Method of Moments Analysis of an Axisymmetric Chiral Radome

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Abstract

An axisymmetric chiral radome has been analyzed numerically by using the method of moments. The chiral body is illuminated by a plane wave and the surface equivalence principle is used to replace the body by equivalent electric and magnetic surface currents. The scattered field outside and the total internal field are produced by these equivalent currents. By using the boundary conditions on the surfaces of the body, eight simultaneous surface integral equations are obtained for four unknown equivalent surface currents. These eight equations are reduced to four by taking linear combinations of them. A Matlab computer program is developed for an axisymmetric chiral radome and examples of numerical calculations are given for a chiral spherical radome and chiral Von Karman radome. Numerical results of the chiral spherical radome are in excellent agreement with the exact ones obtained by the eigenfunction solution.

1. Introduction

A radome is a structure used to protect antennas from outside weather effects such as rain, snow, ice, etc. The construction materials of radomes are chosen such that the electromagnetic signal should be affected minimally. Several shapes of radomes with various materials are built where they can be seen used at many places like airplanes, on the ground covering various antennas, on boats and many more places.

Scattering and radiation from radomes and antenna systems with radomes have been studied in the last couple decades by using different methods, such as the ray tracing technique [1], the method of moments (MoM) [2], the physical optics (PO) method and dielectric physical optics (DPO) technique [3], the finite element method (FEM) [4], precorrected fast Fourier transform methods (P-FFT) [5], and body of revolution (BOR) formulations of MoM [6].

Chiral materials might be used to design anti-reflective structures to control scattering cross section patterns of bodies and scattering angles. We are not aware of any work that uses MoM to analyze an axisymmetric 3-D chiral radome to calculate the internal fields and scattering from it. In this paper, we present results for only two different shapes of axisymmetric chiral radomes.

2. Analysis

A chiral radome with 3 regions is shown in Fig. 1a. A body of revolution and the coordinate system are shown in Fig. 1b. The surface equivalence principle has been used to separate the radome problem into three parts, namely, the region external to surface 1, the region between surface 1 and surface 2, and the region bounded by surface 2. The radome is replaced by equivalent electric and magnetic surface currents, which produce the correct fields. The application of the boundary conditions on the tangential components of the total electric and total magnetic fields lead to coupled equations to be solved. Triangular expansion functions are used for both t-directed and \(\Phi\)-directed currents. The coefficients that multiply these expansion functions are obtained using the method of moments.
Using the equivalence principle, the original problem of Fig. 1a can be reduced to three simpler and equivalent problems shown in Fig. 2a, b, and c.

In the external equivalent problem, electric surface current \( J_1 \) and magnetic surface current \( M_1 \) have been placed on \( S_1 \). These surface currents are radiating in an unbounded medium of \((\varepsilon_1, \mu_1)\) with the same incident field of Fig. 1a. The total field at any point in the external region bounded by \( S_1 \) in Fig. 2a is the same as the total field at the same point of Fig. 1a, while the total field at any point in the internal region bounded by \( S_1 \) in Fig. 2a is zero. In other words,

\[
E_{1\text{tan}}(J_1, M_1) = -E_{1\text{tan}}^i \text{ on } S_1^+ \\
H_{1\text{tan}}(J_1, M_1) = -H_{1\text{tan}}^i \text{ on } S_1^+ \tag{1}
\]

\[
J_1 = n_1 \times H^+_\text{out} \tag{2}
\]

\[
M_1 = E^+_\text{out} \times n_1 \tag{3}
\]

where the superscript on \( S_i \) denotes the surface just inside or outside \( S_i \); “+” indicates the side of \( S_i \) facing the region into which \( n_1 \) points while “−” indicates the other side of \( S_i \), and \( E_i(J_1, M_1) \) and \( H_i(J_1, M_1) \), respectively, denote the electric and magnetic fields produced by the surface currents \( J_1 \) and \( M_1 \) when they radiate in the unbounded medium of \((\varepsilon_1, \mu_1)\). \( n_1 \) denotes the unit outward vector on \( S_1 \) and \((E^+_\text{out}, H^+_\text{out})\) are the total fields just outside \( S_1 \) in Fig. 1a.

The fields internal to the chiral region can be found by using the equivalent problem shown in Fig. 2b. Here the surface currents \( -J_1, -J_2, -M_1 \), and \( -M_2 \) placed on \( S_1 \) and \( S_2 \) radiate in the unbounded medium of \((\varepsilon_2, \mu_2, \xi_2)\). They produce the correct total fields \((E, H)\) at any point in the region bounded by \( S_1 \) and \( S_2 \) and produce zero fields at any point outside the region bounded by \( S_1 \) and \( S_2 \):

\[
E_{2\text{tan}}(J_1, M_1, J_2, M_2) = 0 \text{ on } S_1^+ \tag{5}
\]

\[
E_{2\text{tan}}(J_1, M_1, J_2, M_2) = 0 \text{ on } S_2^+ \tag{6}
\]

\[
H_{2\text{tan}}(J_1, M_1, J_2, M_2) = 0 \text{ on } S_1^+ \tag{7}
\]

\[
H_{2\text{tan}}(J_1, M_1, J_2, M_2) = 0 \text{ on } S_2^+ \tag{8}
\]

\[
J_2 = n_2 \times H^-_{\text{in}} \tag{9}
\]

\[
M_2 = E^-_{\text{in}} \times n_2 \tag{10}
\]

where \( S_1^+ \) and \( S_2^+ \) denote the sides of the surfaces \( S_1 \) and \( S_2 \) facing the regions into which the unit vectors \( n_1 \) and \( n_2 \) point, and \( E_{\text{in}}(J_1, M_1, J_2, M_2) \) and \( H_{\text{in}}(J_1, M_1, J_2, M_2) \), respectively, denote the electric and magnetic fields produced by the equivalent sources when they radiate in the unbounded medium of \((\varepsilon_2, \mu_2, \xi_2)\). \( n_1 \) and \( n_2 \) denote the unit vectors on \( S_1 \) and \( S_2 \), respectively and \((E_{\text{in}}, H_{\text{in}})\) are the total fields on \( S_i^+ \) in Fig. 1a.

The fields in the region bounded by \( S_1 \) in Fig. 1a are found by using the equivalent problem shown in Fig. 2c. Here the surface currents \( J_2 \) and \( M_2 \) placed on \( S_2 \) radiate in the unbounded medium of \((\varepsilon_1, \mu_1)\). They produce the correct total fields \((E, H)\) at any point in the region bounded by \( S_2 \) and produce zero fields outside the region bounded by \( S_2 \):

\[
E_{1\text{tan}}(J_2, M_2) = 0 \text{ on } S_2^- \tag{11}
\]
Formulation of Integral Equations

Equations (1), (2), (5) – (8), (11), and (12) represent eight coupled integral equations for the four unknown surface currents $J_1, J_2, M_1,$ and $M_2$. The combined field formulation reduces these eight equations to four by adding (1) to (5), (6) to (11), (2) to (7) and (8) to (12). These four coupled integral equations are presented below. They are solved numerically by using the method of moments. The details of the solution procedure are available in [7].

$$-E_{1\tan}(J_1, M_1)|S_1^+ = E^i_\tan \text{ on } S_1$$  \hspace{1cm} (13)

$$-E_{2\tan}(J_2, M_2)|S_2^+ = E^i_\tan \text{ on } S_2$$  \hspace{1cm} (14)

$$-H_{1\tan}(J_1, M_1)|S_1^+ = H^i_\tan \text{ on } S_1$$  \hspace{1cm} (15)

$$-H_{2\tan}(J_2, M_2)|S_2^+ = H^i_\tan \text{ on } S_2$$  \hspace{1cm} (16)

4. Results and Conclusion

Numerical results for two chiral bodies, a spherical radome and a Von Karman radome are given in this paper. Figure 3 shows the radomes considered. The bodies are illuminated by a $\theta$-polarized, $z$-travelling incident plane wave where its electric field is $x$-directed.

Internal fields on the $z$-axis and the scattering from the bodies have been computed. Results are given for a spherical radome of outer radius $10\lambda_0$ and inner radius $9.5\lambda_0$, $\varepsilon_r = 1.2$, $\mu_r = 1$, and $\xi_r = 0.2$ for $\theta_{\text{inc}} = 180^\circ$ in Fig. 4, and in Fig. 5, results for a Von Karman radome of $L_1 = 10.1\lambda_0$, $L_2 = 10\lambda_0$, $D_1 = 1.1\lambda_0$, and $D_2 = 1\lambda_0$ with $\varepsilon_r = 1.55$, $\mu_r = 1$, and $\xi_r = 0, 0.3, 0.6, \text{ and } 0.9$ for $\theta_{\text{inc}} = 0^\circ$ are given.

![Fig. 3 a) Spherical radome, b) Von Karman radome](image-url)
Fig. 4 Internal fields on the z-axis and scattering cross section of a spherical radome.

Fig. 5 Internal fields on the z-axis and scattering cross section of a Von Karman radome.

Results of Fig. 4 are in excellent agreement with the exact data. It is observed that, adding chirality to the bodies caused the cross-polarized field component to evolve and the magnitude of the cross-polarized field component to shift the direction of the scattered fields away from the direction of the incident fields. Adding chirality to the bodies affected the inside fields and the scattered fields significantly for the cases studied in this paper.

5. References