Transient Plane-wave decomposition of reflected TE Gaussian-Beam from Moving Dielectric-Magnetic Planar Interface

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Abstract

This paper is concerned with applying plane-wave decomposition for scattering of a pulsed-beam from a fast moving planar dielectric discontinuity under the frame work of Special Relativity.

1 Introduction

Pulsed-beams (PBs) are the basis for the Phase-pace Pulsed-Beam Summation Method which is a framework for analyzing radiation from extended sources as well as scattering from complex media. In this formulation, the field is expanded into a discrete spectrum of pulsed-beams propagators. These propagators emanate from the aperture plane in a given set of points, in a given set of directions and in a given set of delays [1-3]. Such PB solutions have been obtained in generic media profiles such as inhomogeneous [4-6], anisotropic [7-11] and more.

This paper investigates the reflection of an electromagnetic time-dependent PB by a fast moving planar half-space dielectric-magnetic medium (isotropic at the co-moving frame). The PB field is decomposed into its transient plane-wave (PW) constituents which are transformed into the $S'$-frame (co-moving) of reference in which the medium is at rest. The reflected PWs are obtained by applying Maxwell’s boundary conditions in order to obtained a spectral representation of the reflected PB. The resulting spectral integral is transformed back to the $S$-frame of reference (the "laboratory-frame"), yielding a PW spectral representation of the reflected PB. In the present paper we present only the TE PB, but the TM PB can be easily obtained in a similar manner.

2 Lorentz Transformation of EM Field

The moving reference frame’s origin is located at the moving dielectric, $z' = 0$, which is moving with velocity $v = vz$ with respect to the $S$ frame, so that $x' = x, y' = y, z' = z$. The corresponding Lorentz and inverse Lorentz Transformation is given by

\[
ct' = \gamma ct - \gamma \beta \cdot r, \quad ct = \gamma ct' + \gamma \beta \cdot r',
\]
\[
r' = \bar{\alpha} \cdot r - \gamma \beta ct, \quad r = \bar{\alpha} \cdot r' + \gamma \beta ct',
\]
where
\[
\beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \bar{\alpha} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \gamma
\end{bmatrix}.
\]

As outlined in the introduction, the PB is decomposed into there plane wave spectrum constituents. In order to apply Maxwell’s boundary conditions at the $S'$ frame’s $z' = 0$ plane, the propagating PBs are a priori decomposed into TE and TM components with respect to constant $z$ planes at the $S$ frame. Here we present the results for TE PB propagators. Next we transform the fields into the moving $S'$ frame representation using the Lorentz Transformation.
(1) and the Electromagnetic Field Transformation (3)[12].

\[ E' = \gamma \tilde{\alpha}^{-1} \cdot E + c\gamma \beta \times B, \]
\[ B' = \gamma \tilde{\alpha}^{-1} \cdot B - c^{-1}\gamma \beta \times E, \]
\[ D' = \gamma \tilde{\alpha}^{-1} \cdot D + c^{-1}\gamma \beta \times H, \]
\[ H' = \gamma \tilde{\alpha}^{-1} \cdot H - c\gamma \beta \times D, \]

(3)

where

\[ \beta = \frac{v}{c}, \quad \tilde{\alpha} = \tilde{I} + (\gamma - 1)\beta^{-2}\beta. \]

(4)

In (3), the medium in \( z' < 0 \) is assumed to be vacuum, i.e. \( \epsilon = \epsilon_0 \) and \( \mu = \mu_0 \).

3 TE reflected plane wave

The single transient incident PW at the \( S \) frame is given by

\[ E_{PW}^i(\mathbf{r}, t; \kappa_i) = \hat{\mathbf{u}}(\kappa_i) \int [t - c^{-1}(\hat{\kappa}_i \cdot \mathbf{r})], \]
\[ H_{PW}^i(\mathbf{r}, t; \kappa_i) = \eta_0^{-1} \hat{\mathbf{t}}(\kappa_i) \int [t - c^{-1}(\hat{\kappa}_i \cdot \mathbf{r})], \]

(5)

where \( \hat{f} \) denotes some analytic signal[13], \( \hat{\kappa}_i = (\xi_1, \xi_1, \zeta) \) is a unit vector with \( \xi_1 \) and \( \xi_2 \) being the transverse spectral variables and \( \zeta = \sqrt{1 - \xi_1^2 - \xi_2^2} \), and

\[ \hat{\mathbf{u}}(\kappa_i) = \frac{1}{\kappa_i}(\xi_2 \hat{x} - \xi_1 \hat{y}), \]
\[ \hat{\mathbf{t}}(\kappa_i) = \frac{\zeta}{\kappa_i}(\xi_1 \hat{x} + \xi_2 \hat{y}) - \kappa_i \hat{z} \]

(6)

with \( \kappa_i = \sqrt{\xi_1^2 + \xi_2^2} \).

The \( \hat{f} \) argument in (5) is inserted with (1) to produce

\[ t - c^{-1}(\hat{\kappa}_i \cdot \mathbf{r}) = \sigma[t' - c^{-1}(\hat{\kappa}_i' \cdot \mathbf{r}')], \]

(7)

where

\[ \hat{\kappa}_i' = \frac{\hat{\kappa}_i \cdot \tilde{\alpha} - \gamma \beta}{\gamma(1 - \tilde{\kappa}_i \cdot \beta)} = \frac{1}{\sigma}[(1, 0, \gamma(\zeta - \beta)], \quad \sigma = \gamma(1 - \zeta \beta). \]

(8)

Here \( \hat{\kappa}_i' \) is identified as a unit vector in the direction of propagation in the \( S' \)-frame. By inserting (5) into (3) with (7) we obtain the electric incident field at the \( S' \) frame

\[ E_{PW}^i = \frac{\gamma}{\kappa_i}(1 - \beta \zeta)(\xi_2 \hat{x} - \xi_1 \hat{y})f^+ [\sigma(t' - c^{-1}\hat{\kappa}_i' \cdot \mathbf{r}')] \]

(9)

and similarly

\[ H_{PW}^i = \frac{1}{\eta_0} \left[ \frac{\gamma}{\kappa_i}(\zeta - \beta)(\xi_1 \hat{x} + \xi_2 \hat{y}) - \kappa_i \hat{z} \right] f^+ [\sigma(t' - c^{-1}\hat{\kappa}_i' \cdot \mathbf{r}')] \]

(10)

By applying the standard procedure for PW scattering from planar dielectrics, we obtain snell’s law in \( S' \)

\[ \sin \theta_i' = \sin \theta_i, \quad n_2 \sin \theta_i' = \sin \theta_i'. \]

(11)
The resulting reflected PW fields in the $S'$ reference frame are given by

$$E_{PW}' = R' \gamma_{k_t}(1 - \beta \varsigma)(\xi_2 \hat{x} - \xi_2 \hat{y}) f^+[\sigma(t' - c^{-1} \hat{k}' \cdot \hat{r}')],$$

$$H_{PW}' = \frac{R'}{\eta_0 k_t} \left[ \frac{-\gamma_{k_t}(\varsigma - \beta)(\xi_1 \hat{x} + \xi_2 \hat{y}) - k_0^2 \hat{z}}{\eta_0 k_t} \right] f^+[\sigma(t' - c^{-1} \hat{k}' \cdot \hat{r}')]$$

(12)

where

$$R' = \frac{\cos^2 \theta' - \cos^2 \theta'' (n_2^2 - \sin^2 \theta'')}{\cos^2 \theta'' + \cos^2 \theta'' (n_2^2 - \sin^2 \theta'')},$$

(13)

is identified as the reflection coefficient and the propagation angle in $S'$ is given by where

$$\cos \theta' = \frac{\varsigma - \beta}{1 - \varsigma \beta} = \frac{\cos \theta'' - \beta}{1 - \beta \cos \theta''}.$$  

(14)

## 4 Reflected Beam Composition

The TE PB propagator can be defined by its transient PW spectral representation in the form

$$\hat{E}^+(r, t) = -\frac{\partial^2}{(2\pi)^2} \int d^2 k_t k_t \frac{1}{k_t} \hat{\psi}(k_t - k_t, t - c^{-1} k_t \cdot r),$$

(15)

where $\hat{\psi}(k_t, \tau)$ denotes the transient PW spectral distribution of the synthesis window $\frac{1}{k_t} \hat{\psi}(r, t)$ and $\hat{k}$ are the frame directional spectral variables. Next we apply the inverse Lorentz transform in (1) to each reflected transient PW in (9)-(10) and insert the resulting PW in $S$ frame into the spectral representation in (15). The resulting spectral representation of the reflected PB is given by

$$\hat{E}^+(r, t) = -\frac{\partial^2}{(2\pi^2)^2} \int d^2 k_t k_t \frac{1}{k_t} \hat{\psi}^+(r, t; k_t) \hat{n}(k_t) R'$$

(16)

where

$$\hat{\psi}^+(r, t; k_t) = \hat{\psi}(k_t - \hat{k}_t, \rho(t - c^{-1} \hat{k}' \cdot r + \tau'))$$

(17)

with

$$\hat{k}' = \frac{1}{\rho} [\xi_1, \xi_2, \gamma^2 (2c - \varsigma (1 + \beta^2)]$$

with

$$\rho = \gamma^2 (1 - 2 \varsigma \beta + \beta^2), \quad \tau' = \frac{(\beta - \varsigma) z_0}{c \gamma (1 - 2 \varsigma \beta + \beta^2)}.$$  

(18)

Equation (16) with (17) and (18) represent the reflected electric PB field in $S$ frame.

## 5 References


