Algorithm and Preliminary Observations of New Time scale of NPLI

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Abstract

National Physical Laboratory, New Delhi (NPLI) has been maintaining time scale UTC (NPLI) with only one cesium atomic clock. NPLI has recently developed an automatic intercomparison system for the development of better time scale combining all its five clocks. A new algorithm has also been developed for this purpose. Using this algorithm, the analysis has been done based on the preliminary measurement data. Algorithm seems to produce encouraging results. This paper elaborates the algorithm and presents the analytical observations.

1. Introduction

The time scale maintained by National Physical Laboratory (NPL) of India is named as UTC (NPLI). At present UTC (NPLI) is based on only one of its five commercial cesium clocks. A new time scale is being developed to have time scale by optimally using its all cesium clocks. This paper explains the plan of implementing the scheme; discusses the basic system algorithm developed for this purpose and also presents the preliminary results.

2. Formulation

The phase differences of all pairs of clocks are recorded at regular interval of time. An algorithm is developed to make use of these data to generate a smoother time scale than that of an individual clock. This algorithm aims at maintaining UTC (NPLI) as close as UTC in phase and also decides the amount of phase or frequency jump to be given to a particular clock to maintain better stability and accuracy. Jump is normally effected through a micro-phase stepper unit. This section attempts to formulate the algorithm used in this scheme.
The value of x(t) may be estimated based on the previous value of x(t- τ) as

$$\hat{x}_i\ (\mathrm{t}) = x_i\ (\mathrm{t}-\tau) + \tau . y_i\ (\mathrm{t}-\tau) \qquad (1)$$

Similarly the value of y(t) may be estimated as

$$\hat{y}_i\ (\mathrm{t}) = {}^{1}/_{\tau}\ [\, x_i\ (\mathrm{t}) - x_i\ (\mathrm{t}-\tau)]$$

where, x_i (t) is the measured time offset of clock C_i with respect to the ideal clock C_0 at the instant of time t. $x_n(t)$ is the time offset of a virtual clock C_n, ensembling number of clocks of NPLI with respect to the ideal clock C_0, N is total number of participating clocks . $x_{in}(t)$ is the time offset of clock C_i with respect to the virtual clock C_n , $x_{ij}(t)$ is the measured time offset of clock C_i with respect to the clock C_j , and $y_i(t)$ is the frequency offset of clock C_i with respect to the ideal clock C_0 at the instant of time t.

The ideal time is the conceptual time made by accumulation of idealized seconds. So ideally there should not be any change of phase shift of an ideal clock with time. So ideally there should not be any change of phase shift of an ideal clock with time. But the ensembled clock may be derived by averaging the prediction error which is the differenece between the estimated or predicted time (shown in equation 1) and the measured time. So one may write the definition of C_n (i.e. $x_n(t)$) as,

$$x_n(t) = x_n(t - \tau) + \sum \omega_i(t)\ \{x_i(t) - [x_i(t-\tau) + \tau.\ y_i(t-\tau)]\} \qquad (2)$$

$\{x_i(t) - [x_i(t-\tau) + \tau.\ y_i(t-\tau)]\}$ is the difference between actual measured value and estimated value. In ideal case it shuld be zero. But in practical cases it is not zero. Our aim is to minimize this difference. $w_i(t)$ is the weight of the respective clocks. For nomal averaging $w_i(t)$= 1/N.

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$x_i(t)$ is the measurement of time scale for a single clock. But in practical case we can't measure the time and the frequency offset of a single clock . The measured time differences between clocks are the basic elements for the generation of time scale. So to make use of equation (2) it is essential to convert this in terms of practically realizable terms.

Remebering that $\sum \omega_i(t) = 1$ and combining (1) and (2) one gets,

$$x_{jn}(t) = \sum_i w_i(t)\left[x_{ji}(t) + x_{in}(t-\tau) + \tau . y_i(t-\tau)\right] \quad (3)$$

$x_{jn}(t)$ is the parameter which needs to be determined with the help of the measured values like $x_{ij}(t)$. It may be noted that equation (3) is a recursive equation. So the starting of the calculation will be based on initial values which are to be fixed quite carefully.

3. Implementation Scheme

A selector switching system has been designed and developed to select a pair of clocks at a time, and take the measurement sequentially and automatically. A time interval counter (SR680 of Standford Research System) with a resolution of 50 ps has been used to intercompare the clocks.

4. Implementation of Formulation

With help of the above arrangement the data are being recorded regularly. Formulation has been implemented in two different ways. Accordingly some initial preliminary calculations have been made. These calculations are based on the following assumptions to start the analysis.

4.1. Starting Assumptions

Master clock of NPLI is clock number 5. The status of master clock UTC-UTC (NPLI) (as published in Circular T) may be termed as $-x_{5n}(t)$ at the start. So from the starting values of $x_{5n}(t)$, $x_{in}(t)$ may be found as,

$$x_{in}(t) = x_i(t) - UTC = x_{5n}(t) - x_{5i}(t) \quad (4)$$

To find values of $y_{in}(t)$, previous 90 days data have been used , making τ as 90 days .

4.2. Preliminary Observations

One of the ways to assess the usefulness of the algorithm is to find $x_{5n}(t)$ and compare its value with the corresponding data of circular T.

The equation (3) has been used to determine $x_{in}(t)$ with measured data of $x_{ij}(t)$ assuming equal weight (i.e. $w_1 = w_2 = w_3 = w_4 = w_5 = 1/5$).

Using the measured values of $x_{25}(t)$, $x_{25}(t)$, $x_{25}(t)$ and $x_{25}(t)$ five sets of $x_{5n}(t)$ may also be obtained with help of relation (4). These five sets may combined to get one set of $x_{5n}(t)$. This has been done in two ways.

In one case (say, case I), simple average of all $x_{5n}(t)$ has been determined to generate the final time scale. These were compared with the corresponding values of circular T. In case II, unlike of case I, a weighted average method has been adopted. The weights have been determined in the following way.

It has been noted that values $x_{1n}(t)$, $x_{2\,n}(t)$, $x_{3\,n}(t)$, $x_{4\,n}(t)$ and $x_{5\,n}(t)$ have been determined through equation 3. All these sets are linear function of time. One may find the standard deviation SD_i of residual of linear fit for each of $x_{1n}(t)$, $x_{2\,n}(t)$, $x_{3\,n}(t)$, $x_{4\,n}(t)$ and $x_{5\,n}(t)$. These SD_i would represent the noise characteristics of the individual clocks.

So it would quite logical to define the weight (wt_i) of the respective clocks as,

$$wt_i = \frac{SD_i^2}{\sum SD_i^2} \quad (5)$$

It may be noted that double smoothing has been adopted here. In equation 3, the equal weights have been assigned but noise-dependent weights have been assigned in the second averaging. This is where this algorithm different from those adopted in other laboratories as mentioned in [1-2].

Comparison of UTC (NPLI) with time scale by the new algorithm is shown in Fig.1. For first 25 days, time scales by both methods matches quite closely (within 15 ns) with the corresponding values of Circular T. Even after 25days, the time scale of case II still matches quite well with Circular T, whereas time scale for case I deviates largely afterwards.

The stability of the time scale has also been studied by determining the Allan Deviation for the different values of averaging time (τ) and has been illustrated in Fig.2. It is interesting to note that there is hardly any improvement in stability of time scale generated by case I over that by single Cesium clock. But substantial improvement in stability has been observed in case II.

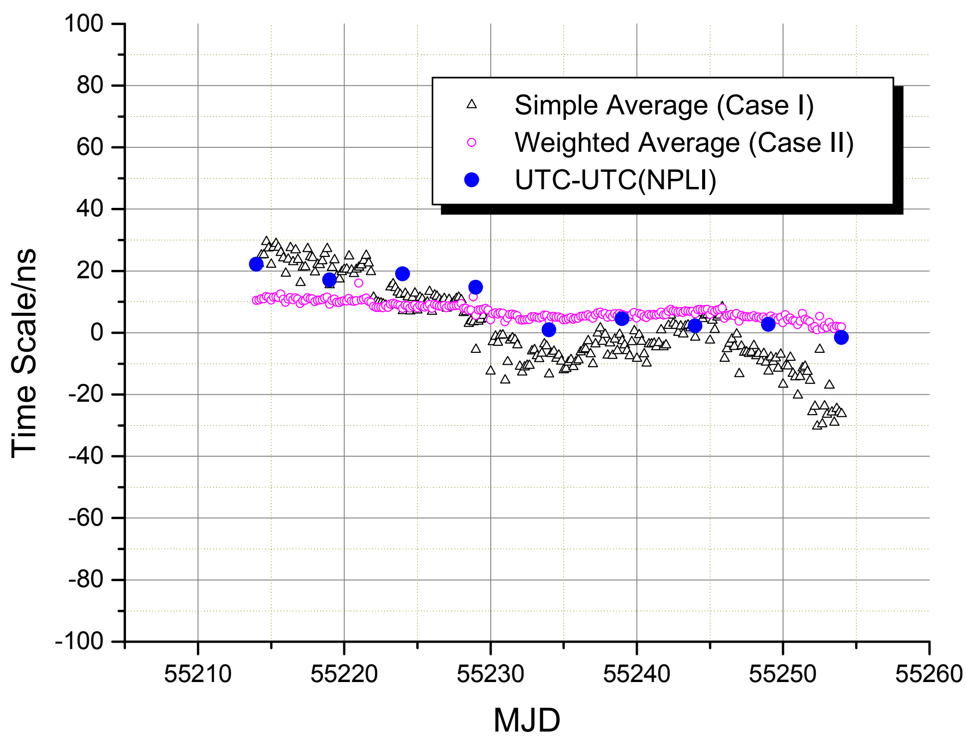

Fig. 1 Status of Time scale vis-à-vis circular T data

5. Conclusions

So the new algorithm of time scale of NPLI shows significant improvement both in accuracy and in stability. Here double smoothing has been applied and first smoothing also helped in assessing independent noise charateristics of individual clocks. Perhaps, by this double smoothing method one gets improvement in time scale even with fewer number of clocks.

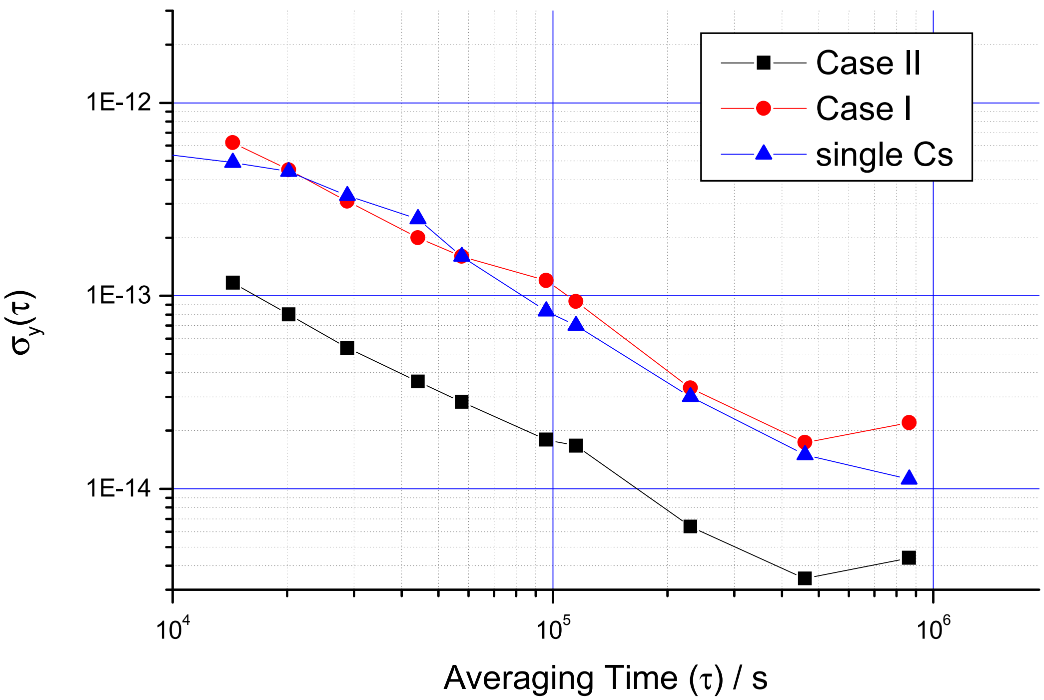

Fig.2. Stability of Corresponding Time Scales

6. References

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