Magnetar Asteroseismology with Magneto-Dipole Radiation

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Abstract

The radiative activity of quaking magnetar undergoing Lorentz-force-driven torsional seismic vibrations with monotonically decaying background field is described. It is shown that depletion of internal magnetic field pressure is accompanied by the loss of vibration energy of the star that causes its vibration period to lengthen at a rate proportional to the rate of magnetic field decay. Particular attention is given to the magnetic-field-decay induced conversion of the energy of differentially rotational Alfvén vibrations of magnetar into the energy of oscillating magneto-dipole radiation with elongating periods.

1 Introduction

The discovery of soft gamma-ray repeaters and their identification with magnetars – quaking neutron stars endowed with ultra strong magnetic fields experiencing decay – has stimulated enhanced interest in the asteroseismology of neutron stars. Of particular interest in this domain of research are the Lorentz-force-driven torsional Alfvén vibrations about axis of dipole magnetic moment of the star whose mathematical treatment rests on equations of magneto-solid-mechanics [1, 2, 3]

$$\rho\, \ddot{\mathbf{u}}(\mathbf{r},t) = \frac{1}{4\pi}[\nabla \times [\nabla \times [\mathbf{u}(\mathbf{r},t)\times \mathbf{B}(\mathbf{r})]]] \times \mathbf{B}(\mathbf{r}). \tag{1}$$

These equations describe Lorentz-force-driven non-compressional vibrations of material displacements $\mathbf{u} = \mathbf{u}(\mathbf{r},t)$ and are appropriate not only to neutron stars but also to white dwarfs [4, 5] and quark stars whose superdense material is too expected to be in solid state[6]. The studied in [1, 2, 3] regime of node-free torsion vibrations under the action of Lorentz restoring force is of some interest in that the rate of differentially rotational material displacements $\dot{\mathbf{u}}(\mathbf{r},t) = [\boldsymbol{\omega}(\mathbf{r},t) \times \mathbf{r}]$ has one and the same form as in torsion elastic mode of node-free vibrations under the action of Hooke's force of mechanical shear stresses [3, 7]. Hereafter $P_\ell(\cos\theta)$ stands for Legendre polynomial of degree ℓ specifying the overtone of toroidal a-mode. The prime purpose of above works was to get some insight into difference between spectra of discrete frequencies of toroidal a-modes in neutron star models having one and the same mass M and radius R, but different shapes of constant-in-time poloidal magnetic fields, provided that the intensity of frozen-in magnetic field B remains unaltered in the course of vibrations. Continuing the above investigations, in this contribution a brief outline is given of theory of vibration powered magnetar which accounts for the impact of magnetic field decay on quake-induced magneto-dipole radiation of magnetar.

2 The model of vibration powered magnetar

In what follows we focus on dependence on time of the intensity of poloidal magnetic field, $B = B(t)$, about axis of which the star undergoes global torsional vibrations and confine our

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consideration to the model of uniform internal field $\mathbf{B}(\mathbf{r},t) = B(t)\,\mathbf{b}(\mathbf{r})$, where $B(t)$ is the time-dependent intensity and $\mathbf{b}(\mathbf{r})$ stands for dimensionless vector-field of spatial distribution of the field inside the star. Scalar product of (1) with the following form of material displacements, $\mathbf{u}(\mathbf{r},t) = \mathbf{a}(\mathbf{r})\,\alpha(t)$, followed by integration over the star volume leads to equation for amplitude $\alpha(t)$ having the form of equation of oscillator with depending on time spring constant

$$\mathcal{M}\ddot{\alpha}(t) + \mathcal{K}(t)\alpha(t) = 0, \quad \mathcal{M} = \rho\, m_\ell, \quad \mathcal{K}(t) = 2P_B(t)\,k_\ell, \tag{2}$$

$$m_\ell = \int \mathbf{a}(\mathbf{r})\cdot\mathbf{a}(\mathbf{r})\,d\mathcal{V},\ \mathbf{a} = A_t\nabla\times[\mathbf{r}\,r^\ell\,P_\ell(\cos\theta)], \tag{3}$$

$$k_\ell = \int \mathbf{a}(\mathbf{r})\cdot[\mathbf{b}(\mathbf{r})\times[\nabla\times[\nabla\times[\mathbf{a}(\mathbf{r})\times\mathbf{b}(\mathbf{r})]]]]d\mathcal{V}. \tag{4}$$

The main subject of our further analysis is the impact of depletion of magnetic-field-pressure on the total energy of Alfvén vibrations $E_A = (1/2)[\mathcal{M}\dot{\alpha}^2 + \mathcal{K}\alpha^2]$ and the discrete spectrum of frequency of the toroidal a-mode

$$\omega_\ell^2(t) = \omega_A^2(t)\,s_\ell^2 = B^2(t)\kappa_\ell^2, \quad \omega_A^2(t) = \frac{v_A^2(t)}{R^2},\ s_\ell^2 = \frac{k_\ell}{m_\ell}R^2, \quad \kappa_\ell^2 = \frac{s_\ell^2}{4\pi\rho R^2}. \tag{5}$$

In the reminder of the paper we focus on the case of torsional Alfvén vibrations in quadrupole ($\ell = 2$) overtone. In so doing we omit index ℓ putting $\omega(t) = \omega_{\ell=2}(t)$.

The most striking consequence of the magnetic-field-pressure depletion is that it leads to the loss of vibration energy at a rate proportional to the rate of magnetic field decay

$$E_A(t) = \frac{\mathcal{M}\dot{\alpha}^2(t)}{2} + \frac{\mathcal{K}(B(t))\alpha^2(t)}{2}, \quad \frac{dE_A(t)}{dt} = \frac{\alpha^2(t)}{2}\frac{d\mathcal{K}(B(t))}{dt}. \tag{6}$$

Of particular interests is the case when quaking magnetar converts the energy of above Alfvén vibrations into the power of magneto-dipole radiation

$$\frac{dE_A(t)}{dt} = -\mathcal{P}(t), \quad \mathcal{P}(t) = \frac{2}{3c^3}\delta\ddot{\boldsymbol{\mu}}^2(t). \tag{7}$$

We consider a model of quaking neutron star whose torsional magneto-mechanical oscillations are accompanied by fluctuations of total magnetic moment preserving its initial (in seismically quiescent state) direction: $\boldsymbol{\mu} = \mu\,\mathbf{n} =$ constant. The total magnetic dipole moment should execute oscillations with frequency $\omega(t)$ equal to that for magneto-mechanical vibrations of stellar matter which are described by equation for $\alpha(t)$. This means that $\delta\boldsymbol{\mu}(t)$ and $\alpha(t)$ must obey equations of similar form, namely

$$\delta\ddot{\boldsymbol{\mu}}(t) + \omega^2(t)\delta\boldsymbol{\mu}(t) = 0, \quad \ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \quad \omega^2(t) = B^2(t)\kappa^2. \tag{8}$$

It is easy to see that these equations can be reconciled if $\delta\boldsymbol{\mu}(t) = \boldsymbol{\mu}\,\alpha(t)$. Given this, we arrive at the following law of magnetic field decay

$$\frac{dB(t)}{dt} = -\gamma\,B^3(t), \quad \gamma = \frac{2\mu^2\kappa^2}{3\mathcal{M}c^3} = \text{constant}, \tag{9}$$

$$B(t) = \frac{B(0)}{\sqrt{1+t/\tau}}, \quad \tau^{-1} = 2\gamma B^2(0). \tag{10}$$

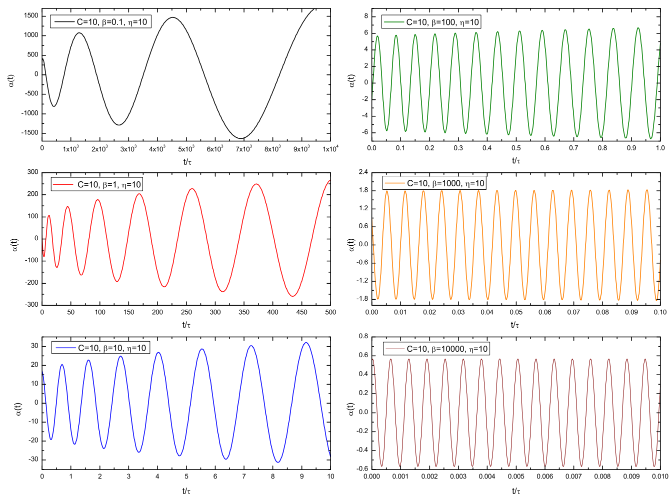

Figure 1: Vibration amplitude $\alpha(t)$ computed at pointed out parameters.

The immediate consequence of above line of argument is the magnetic-field-decay induced lengthening of vibration period

$$P(t) = P(0)\,[1 + (t/\tau)]^{1/2}, \quad P(0) = \frac{C}{B(0)}, \quad C = \frac{2\pi}{\kappa}. \tag{11}$$

The equation for vibration amplitude $\alpha(t)$ permits analytic solution obeying the following boundary conditions $\alpha(t = 0) = \alpha_0,\ \alpha(t = \tau) = 0$, namely

$$\ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \quad \omega^2(t) = \frac{\omega^2(0)}{1 + t/\tau},\ \ \omega(0) = \omega_A\kappa, \tag{12}$$

$$\alpha(t) = C\,[1 + (t/\tau)]^{1/2} \times \{J_1(2\beta\,[1 + (t/\tau)]^{1/2}) - \eta\,Y_1(2\beta\,[1 + (t/\tau)]^{1/2})\}, \tag{13}$$

$$\eta = \frac{J_1(z(\tau))}{Y_1(z(\tau))}, \quad C = \alpha_0[J_1(z(0)) - \eta\,Y_1(z(0)]^{-1}, \quad \alpha_0^2 = \frac{2\bar{E}_A(0)}{K(0)}. \tag{14}$$

where $J_1(2\beta s^{1/2})$ and $Y_1(2\beta s^{1/2})$ are Bessel functions and $\bar{E}_A(0)$ is the energy delivered from burst to the above seismic vibrations. Thus, the depletion of magnetic field pressure resulting in the loss of total energy of Alfvén vibrations of the star causes its vibration period to lengthen at a rate proportional to the rate of magnetic field decay, as is illustrated in Fig.1. It is worth noting that in the considered model, the equation of magnetic field evolution is obtained in similar fashion as equation for the angular velocity $\Omega(t)$ does in the standard model of rotation-energy powered emission [9, 10, 11]. The substantial physical difference between models of rotation-energy and vibration-energy powered pulsating emission of neutron stars is that in the model of quaking neutron star vibrating in toroidal a-mode, the elongation of period of pulses is attributed to magnetic field decay, whereas in the standard model of radio-pulsar the lengthening of period of pulses is ascribed to the slow down of the neutron star rotation.

3 Summary

We found that monotonic depletion of internal magnetic field pressure is accompanied by the loss of vibration energy of the star that causes its vibration period to lengthen at a rate proportional to the rate of magnetic field decay. In this context it is appropriate to remind a seminal work of Hoyle, Narlikar and Wheeler[12] in which it has been pointed out for the first time that vibrating neutron star should operate like Hertzian magnetic dipole deriving radiative power of magneto-dipole emission from the energy of magneto-mechanical vibrations (see, also, [13]). What is newly disclosed here is that conversion of vibration energy into the energy of magneto-dipole electromagnetic radiation can be realized when, and only when, torsional Alfvén vibrations are accompanied by decay of magnetic field. It is hoped that the predicted elongation of pulse period of magnetars should be traced in existing and future observations.

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