Beyond the *LSM* as qualitative *MWI* method: a physical insight into the *far-field* equation and new challenges

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Abstract

The Linear Sampling Method (LSM) is commonly known as a qualitative method able to image the shape of a target by solving a linear equation, namely the *far-field* equation, and by plotting the energy of the solution over an arbitrary grid of points which sample the investigated domain. Starting from the physical interpretation of the LSM as a focusing strategy, the goal of this contribution is to show how this method can *implicitly* provide additional information about the scattering phenomenon through the solution of the *far-field* equation. Then the opportunity to employ such an information in the framework of quantitative reconstruction is also explored by means of a novel hybrid imaging approach.

1 Introduction

The microwave imaging (MWI) problem consists in the retrieval of the electromagnetic and/or geometrical features of an unknown scenario by probing it with an incident radiation, and by measuring and processing the field it scatters. Generally speaking, MWI approaches can be classified as *quantitative* or *qualitative* [1]. Quantitative methods attempt at providing an estimation of both the electromagnetic and the geometrical properties of the targets possibly lying in the investigated scenario, whereas qualitative methods aim at determining only the location and (possibly) the shape of the unknown target.
The Linear Sampling Method (LSM) is a very popular qualitative method, which provides a simple, effective and computationally efficient approach to image the shape of an unknown scatterers, without requiring approximations or a priori information on the target [2]. In particular, due to its very low computational burden, the LSM has been also used as a pre-processing step for hybrid quantitative approaches in which the retrieved shape of the unknown target is exploited to counteract the non linearity and the ill-posedness of the inverse scattering problem [3].

In this contribution, inspired by the analogy which holds between the LSM and the problem of focusing an electromagnetic field in an inhomogeneous medium [4], a new way to exploit the LSM is addressed. In particular, the basic observation is that the LSM equation can be regarded as an attempt to rearrange the incident fields in such a way that the scatterer behaves like a point target located in the sampling point. Hence, by considering as probing wave the one resulting from such a rearrangement of the incident fields, it is possible to obtain, through the solution of the LSM, an approximated expression of the total field in the investigated domain. Then, by repeating this procedure for several positions of the sampling point, we devise a new inversion strategy in which the LSM preprocessing is used to define a set of synthetic primary sources capable of enforcing this peculiar behavior of the target (with respect to different points).

2 Statement of the problem and rationale

We assume a TM polarization and a 2D geometry. Let Ω denote the region under test in which the targets are located. The contrast function $\chi(r) = \epsilon_s(r)/\epsilon_b - 1$ relates the unknown properties of the scatterers to those of the host medium, wherein ϵ_s and ϵ_b are the complex permittivities of the scatterer and of the background medium, respectively. The targets are probed by means of V incident fields coming from different

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incident directions θ_v, $v = 1, ..., V$, and their scattered fields are measured at M locations on a curve Γ, r_m, $m = 1, ..., M$.
Under these hypotheses, the electromagnetic scattering problem is cast as a pair of integral equations:

$$\begin{aligned} E_s(r_m, \theta_v) &= k_b^2 \int_\Omega G(r, r_m)\chi(r)E_t(r, \theta_v)dr \\ &= \mathcal{A}_e[\chi E_t], \quad r_m \in \Gamma, v = 1, ..., V \end{aligned} \tag{1}$$

$$\begin{aligned} E_t(r, \theta_v) - E_i(r, \theta_v) &= k_b^2 \int_\Omega G(r, r')\chi(r')E_t(r', \theta_v)dr' \\ &= \mathcal{A}_i[\chi E_t], \quad r \in \Omega, v = 1, ..., V \end{aligned} \tag{2}$$

where k_b is the background wave-number, E_s, E_i and E_t denote the scattered, incident and total field for the v-th illumination directions, respectively. $G(r, r')$ is the scalar Green's function of the homogeneous background medium, while $\mathcal{A}_e : L^2(\Omega) \rightarrow L^2(\Gamma)$ and $\mathcal{A}_i : L^2(\Omega) \rightarrow L^2(\Omega)$ are the internal and external radiation operators, relating the contrast source χE_t to the scattered field on Γ and in Ω, respectively.

The LSM consists in solving, in each point of an arbitrary grid which samples Ω, the linear system:

$$\mathbf{F}x = \phi(r_s, r_m), \tag{3}$$

where $\mathbf{F}$ denotes the multistatic multiview data matrix, while the right-hand side is the field pattern on Γ radiated by a point-like source located in r_s, i.e. $\phi(r_s, r_m) = -\frac{j}{4}H_0^2(k_b|r_s - r_m|)$, H_0^2 being the 0-*th* order and 2-*nd* kind Hankel function.
The system in eq.(3) is ill-conditioned [1], so that its solution requires the enforcement of a suitable regularization. In particular, by using the Singular Value Decomposition of $\mathbf{F}$ and the Tichonov regularization, the solution of (3) in each sampling point r_s is given as:

$$x(r_s, \theta_v) = \sum_{n=1}^{N} \frac{\lambda_n}{\lambda_n^2 + \alpha^2} \langle \phi(r_s, r_m), u_n(r_m) \rangle v_n(\theta_v) \tag{4}$$

wherein λ_n denotes the singular values of $\mathbf{F}$, while u_n and v_n are its left-hand and right-hand singular vectors respectively; α is the Tichonov parameter and $N = min\{M, V\}$.
According to the LSM theory, the norm of (4), $\sum_{v=1,V} |x(r_s, \theta_v)|^2$, is bounded when the sampling point is inside the scatterer, while it blows when the sampling point belongs to the background region [2].
Therefore, one can introduce the LSM support indicator:

$$\Upsilon(r_s) = 20log_{10}\{\|x(r_s)\|^2 / \|x(r_s)\|^2_{max(\Omega)}\}, \tag{5}$$

whose plot over Ω provides an image of the unknown support.

3 A new linear inversion strategy

A possible way to explain the behavior of the LSM indicator is related to the physics which underlies the method, since the LSM equation can be regarded as an electromagnetic focusing problem [4]. As a matter of fact, given the linear relationship which holds between scattered fields and incident ones, see eqs.(1)-(2), solving (3) is equivalent to determine a linear combination of the incident fields such to induce in the scatterer a contrast source that supports the radiating behavior enforced by the right hand side of (3). Since this latter corresponds to the field of a point-like current centered on the sampling point, one can expect that such a current is the one that will be approximately induced in Ω. Let us now assume that x is the solution of the LSM equation in a sampling point which belongs to the scatterer. Then, we define the incident field:

$$\Psi_i(\mathbf{r}_s, \mathbf{r}) = \sum_{\nu=1}^{N_\nu} x(\mathbf{r}_s, \theta_\nu) E_i^\nu(\mathbf{r}), \tag{6}$$

wherein N_ν is the number of transmitters, while θ_ν denotes the illumination direction. According to the above mentioned physical interpretation, such a primary field is the one that induces, for each sampling point belonging to the support of the scatterer, a contrast source whose far-filed pattern on the measurement curve Γ corresponds to the right hand side of eq. (3), computed as $\psi_s(\mathbf{r}_s, \phi_m) = \mathbf{F}x$. Moreover, the primary field expressed by eq.(6) depends on the particular scatterer at hand and on a particular sampling point, whereas the scattered field that is produced on Γ is always the same regardless the nature of the unknown scatterer.

By taking into account the continuity of the field and by neglecting its evanescent part (which enforces the near-field behaviour of the target), as well as its non radiating component, the above reasonings suggest that the scattered field can be approximated with $\phi(r_s, r)$ where $r \neq r_s$, since the field cannot be singular in Ω. Accordingly, the total field can be expressed as:

$$\widetilde{\Psi}_t(\mathbf{r}_s, r) = \Psi_i(\mathbf{r}_s, r) + \widetilde{\phi}(r_s, r), \tag{7}$$

wherein the second addendum denotes a low-pass filtering of the Hankel function in order to restore the physical consistency of the field. By replicating the above considerations for different sampling points belonging to the scatterer, it is possible to develop a new set $P \leq V$ of scattering experiments in which the incident field and the measured scattered one are provided by equation (6) and the linear combination $\mathbf{F}x$, respectively. Notably, the set P of sampling points take the role of the set V of illumination direction. For this reason we refer to it as multifocus-multistatic scattering experiments by stressing that only a synthetic computation procedure is required to build them, without further collection of measurement data. Finally, we can formulate a multifocus-multistatic inversion method in which the unknown contrast function is retrieved by means of linear inversion of the data scattering equation (1), by minimizing the following cost functional:

$$\Xi(\chi) = \sum_{n_p=1}^{P} \frac{\|\psi_s - A_e(\chi \widetilde{\Psi}_t^p)\|^2}{\|\psi_s\|^2}. \tag{8}$$

Moreover, we exploit a regularization by projections, in which spatial Fourier harmonics are adopted to represent the unknown functions. Furthermore a progressive enlargement of the searched parameters has been employed during the inversion procedure in order to defeat the ill-posedness of the problem [5].

4 Example

In order to show the validity of the formulation we are proposing to solve the inverse scattering data equation, in the following we show a numerical example in which the assessment of the new proposed method has been performed. The unknown scenario consists in a square domain of 1.5λ in free space in which are embedded 9 square shaped scatterers with same size and electromagnetic properties $\epsilon_s = 2$ and $\sigma = 1e-2$ S/m, see Fig.1 (a)-(b). The multiview multistatic data matrix has been collected in 18 equally spaced points on a circumference of radius 1.5λ, by probing the domain with an equal number of line source transmitters. Moreover the scattered field has been corrupted with a gaussian noise with SNR of 20dB. The LSM indicator, see Fig.1 (c), is able to retrieve the number and approximately the shapes of the scatterers system. Thereafter, we reformulate the multifocus-multistatic synthetic scattering experiments by selecting the LSM solutions pertaining to the sampling points as marked in Fig.1 (d). In particular, in order to estimate the total field by means of eq.(6), we adopt a filtering operation which provides a cutoff for all the spatial Fourier harmonics higher than the essential dimension of the scattered field [6]. The results achieved by minimizing the cost functional (8) are depicted in Fig.1 (e)-(f); they show the effectiveness of the new hybrid approach, since the reconstruction error is 25%, which is comparable with the performance of standard non linear approaches. Note the computational burden is extremely low since the algorithm converges after few iteration number.

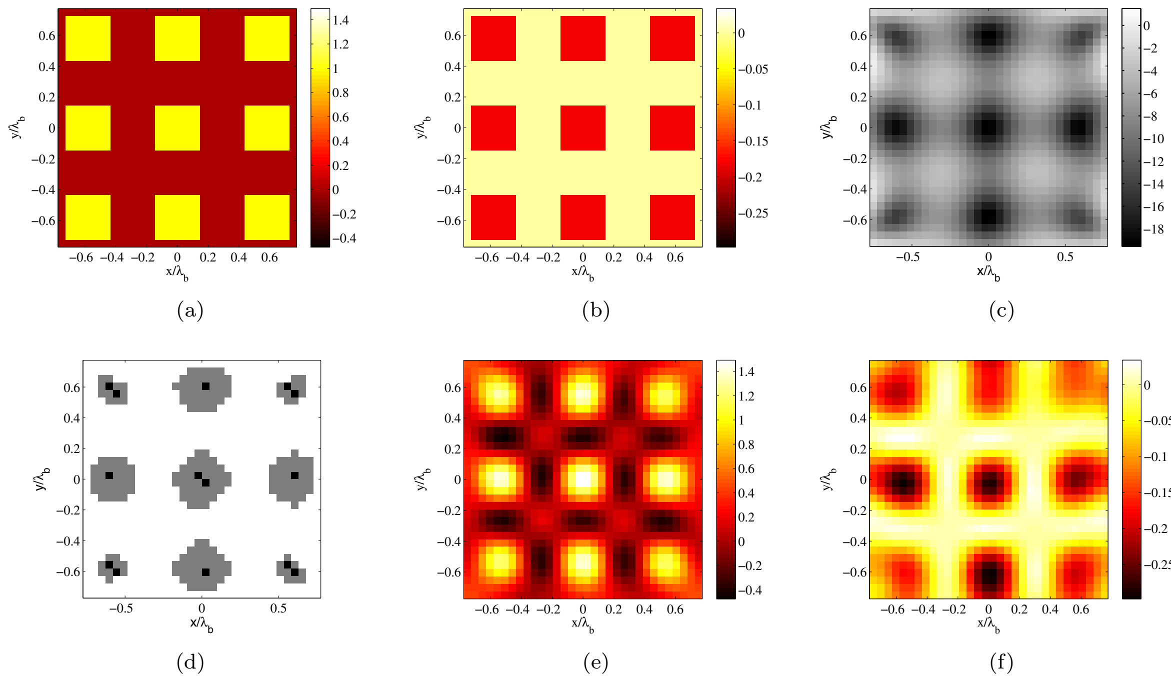

Figure 1: Reference contrast profile: (a) real part; (b) imaginary part. (c) Map of the LSM indicator Υ; (d) binary version of Υ used to select the sampling points (in dark) for the multi-focusing experiment. Reconstructed profile: (e) real part; (f) imaginary part.

5 Conclusion

In this contribution a novel way to exploit the LSM as a pre-processing step for quantitative inversion has been explained. In particular, by introducing the idea of multifocus-multistatic synthetic scattering experiments, the possibility of employing additional information associated with the solution of the far-field equation has been exploited in the context of quantitative reconstruction. Then, a new method has been proposed to achieve quantitative reconstructions by solving only a linear problem. Preliminary results are very satisfactory both in terms of reconstruction error and computational effectiveness.

References

[1] D. Colton and R. Kress, *Inverse acoustic and electromagnetic scattering theory.* 2 ed.: Springer, 1998.

[2] D. Colton, H. Haddar, and M. Piana, "The linear sampling method in inverse electromagnetic scattering theory," *Inverse Problems*, vol. 19, pp. 105–137, 2003.

[3] I. Catapano, L. Crocco, M. D'Urso, and T. Isernia, "On the effetc of support estimation and of a new model in 2-d inverse scattering problem," *IEEE Trans. Antennas Propagat.*, vol. 55, pp. 1895–1899, 2007.

[4] I. Catapano, L. Crocco, and T. Isernia, "On simple methods for shape reconstruction of unknown scatters," *IEEE Trans. Antennas Propagat.*, vol. 55, pp. 1431–1436, 2007.

[5] T. Isernia, V. Pascazio, and R. Pierri, "A nonlinear estimation method in tomographic imaging," *IEEE Trans. Geosci. Remote Sensing*, vol. 35, pp. 910 – 923, 1997.

[6] O. M. Bucci and T. Isernia, "Electromagnetic inverse scattering: retrievable information and measurement strategies," *Radio Science*, vol. 32, pp. 123–2137, 1997.