1/R²-Kernel Cancellation in Classical Marching-On-in-Time Schemes

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Abstract

Cancellation of 1/R²-associated integrals in the numerical solution procedure of the time-domain magnetic field integral equations are explained to halve the computational costs of the classical marching-on-in-time (MOT) schemes in which the delay time is approximated by the barycentric electric distance of the subdomain patches. The new technique improves the accuracy of the MOT as the resulting interpolation coefficients for the calculation of the impedance matrices became independent of the electrical distance of surface elements. Additionally, the analytical closed-form expressions for the evaluation of the remaining 1/R³-associated integrals allow the fast computation of the retarded matrices on demand. The applicability of the scheme to large-scale wave scattering problems is investigated.

1. Introduction

Time-domain surface integral equations, which are perfectly suited for the analysis of electromagnetic wave scattering from three-dimensional objects, are numerically solved using the time-domain boundary element methods [1-7]. Depending on the choice of time discretization approaches, several categories of integral solvers have been attained. Among them, the marching-on-in-time (MOT) schemes, because of the resulting sparse matrices, are yet preferably used when the radiation of electrically large structures are to be modeled. In the classical MOT schemes, to integrate the temporal variable out of the existing double surface integrals, the time that takes for the wave to travel between arbitrary points on the observation and source subdomains (generally shown by R/c) is approximated by the electrical distance between the barycenter of the associated patches. The space quadratures are then evaluated numerically and kept in memory to be used during the construction of the matrix equation in the subsequent time steps.

The kernel of the integral equations can be formulated as a multiplication with the expression $\left[\frac{R}{c} \frac{\partial}{\partial t} + 1 \right]$,

representing the superposition of two contributions. This paper explains how the $\frac{R}{c} \frac{\partial}{\partial t}$ term can be canceled during the interpolation between past solution samples without affecting the late-time stability or accuracy of the MOT schemes. The technique halves the computational costs of the MOT schemes and it can be used in other implementations. In addition, the Wilton's compact formulas for the potentials due to linearly varying source distributions on polygonal patch cells are employed to avoid any numerical quadrature on the source subdomains. This notably facilitates the evaluation of the interaction matrices regardless of the shape of the source (triangular or quadrilateral) subdomains. As a result, performing the scattered field-current multiplication on the fly only for nonzero matrix-vector elements speeds up building the final system of linear equation at every time step and significantly reduces the memory demands of the MOT methods.

2. Formulations

Let S represent the surface of a closed perfect electric conducting body illuminated by an incident plane-wave with the magnetic field $\mathbf{H}^i$. Enforcing the boundary condition $\mathbf{J} = \mathbf{n} \times \mathbf{H}$, which sets the total (incident plus scattered) magnetic field tangential to the exterior surface conformal to S equal to the unknown induced surface current density $\mathbf{J}(\mathbf{r}, t)$, finally results in the time-domain magnetic field integral equation (MFIE) [1-7]. Taking the existing curl operator into the integral and extracting the Cauchy principal value afterwards, the MFIE can be represented by

$$\frac{\mathbf{J}(\mathbf{r}, t)}{2} - \mathbf{n} \times \frac{1}{4\pi} \int_{S_0} \left[\frac{R}{c} \frac{\partial}{\partial t} + 1 \right] \mathbf{J}(\mathbf{r}', \tau) \times \frac{\mathbf{R}}{R^3} dS' = \mathbf{n} \times \mathbf{H}^i(\mathbf{r}, t) \quad (1)$$

where S_0 denotes the surface S from which the contribution of the singularity at $R=0$ has been removed, $\mathbf{n}$ denotes an outward-directed unit vector normal to S , $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ and $R = |\mathbf{R}|$ represents the distance between the observation and source

points $(\mathbf{r}, \mathbf{r}')$ on S_0 , $\tau = t - R/c$ is the retarded time and c is the velocity of propagation of electromagnetic wave in the medium.

To numerically solve (1), the structure is first discretized into triangular (and quadrilateral) patches, $\mathbf{J}(\mathbf{r}, t)$ is then approximated by M divergence conforming vector basis functions $\mathbf{f}_k(\mathbf{r})$, $k=1, 2, \dots, M$ associated with each edge of the mesh and N temporal basis functions $T_n(t)$, $n=1, 2, \dots, N$ as

$$\mathbf{J}(\mathbf{r}, t) = \sum_{k=1}^M \mathbf{f}_k(\mathbf{r}) \left[\sum_{n=1}^N I_{k,n} T_n(t) \right]. \quad (2)$$

The study of delay differential equations shows that the usage of casual and piecewise continuous triangular functions

$$T_n(t) = \begin{cases} 1 - \left| \frac{t}{\Delta t} - n \right|, & \left| \frac{t}{\Delta t} - n \right| \leq 1 \\ 0, & \text{elsewhere} \end{cases} \quad (3)$$

for the time integration and interpolation provides a stable scheme [1]. Due to the presence of the retarded time τ in (1), the necessary knowledge of past solution requires an interpolation between known past solution samples using the subdomain time evolution function (3), which results in the expression

$$\sum_{k=1}^M \left[\frac{R_{mk}}{c} \frac{\partial}{\partial t} + 1 \right] I_{k,n} T_n(t - \frac{R_{mk}}{c}) = \sum_{k=1}^M [n - (r-1)] I_{k,r} - (n-r) I_{k,r-1}. \quad (4)$$

Here, R_{mk} implies the distance between arbitrarily observation and source points respectively on S_m and S_k subdomains and the retarded integer index r is given by the relation

$$r-1 < n - \frac{R_{mk}}{c\Delta t} \leq r. \quad (5)$$

Applying the Galerkin's testing by taking the inner product of the resulting equation with the same spatial bases $\mathbf{f}_m(\mathbf{r})$ $m=1, 2, \dots, M$ and using point matching at time instances $n\Delta t$ yields N sets of matrix equations with size M that have to be solved recursively in time for unknowns $I_{k,n}$

$$[\mathbf{Z}]_{mk,0} [\mathbf{I}]_{k,n} = [\mathbf{V}]_{m,n} - \sum_{r=1}^{n-1} [\mathbf{Z}]_{mk,n-r} [\mathbf{I}]_{k,r}. \quad (6)$$

The outer surface integrals over the observer patches S_m are approximated by the value of the respective integrand at the centroid of the subdomains. Therefore, the vector of interest $(\mathbf{r}' - \mathbf{r}_i) \times \mathbf{R}$, in which $\mathbf{r}_i$ is the free vertex of i^{th} side of S_k , can be substituted by $(\mathbf{r}^c - \mathbf{r}_i) \times \mathbf{R}$ whose $(\mathbf{r}^c - \mathbf{r}_i)$ part is not a function of $\mathbf{r}'$ and can be taken out of the interior space integral. As a result, to construct the interaction matrices only $\mathbf{R}/R^3$ -associated integrals remain. Wilton compact formulas for the potentials due to linearly varying source distributions can be used to avoid numerical integrations on polygonal patches

$$I_{-3} = \int_{S_k} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{r}' = \sum_{i=1}^{3 \text{ or } 4} \hat{u}_i I_i^{-1} + \hat{n} d I_{-3}, \quad (7)$$

where the integrals I_i^{-1} and I_{-3} are available in closed-forms

$$I_i^{-1} = \int_{\partial_i S_k} \frac{1}{|\mathbf{r} - \mathbf{r}'|} dl = \ln \left(\frac{R_i^+ + s_i^+}{R_i^- + s_i^-} \right), \quad I_{-3} = \int_{S_k} \frac{1}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{r}' = \frac{1}{|d|} \sum_{i=1}^{3 \text{ or } 4} \left[\arctan \left(\frac{P_i s_i^+}{R_i^{0^2} + d R_i^+} \right) - \arctan \left(\frac{P_i s_i^-}{R_i^{0^2} + d R_i^-} \right) \right] \quad (8)$$

and $\partial_i S_k$, $i=1, 2, 3$, (or 4) are the edges of the source triangle (or quadrilateral) patches. All the parameters introduced in (8) for analytical evaluation of (7) are obtained by projecting the observation point onto the plane containing the source subdomain, as illustrated in Fig. 1(a).

The storage of all matrices $\mathbf{Z}_{m-r}$ in memory in order to evaluate their convolution product with the past current vectors at each time step causes large memory consumption. Since expression (7) significantly facilitates evaluating the coupling of source elements on near test subdomains, the right hand side of the MOT system equation in (6) can be built by element-wise multiplication on-demand and accumulation with the excitation vector can be handled on the fly. Note that cancelation of the first term in (4) can also be integrated into the numerical solution procedure of the time-domain electric field integral equation (EFIE), using the formulation

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_S \frac{\mathbf{J}(\mathbf{r}', \tau)}{R} dS' + \int_S \left[\frac{R}{c} \frac{\partial}{\partial t} + 1 \right] \nabla_{\mathbf{r}'} \cdot \mathbf{J}(\mathbf{r}', \tau) \frac{\mathbf{R}}{R^3} dS' = \frac{4\pi}{c\eta} \mathbf{n} \times \left(\mathbf{n} \times \frac{\partial}{\partial t} \mathbf{E}^i(\mathbf{r}, t) \right). \quad (9)$$

3. Numerical Results

Fig. 1(b) and (c) illustrate the x -directed transient induced surface current density at the center of the top face of a perfectly conducting cube, 0.2m on a side illuminated by an x -polarized Gaussian plane-wave propagating along the z -direction with a full-width half maximum of 0.7071 m. The cube is discretized into 3072 triangular patches with $M=4608$ common edges and the applied MFIE is solved by the MOT schemes with time step size $\Delta t = 10.3 \min\{R_{mk}\}/c$ purely using adaptive numerical quadratures and Duffy's transformation for singularity cancellation [1] or alternatively the analytic space integration (7). The values are compared with high-resolution finite integration technique (FIT) results. Fig. 1(d) exhibits the difference with respect to the volume discretizing FIT reference solution. The maximum difference generated due to the $\mathbf{R}/R^2$ cancellation by (4) is less than 3×10^{-3} A/m, or 0.44%.

Fig. 2 shows the induced surface current magnitude on a 130.8 m-long free-standing ship at different time instances $t_n = n\Delta t$ illuminated by a z -polarized Gaussian plane-wave traveling along the ship in x -direction with the pulse specifications $cT=40$ m and offset $ct_0=132$ m. The ship is discretized into 21950 triangles and the applied MFIE is solved for $M=32925$ degrees of freedoms using the MOT with safety factor $\alpha=20$ resulting in $c\Delta t=3.0747$ m.

Fig. 1: (a) A typical edge element and the definition of all the parameters needed for analytical evaluation of the interior space integral (7) through projection of the arbitrary observation point $\mathbf{r}$ on the plane of the source. The induced surface current in linear (b) and logarithmic (c) scales, as well as the relative error (d) at the center of top face of the conducting cube under a normal Gaussian plane-wave incidence. The results are obtained solving the MFIE using the MOT with canceled $\mathbf{R}/R^2$ -term during the interpolation and analytical integration only on the remaining $\mathbf{R}/R^3$ -associated kernel.

Fig. 2: The induced surface current magnitude on a 130.8 m-long free-standing ship at different time instances $t_n = n\Delta t$, illuminated by a z -polarized Gaussian plane-wave traveling along the ship in x -direction.

4. Conclusions

Traditional MOT schemes, in which a constant time signature within the time intervals is assumed over the domain of current patches, were manipulated to solve the MFIE efficiently. The new formulation halves the computational effort in calculation of the interaction matrices. The closed-form expressions conventionally used only for singularity extraction in the frequency-domain method of moment were utilized here for the fast calculation of all matrix entries so as to avoid any numerical quadrature in the MOT methods. Summing up the contributions of the scattered fields on the fly in the excitation vector extensively shrinks the memory requirements. Realistic examples confirmed the effectiveness of the proposed scheme for characterizing complex structures.

5. References

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