Enhancing Subspace-Based Inversions through an Efficient Multi-Scaling Scheme

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Abstract

A microwave imaging approach based on the integration of a subspace-base optimization method (SOM) and a multi-focusing procedure is proposed. The scattering equations are expressed within the contrast source formulation of the electromagnetic inverse problem and then solved by a nested procedure which includes an outer multi-resolution loop (dealing with the identification of the regions of interest), a spectrum analysis step (devoted to the reconstruction of the ‘deterministic’ components of the contrast sources) and an inner optimization loop (aimed at the retrieval of their ‘ambiguous’ parts). A numerical analysis is presented to preliminarily assess the features of the proposed approach.

1. Introduction and Motivation

Electromagnetic inverse scattering problems arise in many applications such as radar imaging, non-destructive testing and evaluation, biomedical diagnostics, remote sensing, subsurface inspection, and material characterization [1-5], and several stochastic [1] and deterministic [6,7] techniques has been developed in the last years for their solution.

In this framework, deterministic techniques based on the contrast source (CS) formulation of the inverse scattering problem have received much attention recently [6,7]. Indeed, thanks to the linearity of the “data” equation, higher reconstruction accuracies and robustness with respect to the “contrast field” (CF) formulation have been obtained [6,7]. However, a limitation of CS-methods lies in the non-uniqueness of the associated inverse source problem [8], which is due to the non-radiating (or non-measurable) components of the unknown contrast source [8].

To overcome this drawback, algorithms which retrieve the minimum norm currents and then refine the non-measurable contrast-source components have been proposed [8,9]. In this framework, a subspace optimization method (SOM) has been recently introduced [10,11]. In this approach, the contrast source is subdivided in “deterministic” and “ambiguous” parts [10]. Then, the deterministic currents are computed by the spectrum analysis, whereas the ambiguous ones are determined by minimizing a cost functional including both “data” and “state” terms [10].

The SOM method has been shown to share several merits of CSI-based inversion techniques, while also allowing faster convergences [10]. However, such an approach presents also some limitations. More specifically, it can yield to poor performances if several ‘local minima’ are at hand, since it is based on a CG minimization [10]. Moreover, as the method requires the computation of the singular value decomposition (SVD) of the discretized Green operator, computationally expensive inversions can arise especially when dealing with several unknowns [10]. Furthermore, the choice of the SVD truncation factor can become critical if no information on the signal-to-noise ratio (SNR) is at hand [10].

To overcome these limitations, a new microwave imaging approach based on the integration of a multi-focusing strategy, namely the iterative multi-scaling approach (IMSA) [12-14], with the SOM method [10] is proposed. After having mathematically formulated the problem at hand (Sect. 2), the proposed IMSA-SOM method is briefly introduced in Sect. 3, and a preliminary numerical validation is presented (Sect. 4) to assess the potentialities of this approach in comparison with state-of-the-art SOM methods.

2. Mathematical formulation

Let us consider a two dimensional inverse scattering problem (z is the longitudinal direction) under transverse magnetic (TMz) incidence. In a lossless background with permittivity ε0, a cylindrical scatterer of arbitrary cross section is
successively irradiated by $V$ known monochromatic waves $E_{\text{inc}}^v(r) \hat{z}$, $v=1,...,V$ (whose time dependency factor $\exp(j\omega t)$ is omitted). For each incidence $v$ the scattered field $E_{\text{scatt}}^v(r) \hat{z} = E_{\text{inc}}^v(r) \hat{z} - E_{\text{inc}}^0(r) \hat{z}$ is sampled at $M$ measurement points lying in an observation domain $D_o$ (i.e., $r_o \in D_o$, $m = 1,..,M$) external to the investigation domain $D$. According to the CS formulation, the following ‘data’ equation holds true [10]

$$E_{\text{scatt}}(r_m) = k_o^2 \int_D J^v(r')G(r_m/r')dr'$$

where $k_o$ is the free space wavenumber, $J^v(r) = \tau(r)E_{\text{inc}}^v(r)$ is the contrast source, $E_{\text{tot}}^v(r)$ is the total field, $G(r/r')$ is the 2D free-space Green’s function, $\tau(r) = [\varepsilon(r) - j\sigma(r)/2\pi\varepsilon_0]$ the object function describing the material properties of the nonmagnetic scatterer (assumed constant with respect to $z$), and $\varepsilon(r)$ and $\sigma(r)$ indicate, respectively, the relative dielectric permittivity and electric conductivity inside the scatterer [10]. Moreover, the following ‘state’ equation holds true in the investigation domain [10]

$$\tau(r)E_{\text{scatt}}^v(r) = J^v(r) - k_o^2\tau(r)[J^v(r')G(r/r')dr'$$

where $r \in D_v$, $v = 1,...,V$.

The objective of CS-based inverse scattering techniques is to deduce $\tau(r)$, $J^v(r)$ and $E_{\text{tot}}^v(r)$ in $D$ from the knowledge of $E_{\text{scatt}}^v(r)$ ($v=1,...,V$) and of $E_{\text{scatt}}^v(r_m)$ ($m=1,...,M$, $v=1,...,V$).

3. Multi-Scaling Subspace Optimization Method

In order to solve (1) and (2), an IMSA-SOM method is introduced which is based on the nesting of (a) an external IMSA loop, whose objective is to correctly locate the regions of interest (RoIs) in the investigation domain [12], (b) a truncated SVD step, aimed at computing the “deterministic” currents in the RoIs [10], and (c) a CG-based iterative optimization step, devoted to find the ambiguous currents [10]. More specifically, the following procedure is proposed:

1. **Problem definition** - define $D$, $M$, $V$. For each view $v=1,...,V$, collect the scattered field at the $M$ measurement points $E_{\text{scatt}}^v = [E_{\text{scatt}}^v(r_1),...,E_{\text{scatt}}^v(r_M)]$;

2. **Initialization** - set $s = 1$. Define a (coarse) $N$-sized grid $r_s^{(s)} = [r^{(s)}_1,...,r^{(s)}_N]$ to discretize Eqs. (1) and (2) according to a coupled dipole method (CDM) [10];

3. **Matrix computation** - Evaluate the samples of the incident field $E_{\text{inc}}^v(r)$ as $E_{\text{inc}}^v(r) = [E_{\text{inc}}^v(r_1),...,E_{\text{inc}}^v(r_N)]$. Compute the Green matrices $H^{(s)}$ and $G^{(s)}$ according to

$$H^{(s)} = \begin{bmatrix}
g(r^{(s)}_1,r^{(s)}_1) & \ldots & g(r^{(s)}_1,r^{(s)}_N) \\
\vdots & \ddots & \vdots \\
g(r^{(s)}_N,r^{(s)}_1) & \ldots & g(r^{(s)}_N,r^{(s)}_N)
\end{bmatrix}$$

$$G^{(s)} = \begin{bmatrix}
g(r^{(s)}_1,r^{(s)}_1) & \ldots & g(r^{(s)}_1,r^{(s)}_N) \\
\vdots & \ddots & \vdots \\
g(r^{(s)}_N,r^{(s)}_1) & \ldots & g(r^{(s)}_N,r^{(s)}_N)
\end{bmatrix}$$

where $g(r^{(s)}_p,r^{(s)}_q)$ is the discretized Green operator [10];

4. **Estimation of deterministic currents** - Compute the SVD of $G^{(s)}=U^{(s)}\Sigma^{(s)}V^{(s)^\top}$ ($^\top$ being the transpose operator), select the ‘truncation factor’ $L^{(s)}$ [10] and deduce the deterministic part of the contrast source $I_{\text{det}}^{(s)}$ according to

$$I_{\text{det}}^{(s)} = V^{(s)^\top} (U^{(s)^\top})^{-1} I_{\text{scatt}}$$

where the subscript det identifies the truncated versions of the matrices at the $L^{(s)}$th singular value [10];

5. **Iterative estimation of ambiguous currents and contrast** - Minimize by $T^{(s)}$ iterations of a CG method the cost function

$$F^{(s)} = \sum_{c=1}^V \left( \Delta_{\text{scatt}}^{(s)} \right)^2 / \left\| E_{\text{scatt}}^v \right\| + \Delta_{\text{amb}}^{(s)} \right)^2 / \left\| I_{\text{det}}^{(s)} \right\| ^2$$

where $c$ is the CST.
where $\Delta_{\text{data}}^{(s)}$ and $\Delta_{\text{num.}}^{(s)}$ are the $s$-th view ‘data’ and ‘state’ misfits at the current IMSA step [computed by the discretized versions of (1) and (2)], respectively. Then evaluate the retrieved ambiguous currents and contrast from

$$I_{\text{amb}}^{(s)} = I_{\text{amb}}^{s} = I_{\text{amb}}$$

where $I_{\text{amb}}^{s} = (r_{\text{amb}}^{(s)} \sum_{n} I_{\text{amb}}^{s} (n))$, indicates that although both methods are able to identify the presence of a single hollow scatterer, the multi-focusing method yields to an higher accuracy in terms of retrieved contrast. Moreover, thanks to its zooming procedure, the IMSA-SOM completely avoids artifacts outside the object support, unlike the BARE-SOM [Fig. 1(b) vs. 1(c)].

4. Preliminary Numerical Assessment

A preliminary numerical analysis is presented in this section to assess the features of the proposed IMSA–SOM method with respect to state-of-the-art SOM approaches. Towards this end, an investigation domain of size $2.4\lambda \times 2.4\lambda$ (free-space background) is illuminated by $V=24$ TM plane waves. The total field is calculated through a MoM-based solver and an AWG noise (SNR=10 dB) is added to the its samples collected at $M=24$ measurement points (located on a circle of radius $1.8\lambda$). In all the inversion procedures, $T^{(b)}=100$ and $L^{(b)}$ has been selected so that the 90\% most significant singular values of the $G^2$ spectrum are included in the ‘deterministic’ current computation [Eq. (5)]. Moreover, the maximum number of IMSA steps has been set to $S=4$.

![Figure 1](image1.png)

Figure 1. [Hollow Cylinder, $\tau = 0.4$, SNR=10 dB] Actual (a) and retrieved $\tau$ by (b) BARE-SOM and (c) IMSA-SOM.

In the first numerical example, a lossless hollow scatterer with $\tau = 0.4$ is considered [Fig. 1(a)]. The plots of the retrieved dielectric profiles by a standard SOM method [BARE-SOM - Fig. 1(a)] and by the IMSA-SOM approach [Fig. 1(c)] indicates that although both methods are able to identify the presence of a single hollow scatterer, the multi-focusing method yields to an higher accuracy in terms of retrieved contrast. Moreover, thanks to its zooming procedure, the IMSA-SOM completely avoids artifacts outside the object support, unlike the BARE-SOM [Fig. 1(b) vs. 1(c)].

![Figure 2](image2.png)

Figure 2. [Parallel Cylinders, $\tau = 0.6$, SNR=10 dB] Actual (a) and retrieved $\tau$ by (b) BARE-SOM and (c) IMSA-SOM.
Similar conclusions hold true also when dealing with multiple scatterers. Indeed, the next numerical experiment, concerned with two parallel cylinders with \( \tau = 0.6 \) [Fig. 2(a)], confirms that the IMSA-SOM is able to correctly retrieve the position, shape and permittivity of the inspected objects, while the BARE methodology yields to a lower accuracy. It is also worth remarking that, despite the exploitation of a single-region focusing methodology, the two objects are correctly separated by the IMSA method. Moreover, a multi-region version of the IMSA-SOM approach able to efficiently deal with multiple distant objects is at present under development.

5. Conclusion

The integration of a multi-focusing procedure and a subspace-base optimization method (SOM) is introduced within the contrast source formulation of the electromagnetic inverse problem. A nested procedure is implemented for the solution of the arising inverse problem. The presented preliminary numerical validation has shown the effectiveness and accuracy of the method in comparison with a state-of-the-art SOM approach.

6. References