# Recent Advances in the Incremental Theory of Diffraction for Complex Source Point Illumination

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## 1 Introduction

The accurate prediction of the far field radiated or scattered by large structures, such as large reflector antennas, requires efficient techniques for representing the illuminating field. Complex Source Points (CSP) inherently contain information about the source directivity, hence they can be used as the basis function to expand a given, but arbitrary, radiating wave field [1–3], such as the field incident on an antenna or a more general complex structure. As a consequence, a CSP field representation, when combined with the analytic continuation in complex space of typical ray-techniques such as the Geometrical and the Uniform Geometrical Theory of Diffraction (GTD/UTD), may provide a very efficient tool to estimate the fields radiated by large objects [4]. In this framework, an extension of the Incremental Theory of Diffraction (ITD) formulation for the scattering by wedges illuminated by CSP has been introduced [5], which essentially overcomes the typical impairments of the GTD/UTD ray techniques associated with possible ray caustics and with the difficulties of ray tracing in complex space. On the other hand, when dealing with the description of the field radiated by large structures, many of the existing electromagnetic codes resort to a Physical Optics (PO) representation also with an arbitrary incident field. It is however well known that the PO approach does not always produce accurate field predictions [6]. A significative augmentation of the PO field estimate can be achieved by including along the structure's edges a line integration of an incremental fringe field, that acts as a correction term for the field estimate. Several techniques have been published to derive these elementary contributions, leading to Physical Theory of Diffraction (PTD), Elementary Edge Waves/Incremental Length Diffraction Coefficient (EEW/ILDC), and ITD. In this work we discuss some recent advances in the incremental formulation for the field diffracted by edges in perfect electric conductor (PEC) objects when illuminated by a CSP beam expansion, with application to the analysis of large reflector antennas. A fringe formulation of the field diffracted by wedges with PEC faces when illuminated by a single CSP has been recently presented [7]. At each point on the edge the incremental fringe correction term is deduced from tangential canonical problems as the difference between the local ITD diffracted field [5] and an appropriate incremental end-point PO field (IEPO) scattered by the half-lit plane tangent to the edge [8]. The total spurious effects due to the presence of the edge are corrected by adiabatically distributing and integrating the local incremental fringe field coefficients along the line of the edges. This formulation yields more accurate predictions of the radiated field. For configurations in which several metallic edges are present and for grazing aspects of incidence and observation, the correct interactions between the edges in the problem need to be properly accounted for. Hence we introduce correct incremental double-diffraction coefficients for CSP illumination in the first-order fringe formulation [9]. These incremental coefficients have been derived by a proper analytical continuation of their real counterparts [10]. The formulation provides an accurate first-order asymptotic description of the interaction between two edges, which is valid both for skewed separate wedges and for edges joined by a common PEC face. It also includes a double incremental slope diffraction augmentation, which provides the correct dominant high-frequency incremental contribution at grazing aspects of incidence and observation. The total doubly-diffracted field is obtained from a double spatial integration along each of the two edges on which consecutive diffractions occur. In the application to the analysis of large reflector antennas the first-order fringe correction to the PO scattered fields tends to fail in those directions parallel to the aperture plane. Here, the addition of the incremental double diffracted field provides the correct estimation of the radiated field.

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## 2 The fringe formulation for CSP illumination

Referring to Fig. 1(a), let us consider a wedge scatterer with edge $l$ illuminated by a single CSP with electric dipole moment $\mathbf{p}$, that produces an incident far field $\mathbf{E}^i(\mathbf{r})$ given by

$$\mathbf{E}^i(\mathbf{r}) \simeq jk\zeta\hat{R} \times \hat{R} \times \mathbf{p}\frac{e^{-jk\widetilde{R}}}{4\pi\widetilde{R}}, \tag{1}$$

where $\widetilde{R} = \sqrt{(\mathbf{r}-\tilde{\mathbf{r}}')\cdot(\mathbf{r}-\tilde{\mathbf{r}}')}$ is the complex distance from the complex source position $\tilde{\mathbf{r}}' = \mathbf{r}' - jb\hat{r}'$, and $b > 0$ defines the beam-collimation distance [11]. The incremental end-point PO contribution $d\mathbf{E}_d^{\mathrm{PO}}$, associated with the abrupt truncation of the PO currents at the Shadow Boundary Lines (SBL) on the object (i.e., the edge of the scatterer) is computed first. This contribution is not correct because it is based on a locally-tangent canonical configuration given by a half-lit infinite plane, which is not an accurate model of the actual object in the neighborhood of the SBL. The contribution has the high-frequency dyadic closed-form expression

$$d\mathbf{E}_d^{\mathrm{PO}}(\mathbf{r}) = \underline{\underline{D}}^{\mathrm{PO}}(\tilde{\nu};\phi,\beta,\tilde{\phi}',\tilde{\beta}')\cdot\mathbf{E}^i(Q_l)\frac{e^{-jkr}}{4\pi r}\,dl, \tag{2}$$

where each entry $D_{il}^{\mathrm{PO}}$ of the dyadic $\underline{\underline{D}}^{\mathrm{PO}}$ is found as

$$D_{il}^{\mathrm{PO}}(\tilde{\nu};\phi,\beta,\tilde{\phi}',\tilde{\beta}') = \left(\cos\beta\cos\tilde{\beta}'\right)^{\frac{l-i}{2}}\left\{\frac{1}{2}\left[\frac{\sin(\phi-\tilde{\phi}')}{\cos\tilde{\nu}+\cos(\phi-\tilde{\phi}')} + \frac{(-1)^{\frac{(i+l)}{2}}\sin(\phi+\tilde{\phi}')}{\cos\tilde{\nu}+\cos(\phi+\tilde{\phi}')}\right]\right\}^{i-l+1}, \tag{3}$$

except for $D_{21}^{\mathrm{PO}}(\tilde{\nu};\phi,\beta,\tilde{\phi}',\tilde{\beta}') = 0$. Here, $\tilde{\nu} = -j\log\left[\tan(\beta/2)/\tan\left(\tilde{\beta}'/2\right)\right]$ and $\tilde{\beta}'$, $\tilde{\phi}'$ are the analytic continuation in complex space of the corresponding real angles [5]. The above incremental field is then subtracted from the ITD field $d\mathbf{E}_d^{\mathrm{ITD}}(\mathbf{r})$ to obtain the desired fringe incremental field

$$d\mathbf{E}_d^{\mathrm{IFF}}(\mathbf{r}) = d\mathbf{E}_d^{\mathrm{ITD}}(\mathbf{r}) - d\mathbf{E}_d^{\mathrm{PO}}(\mathbf{r}). \tag{4}$$

The formulation (2)-(4) provides a description of the field, which is well-behaved at any incidence and observation aspects, including $\tilde{\beta}' = 0$ or $\beta = 0$. The expected transitional behavior of the field is reconstructed by numerical integration of the incremental field contributions (4) along the curved contour $l$ of the actual scatterer. The latter, when distributed and integrated along the actual edge discontinuity, provides a correction of the PO estimate of the scattered field

$$\mathbf{E}_s(\mathbf{r}) \sim \mathbf{E}_s^{\mathrm{PO}}(\mathbf{r}) + \int_l d\mathbf{E}^{\mathrm{IFF}}(\mathbf{r}). \tag{5}$$

The field representation (5) is particularly suitable for numerical calculations, since the integrand in (5) provides a precise cancellation of the singularities that occur at the shadow boundary aspects of the GO incident and reflected fields for both the IEPO and the ITD incremental coefficients. As an example for the application of the proposed technique, the far field radiated by a large reflector antenna is shown in Fig. 2(a). The circular parabolic reflector (diameter $d = 20\lambda$; focal distance $f = 30\lambda$) is illuminated by a defocused horn antenna with phase center located at $\mathbf{r}' = (-30\lambda\sin 20°, 0, 30\lambda\cos 20°)$, and tilted so that the main beam axis still points to the center of the parabolic dish, as shown in Fig. 2(a). The horn beam is represented using a superposition of a sufficient number $N$ of CSPs located on a sphere of radius $a$ that most tightly surrounds the feed source. Observation is made at $r = 60\lambda$ from the center of the aperture at $\phi = 45°$. The curves show the total electric field $|E_\theta|$ obtained from MoM simulations (dotted line) and compared with the PO (dashed line) and PO+IFF correction (continuous line). An overall significantly improved accuracy is observed when introducing the fringe corrections in the PO formulation, and this is more evident at observation angles far from the horn boresight.

## 3 The incremental double diffraction for CSP illumination

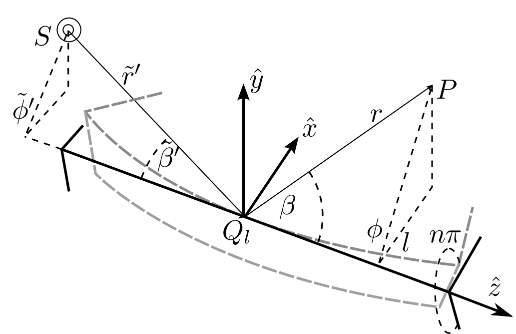
Field contributions associated with multiple interacting edges need to be introduced to improve the field estimate. When the dominant field of a singly-diffracted ray is shadowed by a second edge, a discontinuity may occur in the field estimate mostly at grazing incidence and observation aspects. Thus, the introduction of a double diffraction field compensates for such a discontinuity [10]. In this framework, a formulation for the incremental double diffraction problem for the case of CSP illumination has been recently proposed [9]. Referring to Fig. 1(b) let us consider a pair of wedges with curved edges $l_1$ and $l_2$, arranged in an arbitrary configuration and illuminated by a CSP beam at $S$. To obtain a doubly-diffracted field contribution which explicitly satisfies reciprocity, we developed a proper formulation based on a particular formulation of the reaction principle for the scattering by metallic objects. We consider the metallic wedge with edge $l_2$ as a scatterer which is illuminated by two sets of sources: the equivalent filamentary current sources distributed on the edge $l_1$ (labeled as $a$) that radiate the diffracted field by the first wedge, and a local $\hat{z}$-directed unit dipole located at the observation point $P$ (labeled as $b$). If we limit our analysis to the field generated by the equivalent filamentary electric currents $\mathbf{I}_e$ flowing along the edges and by electric dipoles, it is found that the total double diffracted field $\mathbf{E}_{12}^{dd}(P)$ is given by

$$\mathbf{E}_{12}^{dd}(P) = \int\limits_{l_2} \left[\mathbf{E}_i^a(Q_{l_2}) \cdot \mathbf{I}_e^b(Q_{l_1}, Q_{l_2}, P)\right] dl_2, \tag{6}$$

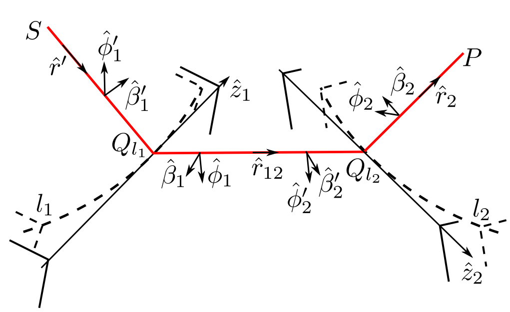
where $\mathbf{E}_i^a(Q_{l_2})$ is the diffracted field incident at $Q_{l_2}$ due to the equivalent currents at $Q_{l_1}$, and $\mathbf{I}_e^b(Q_{l_1}, Q_{l_2}, P)$ is the equivalent free-space currents on $l_2$ that radiate the diffracted field at $Q_{l_1}$ when the second wedge is illuminated by a local $\hat{z}$-directed unit electric dipole placed at $P$. When both the explicit expressions $\mathbf{E}_i^a$ and $\mathbf{I}_e^b$ are inserted in (6), the incremental doubly-diffracted field contribution at $P$ is obtained as

$$d\mathbf{E}_{12}^{dd}(P, Q_{l_2}, Q_{l_1}) = \underline{\underline{D}}_2(-\nu_2; \phi_2', \phi_2) \cdot \underline{\underline{M}}(\gamma_{12}) \cdot \underline{\underline{D}}_1(\tilde{\nu}_1; \phi_1, \tilde{\phi}_1') \cdot \mathbf{E}^i(Q_{l_1}) \, \frac{e^{-jkr_{12}}}{2\pi r_{12}} \frac{e^{-jkr_2}}{2\pi r_2} \, dl_2 \, dl_1. \tag{7}$$

In (7), $\underline{\underline{D}}_1(\tilde{\nu}_1; \phi_1, \tilde{\phi}_1')$ and $\underline{\underline{D}}_2(-\tilde{\nu}_2; \phi_2, \phi_2')$ are the two diffraction dyadics associated with the incremental single diffraction at the first and second edges, respectively, and $\underline{\underline{M}}(\gamma_{12})$ is a transformation matrix between the two local reference systems associated with the two edges. To obtain a double diffraction description that provides the proper asymptotic order in all the different transitions that may occur, it is best to retain the product of both the even and the odd parts of each cotangent associated with the spectral Green's Function of a single wedge [10]. An example where the introduction of the double diffraction results in a better estimation of the radiated field is the problem of a circular metallic disc illuminated by a single CSP. A horizontal $\hat{y}$-directed CSP electric dipole is placed at a distance $h = 8\lambda$ from a PEC disc (radius $a = 2.5\lambda$). The beam axis is $\mathbf{b} = \lambda(\sin(8/9\pi), 0, \cos(8/9\pi))$, so the beam points toward the edge of the disc. The observation is made on a circular scan at a distance $r = 10\lambda$ in the $yz$ plane. In Figure 2(b), the $\hat{\theta}$-component of the electric field obtained from the fringe formulation including the double diffraction mechanism (continuous line) is compared with those from the first order fringe estimation (dashed line), and from a Methods of Moments prediction (dotted line). Good agreement between MoM and PO+IFF correction is found everywhere except for the region close to grazing observation, where double diffraction coefficients need to be introduced, as expected.

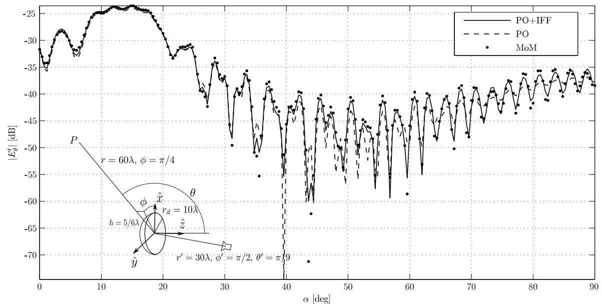



(a) Diffraction at a locally tangent wedge

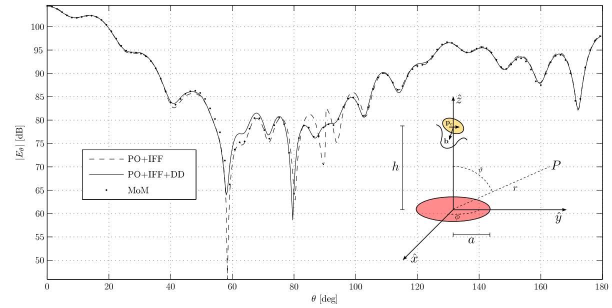



(b) Double diffraction between wedges

Figure 1: Geometry for the canonical configuration problem: (a) single diffraction (b) double diffraction




(a) $|E_\theta|$ component of the total electric field.




(b) $|E_\theta|$ component of the total electric field.

Figure 2: Numerical examples: (a) single diffraction (b) double diffraction

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