Hybrid Methods Based on Generalized Scattering Matrices

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Abstract

A general hybrid technique based on the use of generalized scattering matrices is presented for the electromagnetic analysis of complex antenna and/or scattering problems. The analysis domain is decomposed into separate subdomains, which are independently studied through the most appropriate technique and characterized by a generalized scattering matrix (GSM), where the ports are associated with a proper set of wave objects. Finally, the interactions among the subdomains are described by imposing the proper subdomain connections. Two particular implementations of the general technique are illustrated. In the first one, the wave objects are complex point source (CPS) beams. Thanks to the angular selectivity of the CPS beams, only a small fraction of the beams are involved in the final system, thus, leading to an efficient numerical procedure. In the second one, spherical waves are used as wave objects. This choice offers the advantage of direct interfaceability with the output of spherical near-field measurements or numerical simulations.

1. Introduction

In electromagnetic problems, complex multiscale environments cannot be treated with a unique technique, since a prohibitive number of unknowns is in general required due to the large electrical size of the overall domain. Furthermore, the simultaneous presence of electrically large structures and small features may lead to ill conditioned systems. For these reasons, it is usually necessary to combine different methods to obtain an effective numerical procedure. A convenient approach for the solution of the overall problem consists therefore in subdividing the analysis domain into subdomains, analyzing the subdomains separately and then combining the results by imposing a proper connection among the different subdomains [1]. Typically, the electromagnetic fields in each subdomain are expanded in a proper set of wave objects, which may consist of basis functions of a purely numeric approach, of guided modes or of radiated modes or beams. Guided modes are well suited for representing microwave circuits, while radiated modes and beams are more useful to study antenna and scattering problems.

In this work, a general procedure for the efficient unification of different techniques in a single framework is proposed. First, the analysis domain is decomposed into separate subdomains and each subdomain is characterized with the most appropriate technique through a generalized scattering matrix (GSM). Then, the interactions among different subdomains are described through a network formalism, where the ports are associated with properly chosen wave objects. Finally, a linear system is constructed, where the excitation vector is given by the coefficients of the wave object expansion of the primary sources. Two different kinds of wave objects are considered in the following: complex point source (CPS) beams [2] radially emerging from the subdomain boundaries and spherical waves (SW) [3]. Thanks to the angular selectivity of the CPS beams, the subdomain interactions only involve a small fraction of the beams; thus, yielding sparse moderate size linear systems. On the other hand, the choice of spherical waves allows one to directly use the output of spherical near-field measurements or numerical simulations as an input for the GSM based approach.

The proposed formulation is a general architecture where one can employ an arbitrary approach to construct the scattering matrices; this means that the most appropriate technique can be used for the analysis of each subdomain. The resulting procedure is extremely efficient when applied to problems of antenna installation or positioning, to parametric studies or to the analysis of structures containing geometric repetitions, since the single scattering matrices can be reused as building blocks in successive computations.

In the next Section, the formulation of the problem in terms of generalized scattering matrices is summarized for a generic choice of the wave object associated with the ports. Then, the implementation with CPS beams and spherical waves as wave objects are described in Section 3 and 4, respectively. Finally, Section 5 presents some numerical results, and conclusions are drawn in Section 6.

2. The Generalized Scattering Matrix Approach

The overall analysis domain is decomposed into disjoint subdomains, each of which is delimited by an equivalence surface enclosing a source, a scatterer or a group of scatterers. In each subdomain a set of wave objects (i.e.

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functions satisfying the wave equation) is introduced. These wave objects are used to construct series expansions of the incident, radiated or scattered electric field. Then, the following steps are performed:

- each subdomain is described as a multiport junction by associating a port to each wave object and defining the relevant generalized scattering matrix (GSM). In analogy with the microwave theory, the Generalized Scattering Matrix (GSM) $\underline{\underline{\mathbf{S}}}^{(n)}$ of the generic subdomain n is the matrix relating the coefficient vectors of the incident and the reflected wave objects when all the other sources or scatterers are removed (Fig. 1) $\underline{\mathbf{b}}^{(n)} = \underline{\underline{\mathbf{S}}}^{(n)} \underline{\mathbf{a}}^{(n)}$. The element $S_{ij}^{(n)}$ is found by illuminating the subdomain with the j-th wave object, calculating the scattered field and determining the coefficient of the i-th wave object of its expansion. The subdomain must be isolated in free-space to ensure the absence of reflections from outside, which corresponds to the matched load condition for microwave devices. The generalized scattering matrix completely characterizes the scattering properties of one subdomain.
- the proper relationships among the field expansions in the different subdomains are established on the basis of the problem topology though the Generalized Transport Matrices (GTM). The GTM $\underline{\underline{\mathbf{T}}}^{(n,m)}$ from subdomain m to subdomain n relates the coefficient vector of the field incoming to the subdomain n to the coefficient vector of the field outgoing from the subdomain m (Fig. 1) as follows $\underline{\mathbf{a}}^{(n)} = \underline{\underline{\mathbf{T}}}^{(n,m)} \underline{\mathbf{b}}^{(m)}$. The element $T_{pq}^{(n,m)}$ is the p-th coefficient in the expansion in the subdomain n of the field of the q-th wave object of the subdomain m when all the subdomains are filled by free space. Vanishing elements of the transport matrix correspond to couples of non-interacting wave objects, or unconnected ports in the network formalism.
- a linear system involving the expansion coefficients relevant to all the subdomains is set up by expressing the total incident field to the generic subdomain n as the superimposition of the contributions of all the other subdomains. This leads to equations of the form

$$\underline{\mathbf{a}}^{(n)} = \sum_{m \neq n} \underline{\underline{\mathbf{T}}}^{(n,m)} \underline{\underline{\mathbf{S}}}^{(m)} \underline{\mathbf{a}}^{(m)} + \underline{\underline{\mathbf{T}}}^{(n,0)} \underline{\mathbf{b}}^{(0)} \quad (1)$$

  where $\underline{\mathbf{b}}^{(0)}$ is the vector containing the coefficients of the field radiated by the primary source. The collection of the equations for all the subdomains leads to a linear system whose solution provides the coefficients of the incoming wave objects for all the subdomains. The coefficients of the corresponding outgoing wave objects can then be easily obtained by using the generalized scattering matrices.

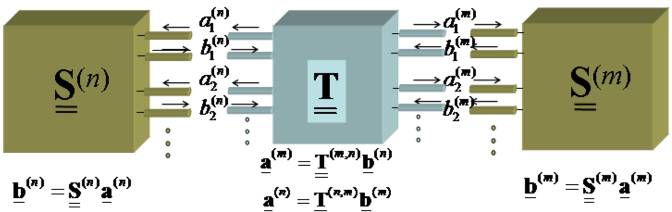


Fig. 1. Connection of two subdomains through generalized scattering and transport matrices

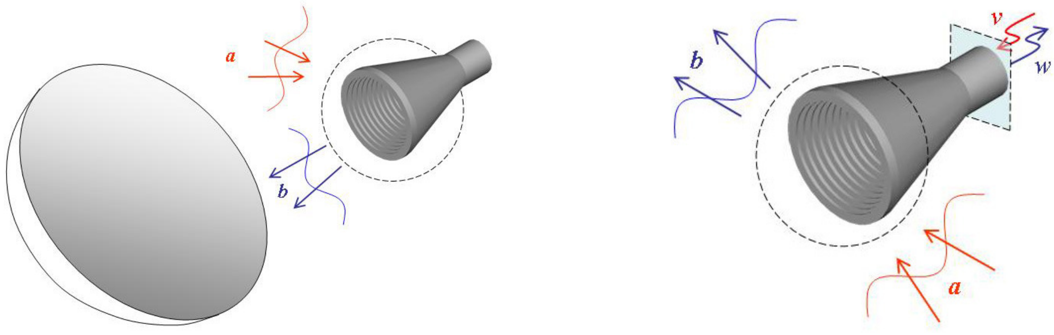
3. Complex Source Point Beams as Basis Functions

A key issue for the efficiency of the approach presented in the previous section is the minimization of the number of interacting wave objects, which determines the actual dimension of the final system. An effective way to reach this objective consists in using directive beams as wave objects. In fact, due to their angular selectivity, only a small number of beams provide a significant contribution in a given region of space. This leads to the choice of a set of translated and tilted CPS beams as bases for the field description.

The CSPs used for the field expansion have their real part uniformly distributed over the subdomain boundary and the imaginary part directed along the radial direction. Since a point source located at the complex location $\mathbf{r}_0$-$j\mathbf{b}$ produces a beam-type field with axis parallel to $\mathbf{b}$, which reduces to the field of an electromagnetic Gaussian beam with minimum waist at $\mathbf{r}_0$ in the paraxial region [2], the fields associated with the CSP described above are beams radially emerging from the subdomain boundary. The directivity is the same for all the beams, and depends on the electrical size of the relevant subdomain. The expansion coefficients are numerically determined by imposing the matching between the tangential components of the electric field to be expanded and its CSP expansion [4]. Only electric-type CSPs are used; this is possible because the CSP expansion is solely required to represent the original field either externally or internally to the subdomain boundary, as demonstrated in [5].

4. Spherical Waves as Basis Functions

The use of spherical waves as wave objects in the GSM approach is particularly useful when, as often occurs, SW coefficients are available from measurements or numerical simulations. As an example, we consider the case of a horn radiating in the presence of a nearby scatterer (*e.g.* a reflector), as shown in Fig. 2(a).



(a) (b)

Fig. 2. Geometry for the problem and representation of the field outside the antenna's minimum sphere in terms of inward and outward spherical waves and incident and reflected waveguide modes.

The isolated antenna is described by using the antenna total scattering matrix formalism, as in [3]. According to this model, all the transmitting, receiving or scattering properties of a general antenna, with an associated spherical coordinate system, are described by a linear relationship between guided waves at the waveguide input ports and incoming and outgoing spherical waves outside the antenna's minimum sphere, which define the "radiation ports" (see Fig.2). This relationship can be written as follows:

$$\begin{bmatrix} w \\ \underline{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \Gamma & \underline{\underline{\mathbf{R}}} \\ \underline{\underline{\mathbf{T}}} & \underline{\underline{\mathbf{S}}} \end{bmatrix} \begin{bmatrix} v \\ \underline{\mathbf{a}} \end{bmatrix} \tag{2}$$

where v and w are the coefficients of the incident and reflected waveguide modes, respectively, $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$ the coefficients of the incoming and outgoing spherical waves, Γ is the modal reflection coefficient at the isolated antenna's input port, $\underline{\underline{\mathbf{S}}}$ is the antenna scattering matrix and $\underline{\underline{\mathbf{R}}}$ and $\underline{\underline{\mathbf{T}}}$ are the receiving and transmission matrices, respectively. If the SW expansions of the different subdomains are defined in different reference systems, translation and rotation formulas [3] can be used to obtain a closed-form expression of the GTM. However, for certain applications it is more convenient to define all the SW expansions in the same reference system. Consider for instance the case illustrated in Fig. 2: we are interested in estimating the perturbation induced on the reflection coefficient of an antenna by the presence of a nearby scatterer. If the multiple interactions between the antenna and the obstacle are neglected and all the SW expansions are defined in the antenna reference system, then the relationship between the coefficients $\underline{\mathbf{b}}$ of the SWs impinging on the obstacle (leaving the antenna) and the coefficients $\underline{\mathbf{a}}$ of the SW leaving the obstacle (coming to the antenna) can be condensed in the obstacle's generalized scattering matrix (GSM) $\underline{\underline{\mathbf{D}}}$, such that $\underline{\mathbf{a}} = \underline{\underline{\mathbf{D}}}\underline{\mathbf{b}}$. The m^{th} column of the obstacle SW-GSM contains the SWE coefficients of the field scattered by the obstacle when it is illuminated by the m^{th} spherical mode. Since all the SW expansions (SWE) are referred to the feed antenna minimum sphere, the number of the used SWs is related to the electrical size of the minimum sphere surrounding the feed and it is independent of the obstacle's size. Once the SW-GSM of the obstacle is known, the first-order backscattering effect on the antenna feed's input impedance is estimated by the formula $\Gamma' = w / v = \Gamma + \underline{\underline{\mathbf{R}}}\,\underline{\underline{\mathbf{D}}}\,\underline{\mathbf{b}} \big/ v$.

5. Numerical Results

An example of the use of CPS beams as wave objects is illustrated in Fig. 3. It is relevant to the mutual interactions between a couple of $2\lambda \times 2\lambda$ metallic plates. The excitation is given by an elementary electric dipole. A set of 486 CSPs are defined on a complex sphere of radius $r \cong (2.3 - j2.7)\lambda$ enclosing the first plate and the relative scattering matrix has been evaluated by using a PO approach. The same matrix, properly translated and rotated, is then re-used for the second scatterer. Fig. 3 compares the results provided by the GSM approach (solid line) with those obtained by a commercial software (FEKO) through a direct PO approach. Dashed line is relative to PO without considering interactions between the plates, while the circles represent the solution that takes into account the multiple reflections up to the sixth order. As it is apparent, the proposed procedure correctly predicts the effects of all the multiple PO interactions. As a second example, the GSM approach based on spherical waves has been applied to estimate the reflection coefficient of a rectangular horn radiating in the presence of a metallic disk, as shown in Fig. 4. The scattering from the obstacle is analyzed by using a PO approach and the multiple interactions between the feed horn and the reflector are assumed to be negligible. The results obtained by applying the proposed approach have been compared with a full-wave simulation carried again with FEKO. A good agreement is observed in the considered frequency range.

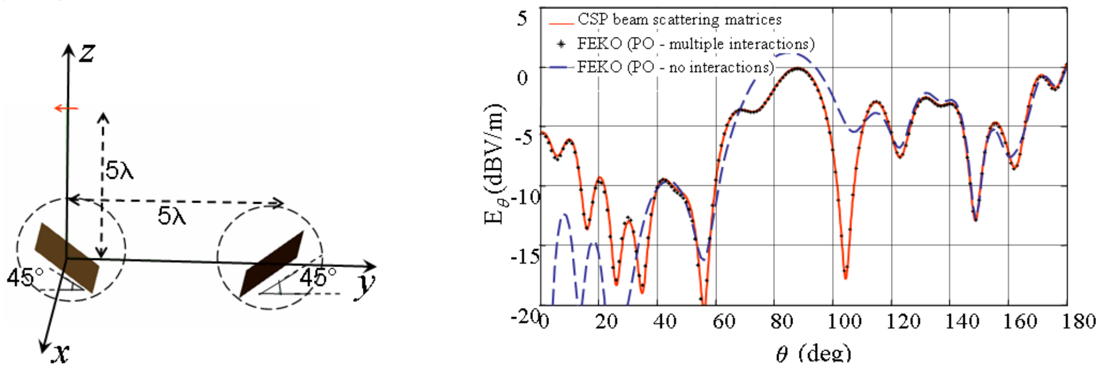


Fig. 3. Two plates scattering problem. Geometry for the problem (on the left) and scattered field (on the right).

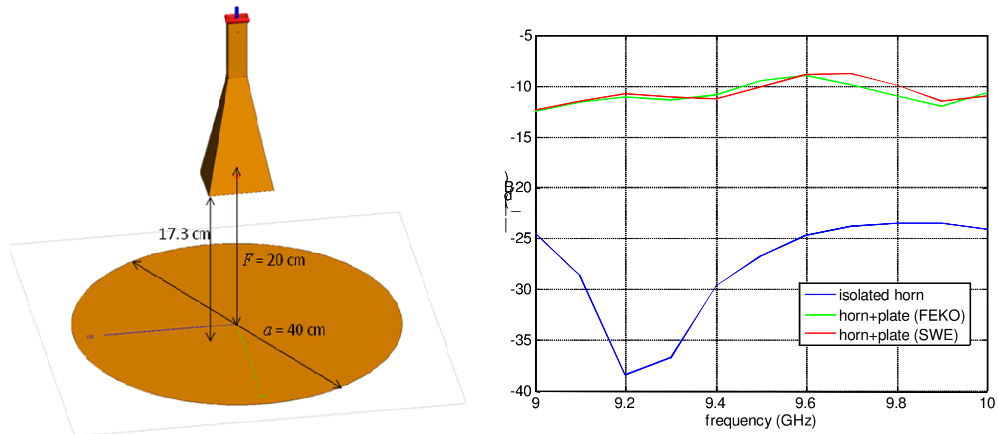


Fig. 4.Horn near a round metallic plate located at 20 cm from the horn phase center. Geometry for the problem (left). Reflection coefficient (right).

6. Conclusion

A general procedure for the analysis of complex antennas or scattering problems has been presented. The method allows for an efficient hybridization of different analysis techniques, since the overall domain is subdivided into separate subdomains which are analysed independently, and the mutual interactions are described through the Generalized Scattering Matrix formalism, with ports associated to wave objects. The re-usability of the GSMs renders the proposed approach particularly efficient in the context of parametric studies or antenna installation problems.

7. References

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