

# Near-field Estimation using a Reduced Basis Expansion of Induced Modes in a Human Head Model from Equivalent Sources

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## Abstract

A new approach to evaluate near-field induced by a cellular phone in a human head model using an E-field expansion into a basis is proposed. This technique is first based on Huygens principle using equivalent currents on a closed surface which makes it valid for any cellular phone. Then, using Singular Value Decomposition, the generated induced E-field by any cellular phones would be expanded with the orthonormalized modes excited by the equivalent sources in a human head model. Finally, we estimate the error reconstruction of E-field using a reduced number of modes.

## 1. Introduction

In wave interaction with human bodies, the evaluation of near electric field can be of great interest for verification of compliance of radiofrequency base stations, or of cellular phones [1]. In this last case where we are focused in, the most accurate technique to measure the field induced in a Specific Anthropomorphic Mannequin (SAM) is to use a robot scanning all the volume equipped with a probes. Unfortunately, this technique takes a long time to estimate the Specific Absorption Rate (SAR) of a cellular phone deduced from the E-field square induced in the SAM and physical parameters (conductivity and density). Techniques, using data interpolation, or parametric approach to reconstruct E-field have been proposed to reconstruct E-field from a scan on a reduced number of measurements [2]. More recently, a technique was studied by using integral formulation of equivalence principle in a flat phantom to expand the E-field [3]. In this paper, we propose a new technique suited to a curved phantom as SAM, using an equivalence principle to expand the induced E-field of any cellular phone into modes. The induced modes ensure an optimal decomposition i.e. with a reduced number of modal coefficients.

## 2. Equivalence Principle

The proposed technique starts with an equivalence principle. Any cellular phone enclosed by a surface S can be described from a field point of view by an equivalent current on surface S (Fig.1).

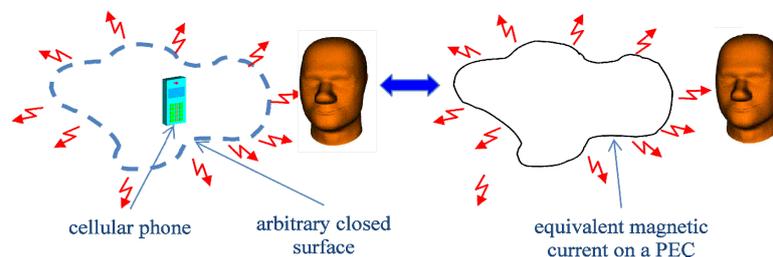


Fig.1: Equivalence theorem applied with a SAM phantom

We are interested in the E-field outside the arbitrary closed surface, and more precisely inside the human head model. There exists a distribution of equivalent currents over the surface emitting the same induced E-field in the human head model than the devices. Instead of using both electric and magnetic surface currents; we use only magnetic

surface current on a Perfect Electric Conductor (PEC) [4]. According to this, the equivalence current produces the same field in region 2 ( $\mathbf{E}_2, \mathbf{H}_2$ ) than the device under test (DUT).

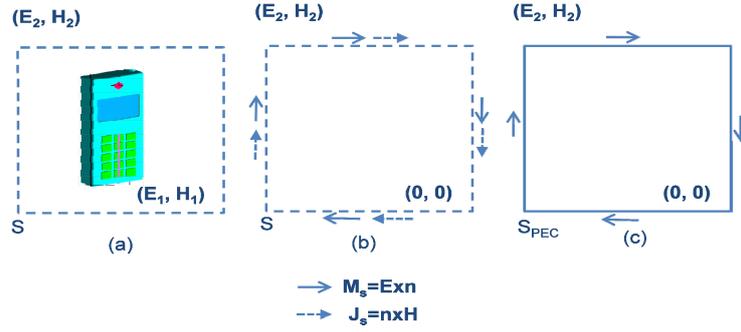


Fig.2: The induced E-field is the same in (a), (b), and (c)

This equivalence principle (Fig.2-c) is applied on a FDTD grid. The surface enclosing the cellular phone corresponds to a surface mesh with the edges supporting the electric field. It means that each edge  $i$  of the mesh surface is a voltage corresponding to the magnetic current  $M_i$ , while the others edges are short-circuited.

The device illuminating the phantom can be then replaced by a distribution of magnetic current, and the E-field induced by the DUT in the media can be written as a linear combination of induced field by these equivalent elementary sources in the same media (3).

$$E_{DUT} = \sum_{i=1}^N c_i E(M_i) \quad (1)$$

$N$  is the number of edges on the Huygens surface surrounding a device,  $E(M_i)$  is the induced field by the equivalent source  $M_i$  at the grid points  $i$  and  $c_i$  the corresponding coefficients.

### 3. A reduced decomposition based

The equation (1) can be rewritten in a matrix formulation (2):

$$E_{DUT} = Mc \quad (2)$$

Where  $M = (E(M_1) \dots E(M_N))$  and  $c$  is the vector of the coefficients  $c_i$ .

The minimum number of degrees of freedom, corresponding to the optimum number of basis function to expand  $E_{DUT}$  can be reached by a Singular Value Decomposition (SVD) of the matrix  $M$ . The SVD factorises the rectangular and complex matrix  $M$  into three matrices (3).

$$M = UDV^* \quad (3)$$

The columns of  $V$  form a set of orthonormal "input" basis vector directions for  $M$  corresponding to the excited sources on the closed surface and the columns of  $U$  form a set of orthonormal "output" basis vector for  $M$  corresponding to the induced singular modes in the media by the excited sources. The element  $s_i$  of the diagonal matrix  $D$  contains the  $N$  decreasing singular value and corresponds to the weight of each associated singular modes.

In our study case, we use a set of  $N=6468$  (the number of edges on the surface  $S$ ) elementary equivalent sources to create the basis of decomposition. After a singular value decomposition (5), we may obtain an optimal reduce basis of decomposition which can be set to a number of modes equal to the rank of  $M$ .

The E-field  $E_{DUT}$  can be then decomposed into a reduced number of modes  $U$  (4, 5).

$$E_{DUT} = \sum_{i=1}^n d_i U_i \quad (4)$$

$$E_{DUT} = Ud \quad (5)$$

With  $d$  the modal coefficient and  $n$  the number of significant modes. Some of them are presented in figure 3 associated to their respective excited equivalent sources on the closed surface in figure 4.

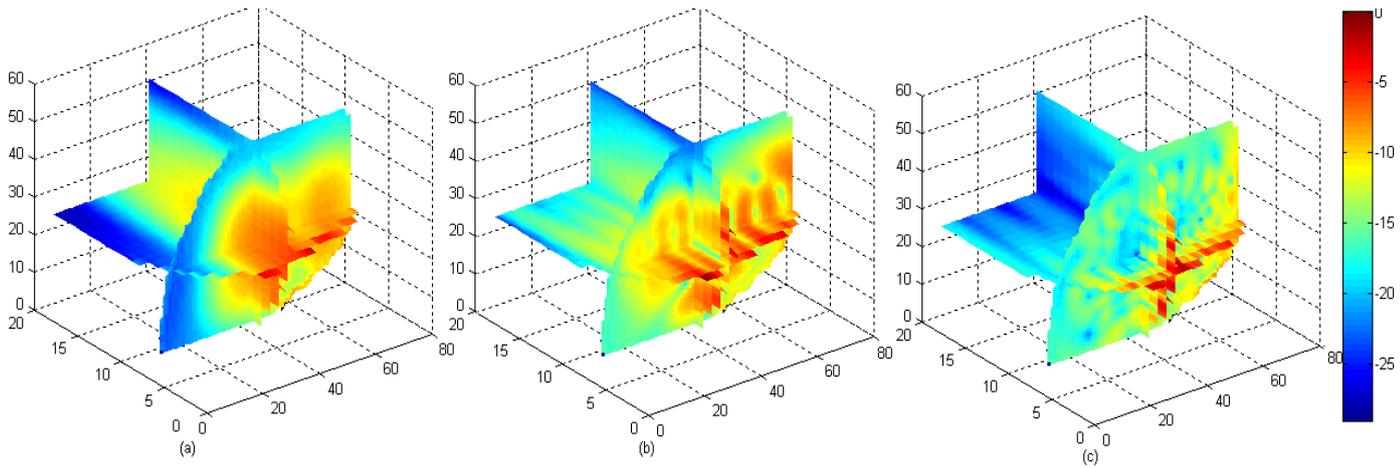


Fig.3: Representation in amplitude (dB) of orthonormal modes: 1<sup>st</sup> (a), 30<sup>th</sup> (b), and 200<sup>th</sup> (c).

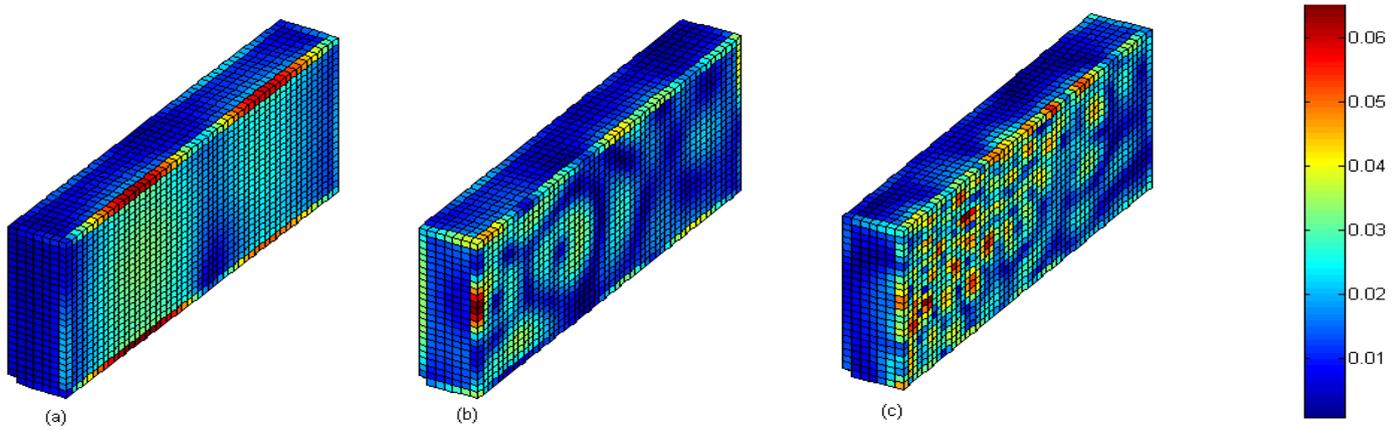


Fig.4: Representation in amplitude (dB) of excited equivalent sources on the closed surfaces generating the 1<sup>st</sup> (a), 30<sup>th</sup> (b), and 200<sup>th</sup> (c) modes.

#### 4. E-field decomposition

We worked in a case of induced E-field  $E_{DUT}$  in a volume inside a human head model at the vicinity of a cellular phone emitting at 1800 MHz. All the orthonormal modes were generated in the same condition of exposure, and we study the decomposition of this induced E-field into these modes. In figure 5 we represent  $E_{DUT}$  and its signature in the orthonormal basis function  $U$  using (7).

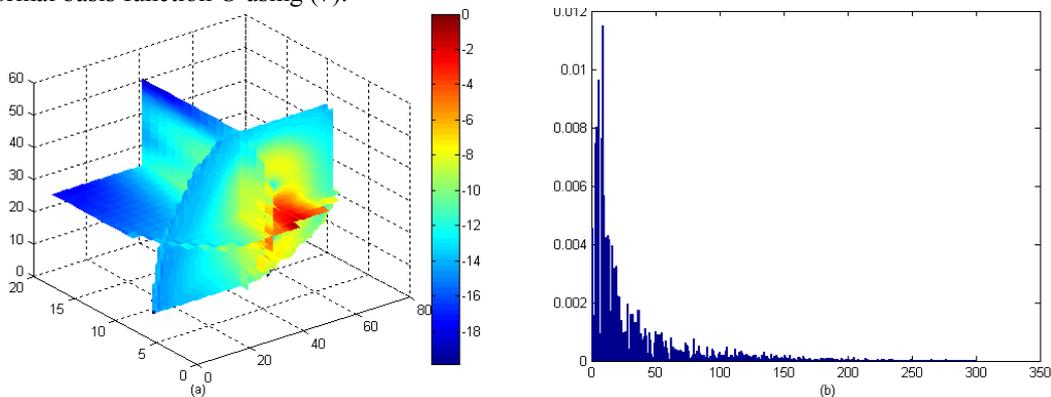


Fig.5: Amplitude (dB) of  $E_{DUT}$  representation in the volume of study (a), and amplitude of coefficient (b).

As we see, we maybe don't need to recover  $E_{DUT}$  using all modes. In figure 7, we have the L<sup>1</sup> and L<sup>2</sup> relative error norm (8) versus the number of modes.

$$L^p(d_{DUT}) = 100 \times \frac{\|U.d_{DUT} - E_{DUT}\|_p}{\|E_{DUT}\|_p}, p=1,2 \quad \|E\|_p = \left( \sum_i |E_i|^p \right)^{1/p}, p=1,2 \quad (8)$$

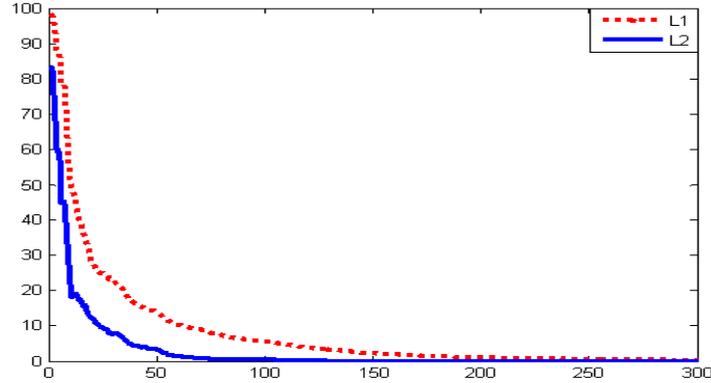


Fig.7: relative error norm in % L1 and L2.

## 5. Conclusion

In our approach, we are able to know physically the number of significant modes from where we can expand an induced E-field on a human head model. This knowledge is of great importance in the case where sparse measurements are envisaged to reconstruct the volumetric data inside a curved phantom.

## 6. Acknowledgments

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## 7. References

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