n=0}^{N} \sum_{m=-n}^{n} \sum_{k=1}^{K} \frac{s_{n,m,k}}{p - s_{n,m,k}} i_n(p t_h)\, Y_n^m(\vartheta,\varphi)\, \boldsymbol{\rho}_{n,m,k}, \tag{11}$$

where $k_0 = -jp/c_0$ is the free-space complex wave-number. By using (10) and (11), one can determine the antenna far-field response to any arbitrary excitation.

3 Pole/Residue Modeling of Bow-tie Antennas

The developed SEM-based modelling approach has been applied to the investigation of the radiation properties of canonical bow-tie antennas, whose electromagnetic characteristics are well-known in the scientific literature. To this end, the near-field analysis of the considered structures has been carried out by means of a suitable full-wave finite-difference time-domain ($FDTD$) procedure. Each antenna, featuring a flair angle $\Theta = 80°$ and total length ℓ, has been meshed on a suitable uniaxial-perfect-matched-layer-backed non-uniform computational grid, and excited by the Gaussian voltage pulse:

$$v_g(t) = V_g \exp\left[-\left(\frac{t - T_0}{T_g}\right)^2\right] u(t), \tag{12}$$

with $V_g = 1\ V$ and $T_0 = 10\ T_g$, the characteristic time constant T_g being selected in order to have the source significant energy in the frequency band from up to $f_{\max} = 6\ GHz$, namely $T_g = \sqrt{\ln 10}/(\pi f_{\max}) \simeq 0.08\ ns$. The voltage generator, having internal resistance $R_g = 50\ \Omega$, is coupled into the $FDTD$ update equations of the electric field distribution within the dipole feeding delta gap, and the normalized incident/reflected waves excited at the input terminals of the antenna are used to evaluate the relevant return loss, whose frequency-domain behavior, as function of the flair length, is shown in Fig. 1a. By using the proposed formulation, the time-variant spherical harmonic expansion vector coefficients $\left\{\boldsymbol{\psi}_{n,m}(t)\right\}$ of the surface equivalent currents excited on the Huygens sphere S_h with radius $R_h = \ell$ have been computed on-the-fly in step with the $FDTD$ numerical simulation. In doing so, a suitable local inverse distance weighting (IDW) interpolation technique has been adopted to derive a cardinal series (CS) representation of the electromagnetic field distribution over S_h [1]. Truncation errors arise when limiting the order of the CS expansion to a finite discrete angular bandwidth N. To mitigate such problem, the order N in (3) is to be specified, according to the theory of the optimal interpolation of the radiated electromagnetic fields over a sphere, as:

$$N = \left\lceil \frac{2\pi\chi R_h}{c_0 \Delta_g} \right\rceil, \tag{13}$$

$\lceil\cdot\rceil$ being the usual ceiling function, and where Δ_g denotes the source pulse width. In (13) the excess bandwidth factor χ is used to control the approximation error which decreases more than exponentially with $\chi - 1$. So, upon assuming $\chi = 1.1$ and $\Delta_g = 5\ T_g \simeq 0.4\ ns$ the optimal order is found to be $N \simeq \lceil 55\pi\ell/3 \rceil$. Afterwards, the time-variant current coefficients have been fitted to the pole/residue expansion (4) with order $K = 10$ selected heuristically in such a way as to guarantee an adequate degree of accuracy in the modelling of the natural resonant processes occurring in the structure. The location of the antenna poles in the complex frequency domain is shown in Fig. 1b for different lengths of the radiating flairs. The dominant poles featuring the smallest damping coefficient $\sigma_{n,m,k} = -\mathrm{Re}\{s_{n,m,k}\}$ can be readily noticed. As expected from theory, the relevant spherical harmonic mode, having orders $n = 1$ and $m = 0$, is characterized by resonant radian frequency $|\mathrm{Im}\{s_{n,m,k}\}| = |\omega_{n,m,k}| \simeq \pi c_0/\ell$, where the electrical length of each dipole flair is about $\pi/2$. It has been found out that the damping coefficient of the dominant poles tends to increase as ℓ becomes larger. The reason for the observed behavior lies on the fact that longer bow-tie dipoles transmit longer-in-time radio signals. As it can be easily inferred, the dominant poles affect strongly the late-time radiation properties of the antenna, whereas the higher-order poles with larger attenuation coefficients account for the very early transient characteristics. The presented non-uniform spherical wave representation (10) has been

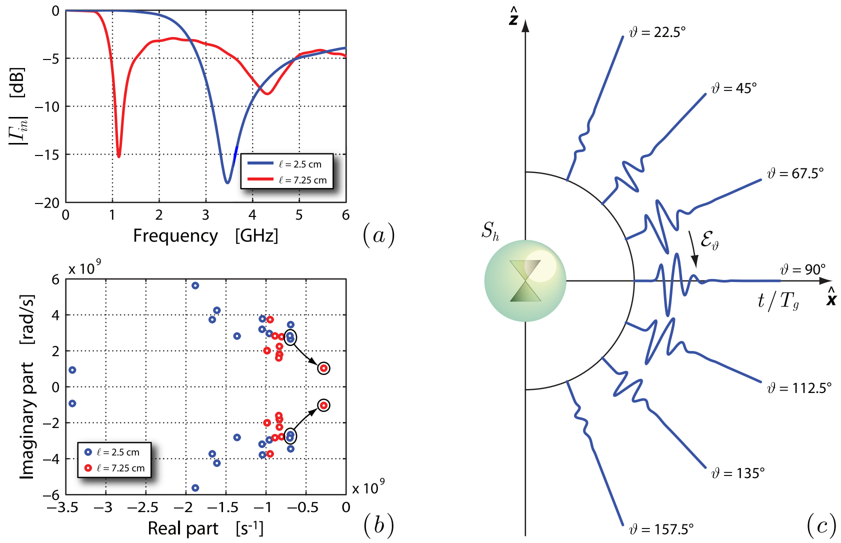

Figure 1: Frequency behavior of the input reflection coefficient (a), and location in the complex plane of the dominant resonant poles (b) of a bow-tie antenna having flair angle $\Theta = 80°$ and length ℓ. The space-time distribution of the radiated $\mathcal{E}_\vartheta$ field component excited by the antenna with $\ell = 2.5\ cm$ is also shown (c).

finally applied to evaluate the transient behavior of the radiated electromagnetic field. In this way, one can observe the different wave-fronts propagating in the air region $r > R_h$ outside the Huygens sphere enclosing the antenna. As it appears from Fig. 1c, the wave-front related to the radiation process occurring at the dipole feeding point is nearly a sphere meaning that, at any observation point, the radio wave contribution due to the instantaneous effect of the driving voltage arrives at the same time. The interaction of this wave with the open ends of the antenna is responsible for the excitation of a diffracted field resulting in two symmetrical wave-fronts which interfere constructively along the broadside direction and propagate back to the feed point, hence generating secondary emission/diffraction phenomena. This process repeats resulting in the well-known ringing effect. Finally, the electromagnetic field behavior in the frequency domain can be determined by using (11) with $p = 2j\pi f$. In this way, it has been pointed out that the magnitude of the radiated electric field exhibits the typical $1/r$ spatial decay with global maximum for $\vartheta = 90°$.

4 Conclusion

An accurate and computationally efficient *SEM*-based approach has been developed for the unified time- and frequency-domain characterization of complex antennas[1]. By the use of a dedicated two-step vector fitting procedure, a pole/residue expansion of the transient electromagnetic field distribution radiated in the Fraunhofer region has been derived. In this way, the far-field antenna response has been expressed analytically in terms of incomplete modified spherical Bessel functions as the superposition of propagating non-uniform wave contributions accounting for the complex resonant processes occurring in the structure under analysis. The developed procedure has been applied to the analysis of the radiation properties of canonical bow-tie antennas. In particular, the relation between the dominant poles and the antenna geometry has been thoroughly investigated. The antenna transient response evaluated by means of a minimal pole/residue model has been found to be in good agreement with theory.

5 References

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