Numerical Computation of Corner Diffraction Coefficients for a Plane Angular Sector

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Abstract

Numerical diffraction coefficients are derived for currents associated with vertex diffraction on an infinitely-thin, perfectly conducting semi-infinite plane angular sector. The vertex-diffracted currents are formulated as the difference between the exact and PTD currents. The difference current is expressed as a traveling wave originating from the tip of the plane angular sector with unknown amplitude and decay factors. The unknown factors are then calculated by using least squares fit approximation. The discontinuities of the current density along the shadow boundaries are removed by the inclusion of the vertex-diffracted currents. The resulting current density also behaves as suggested by the edge condition. It is also demonstrated that incorporation of the vertex-diffracted currents improves the accuracy of the RCS pattern.

1. Introduction

The exact solution of the problem of electromagnetic scattering from a perfectly-conducting plane angular sector was first developed by Satterwhite and Kouyoumjian [1]. In this solution, fields and currents are expressed in the form of eigenfunction expansions that are the solutions of a two-parameter eigenvalue problem of coupled spherical Lamé differential equations and spherical Bessel functions. These expansions are slowly convergent and not suitable for practical asymptotic evaluation. The series expansions were employed to obtain a tractable approximation that can be used in the high frequency modeling of the angular sector problem [2], [3]. However, the eigenfunction expansions are difficult to evaluate especially for increasing distance from the tip for plane wave incidence. A uniform geometrical theory of diffraction (UTD) solution [4] for describing corner-diffracted fields was developed by Hill and Pathak [5]. The corner diffraction coefficients were derived in the UTD format by asymptotically evaluating the plane wave spectral (PWS) representation of the total field scattered from a perfectly-conducting plane angular sector. Asymptotic evaluation of the PWS integral provides the Geometrical Optics (GO) edge- and corner-diffracted fields. The corner-diffracted fields compensate for the discontinuities in the edge-diffracted fields. A similar approach based on the incremental theory of diffraction (ITD) was used in [6] to calculate the end-point contributions from the finite edges of the plane angular sector. However, this method leads to inaccurate results when nose-on incidence is approached.

In this paper, we present the derivation of numerical diffraction coefficients for describing the fields and currents diffracted from the vertex of a plane angular sector. We consider an infinitely-thin, perfectly conducting semi-infinite plane angular sector illuminated by a plane wave depicted in Fig. 1. Within the framework of high-frequency techniques, the current density induced on the angular sector can be approximated using the physical theory of diffraction (PTD) [7]. The total current density induced on the surface is calculated using the exact eigenfunction expansion [1]. The current density associated with vertex diffraction is then defined as the difference between the known exact current density and the PTD currents. It is observed that the vertex-diffracted current density can be expressed as a traveling wave originating from the tip of the plane angular sector with unknown amplitude and decay factors. The unknown factors are then calculated by using least squares fit approximation. This approach was previously used by Brinkley [8] for the same purpose. However, only the first-order fringe-wave contributions were used in calculating the vertex-diffracted currents. As a consequence, the solution becomes inaccurate when the total current on the angular sector is dominated by higher-order edge-to-edge interactions. An important aspect of the present solution is the inclusion of higher-order edge-to edge interactions in the calculation of the PTD currents. This yields an accurate representation of the vertex-diffracted currents for arbitrary directions of incidence. Thus, the resulting expressions for corner currents can be used in high-frequency modeling of finite scatterers to calculate the corner-diffracted fields. It is observed that incorporation of the vertex-diffracted currents provides a substantial improvement in the accuracy of the RCS patterns in off-specular directions. Further details will be reported in [9].

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2. Vertex-Diffracted Currents

The current density on the plane angular sector can be expressed as the sum of the physical optics (PO) current, the fringe wave (FW) current and the vertex-diffracted current. The vertex diffracted current is defined as

$$\mathbf{J^c} = \mathbf{J^e} - (\mathbf{J^{PO}} + \mathbf{J_1^{fw}} + \mathbf{J_2^{fw}} + \mathbf{J_{12}^{fw}} + \mathbf{J_{21}^{fw}}) \tag{1}$$

where $\mathbf{J^e}$ is the exact current and $\mathbf{J^{PO}}$ is the PO current. $\mathbf{J_1^{fw}}$ and $\mathbf{J_2^{fw}}$ are the fringe-wave contributions due to the edges of the angular sector. $\mathbf{J_{12}^{fw}}$ and $\mathbf{J_{21}^{fw}}$ denote the edge-to-edge double-diffracted current contributions. The first-order contributions can be calculated using the well-known expressions for the fringe-wave currents arising from the edge of a half plane [7]. The double-diffracted currents can be expressed in terms of the first-order diffracted current at the second-order diffraction point [10]. Consider the diffraction path depicted in Fig. 1(b). The field diffracted at Q_1 is re-diffracted by the other edge at Q_2. The direction of propagation of the diffracted rays ($\hat{s}_1$ and $\hat{s}_2$) is defined by the intersection of the Keller cone and the plate surface. Assuming that Q_2 is illuminated by a plane wave, the double-diffracted current at P can be calculated by using the first-order diffraction expressions with $\phi_i = 0$. The double-diffracted FW current at P can be conveniently expressed in terms of the first-order FW current at Q_2 as

$$\begin{aligned}\mathbf{J_{12}^{fw}}(s_2) = \frac{-e^{-j\pi/4}}{\sqrt{\pi k s_2(1-\hat{s}_2\cdot\hat{s}_1')}} e^{-jks_2}\Big\{ & \hat{x}_2(\mathbf{J_1^{fw}}(Q_2)\cdot\hat{x}_2)F(ks_2(1-\hat{s}_2\cdot\hat{s}_1')) \\ & -\hat{e}_2(\mathbf{J_1^{fw}}(Q_2)\cdot\hat{e}_2)[2-F(ks_2(1-\hat{s}_2\cdot\hat{s}_1'))]\Big\}\end{aligned} \tag{2}$$

where

$$F(x) = 2j\sqrt{x}e^{jx}\int_{\sqrt{x}}^{\infty} e^{-j\tau^2}d\tau.$$

$\hat{x}_2$ is the surface tangent at Q_2 and s_1, s_2 are measured from their respective diffraction points. It should be noted

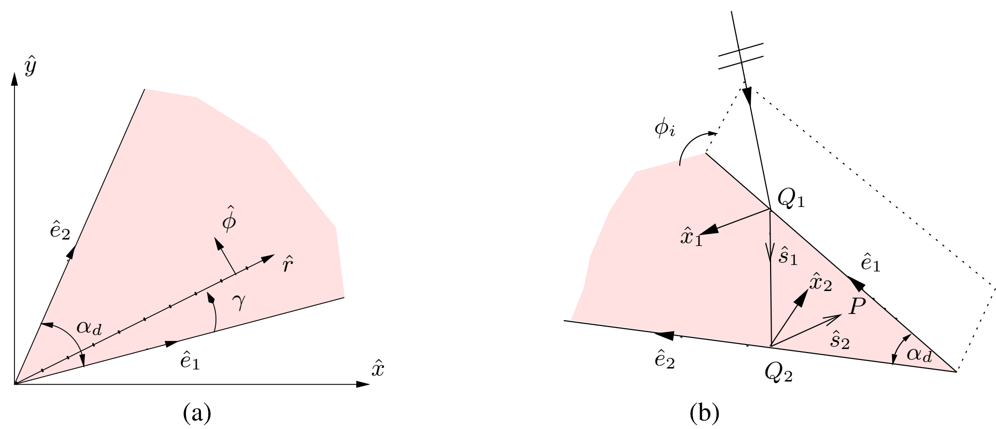


Figure 1: (a) Plane angular sector of $\alpha_d \leq 90°$. The angular sector is centered in the first quadrant of the x-y plane (i.e. $\hat{e}_1\cdot\hat{x} = \hat{e}_2\cdot\hat{y}$). (b) Rectangular plate illuminated by a plane wave.

that the first-order diffracted current ($\mathbf{J_1^{fw}}$) has a component normal to $\hat{e}_2$ at Q_2. It can be shown using (2) that this component is canceled by the second-order diffracted current $\mathbf{J_{12}^{fw}}$. Therefore, the edge condition for the current density is satisfied by the introduction of double-diffracted currents.

In order to investigate the behavior of the vertex-diffracted currents, we consider the plane angular sector shown in Fig. 1(b). It was observed that the vertex-diffracted current given by (1) behaves like a traveling wave originating from the tip of the plane angular sector. Then, the vertex-diffracted current can be formulated as

$$J^c = K(\gamma)(kr)^{-\alpha(\gamma)}e^{-jkP(\gamma)r} \tag{3}$$

where J^c represents the $\hat{r}$ or the $\hat{\phi}$ component of the current. $K(\gamma)$, $\alpha(\gamma)$ and $P(\gamma)$ are the unknown functions to be obtained numerically. $\alpha(\gamma)$ and $P(\gamma)$ are real-valued and $K(\gamma)$ is complex-valued. The unknown factors are functions of the angular position of the observation point on the plane angular sector for a given direction of incidence and polarization. Therefore, (3) must be solved for various radial cuts to cover the entire surface for the given direction of incidence. In order to achieve this, the vertex-diffracted current obtained using (1) is uniformly sampled along specific radial cuts as depicted in Fig. 1(a). The unknown factors are calculated for each γ by solving (3) using linear least-squares fitting. The difference current and the least-squares fit (LSF) approximation for $(\theta_i, \phi_i) = (30°, 225°)$ are shown in Fig. 2(a), (b). The LSF current density agrees well with the difference current of (1). Fig. 2(c) shows the total current density for $(\theta_i, \phi_i) = (60°, 225°)$ at $r = 0.5\lambda$. The FW currents exhibit discontinuities at the shadow boundaries. These discontinuities are removed by the inclusion of the vertex diffracted currents and the resulting current density agrees well with the exact current. Fig. 3 shows the bistatic RCS of a square plate of width 5λ for a

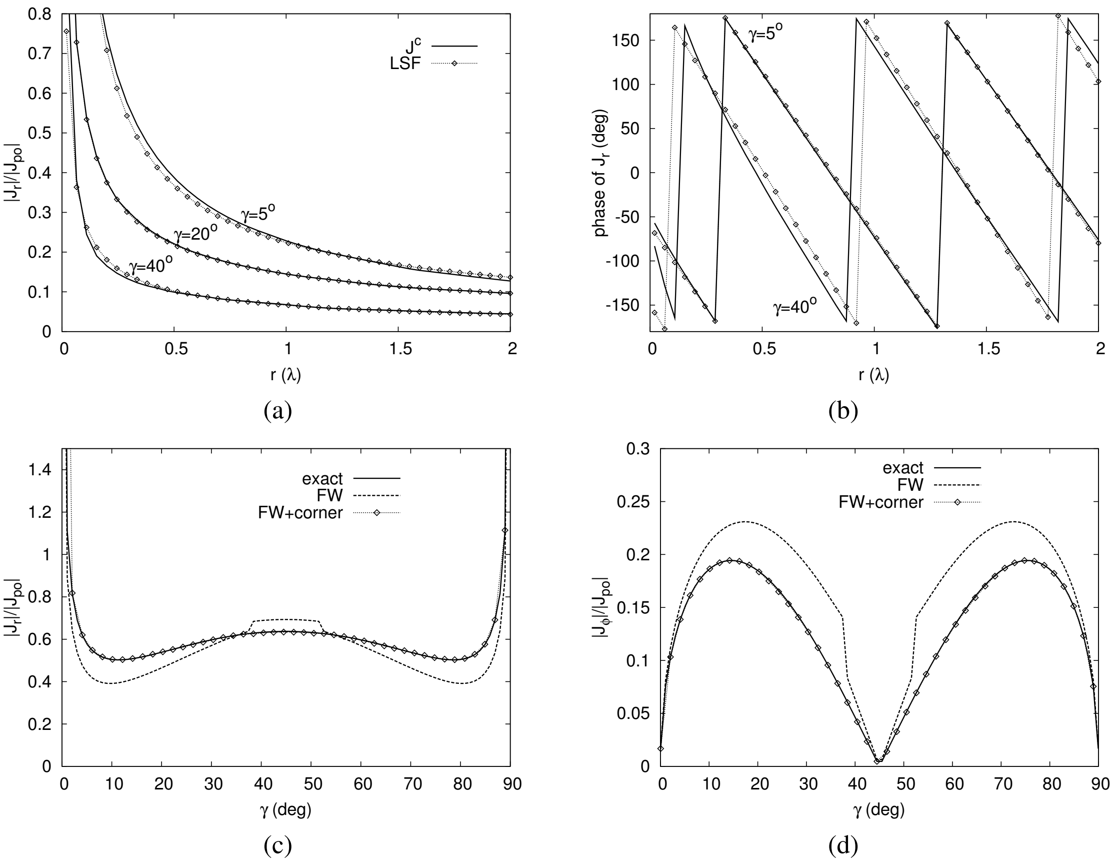
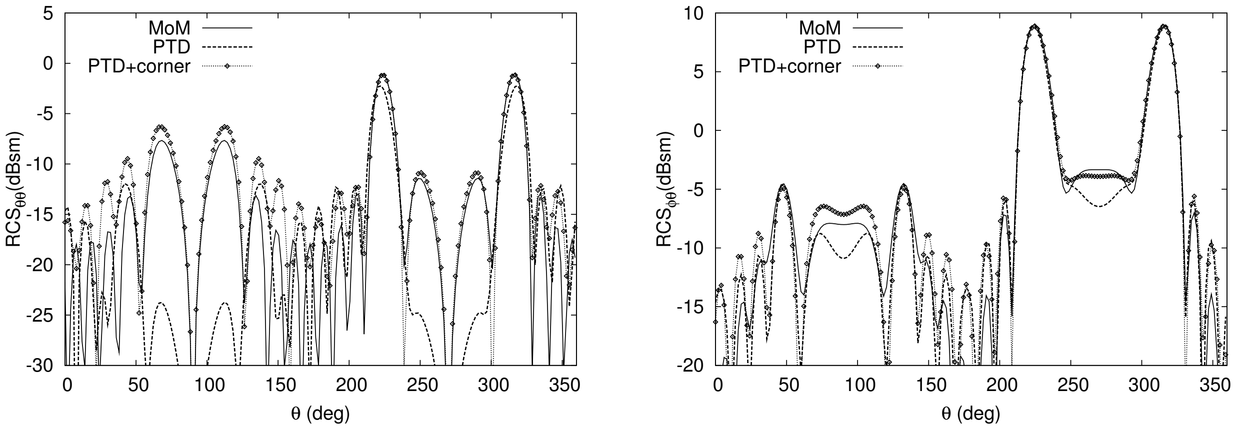


Figure 2: (a), (b) Magnitude and phase of J_r^c and the least-squares approximation on a $90°$ angular sector along various radial cuts for $\hat{\theta}_i$ polarized plane wave incident from $(\theta_i, \phi_i) = (30°, 225°)$ (c), (d) $\hat{r}$ and $\hat{\phi}$ components of the total current density on a $90°$ angular sector at $r = 0.5\lambda$ for a $\hat{\theta}_i$ polarized plane wave incident from the direction $(\theta_i, \phi_i) = (60°, 225°)$.

$\hat{\theta}_i$ polarized field incident from the direction $(\theta_i, \phi_i) = (80°, 45°)$. The plate is placed in the x-y plane and the RCS pattern is calculated at $\phi_s = 90°$. It is important to note that even though up to second-order edge-to-edge interactions are included, the PTD pattern does not agree well with the MoM in the low-level regions. The corner contribution is rather significant over these regions. The "PTD+corner" pattern agrees well with the Method of Moments results.

3. Conclusion

Numerical diffraction coefficients for currents close to the tip of a plane angular sector were calculated. The vertex-diffracted current was modeled as a wave traveling away from the corner. It was observed that multiple interactions among the edges are important and should be included in defining the vertex-diffracted currents. Up to



(a) (b)

Figure 3: Bistatic RCS of a 5λ square plate. $(\theta_i, \phi_i) = (80°, 45°)$, $\phi_s = 90°$. The incident field is $\hat{\theta}_i$ polarized. (a) $RCS_{\theta\theta}$ and (b) $RCS_{\phi\theta}$.

second-order edge-to-edge interactions are included in our derivations. It was demonstrated that the inclusion of vertex diffraction provides accurate results for the current density induced on the plane angular sector. The diffraction coefficients were also used to calculate the RCS of a rectangular plate. The accuracy of RCS patterns in the low-level regions improves substantially by the inclusion of vertex-diffracted fields.

4. References

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