# Impedance Matched Hyperlens

***Zubin Jacob**, Alexander V. Kildishev and Evgenii E. Narimanov*

Birck Nanotechnology Center, School of Computer and Electrical Engineering, Purdue University, West Lafayette, IN 47907, USA

**Abstract**

We develop an imaging system capable of magnification, subwavelength-resolution and impedance matching, which minimizes reflection losses. We propose a practical design of the system based on available materials and existing fabrication technologies.

**Introduction**

Advances in metamaterial technology, has led to the experimental demonstration of imaging devices such as the superlens [1] and hyperlens [2,3], that can overcome the long standing problem of the diffraction limit undermining conventional optics. In particular, the hyperlens [2,3] is capable of projecting into the far field the magnified image of a subwavelength object [4,5]. However, the high impedance mismatch between the air and the metamaterial forming the hyperlens, severely limits the potential applications of the device. Not only does this high reflectivity lead to low light throughput, but it also affects the minimum achievable subwavelength resolution. With these limitations, the original design of the hyperlens [2,3] cannot fulfill the expectations of revolutionizing bio-imaging and nanolithography.

To address this critical issue, in the present paper we develop a new approach to hyperlensing using the methods of transformation optics [6]. Within this framework, we demonstrate the generalized hyperlens with matched impedance that can significantly reduce and even eliminate reflection losses [7]. Furthermore, we propose a design for the practical realization of this impedance matched hyperlens in the visible as well as mid IR, based on available materials and existing fabrication technologies .

An ideal imaging device will translate the field from its input interface to the output interface, a trivial example of which is a ring with very small thickness. If we transform this thin ring to a physical domain with nonzero dimensions, preserving the field pattern at the boundaries, the resulting device would enable perfect imaging. The simplest map achieving this is based on a linear radial transformation, $\rho(r)$, where a point r on the initial virtual space is mapped onto a point with radius $\rho$ in the physical world with the same azimuthal position $\varphi$ [Fig. 1(a)]. We define the hyperlens spatial transform which maps a thin circular ring A in the virtual domain, $a \le r \le b$, onto a thicker ring B, $a \le \rho \le l$ . Using the laws of transformation optics, we obtain the material parameters needed to achieve this transformation for fields which have the magnetic field perpendicular to the plane of the physical domain [7],

$$\varepsilon_\varphi = \rho/(r\tau),\ \varepsilon_\rho = 1/\varepsilon_\varphi,\ \mu_z = r/(\rho\tau) \tag{1.1}$$

where $\tau$ is a constant. A non-magnetic material with $\mu_z = 1$ and $\varepsilon_\varphi = \rho/r$, $\varepsilon_\rho = r\tau/\rho$ has the same material dispersion, hence imaging characteristics as (1.1) and achieves impedance matching at the input interface.

We now consider a metamaterial design of the impedance matched hyperlens (IMH) where a three material meta-layer forms the basic building block to achieve the desired dielectric response. The fill fractions of the three constituents, which are continuous functions of $\rho$, are approximated by 25 meta layers [Fig. 1(b)] where each meta layer is made of three layers consisting of SiC/air/$SiO_2$ [Fig. 1(b) inset]. At $\lambda = 10.6$ µm, $\varepsilon_{SiO2} \approx 4$ and $\varepsilon_{SiC} \approx -1.4 + 0.14i$. where the dielectric permittivity of SiC is obtained from the dispersion relation $\varepsilon_{SiC} = \varepsilon_\infty \frac{\omega^2 - \omega_{LO}^2 + i\gamma\omega}{\omega^2 - \omega_{TO}^2 + i\gamma\omega}$

$\omega_{TO} = 796$ cm$^{-1}$, $\omega_{LO} = 972$ cm$^{-1}$, $\varepsilon_\infty = 6.5$, $\gamma = 5$ cm$^{-1}$ [8]. Note the realistic losses in the system have been taken into account in our design. To study the imaging characteristics of the mid IR system, we numerically simulate the field due to two point sources separated by a distance ($\lambda$ /3) placed within the IMH. The resulting intensity in the system and surrounding area is shown in Fig. 2(a) The two sources are clearly resolved [Fig. 2 (b)] even though the distance between them ($\lambda$ /3) is clearly below the diffraction limit. For a design in the visible spectrum we use a

Silver/Air/$Al_2O_3$ system as the basic materials comprising the layered realization of the device. At $\lambda \approx 350$ nm, $\varepsilon_{Ag} \approx -1.5+0.3i$, $\varepsilon_{Al2O3} = 3.217$, and can be used to achieve the desired dielectric response. Simulations show that this system achieves a similar performance as the presented SiC/air/$SiO_2$ system. It has to be noted that the very nature of the non-resonant transformation optics design makes the impedance matched hyperlens less sensitive to losses. This leads to a viable practical implementation of the impedance matched hyperlens. Once beyond the device the magnified image of the object with subwavelength features is available for post-processing by conventional optics.

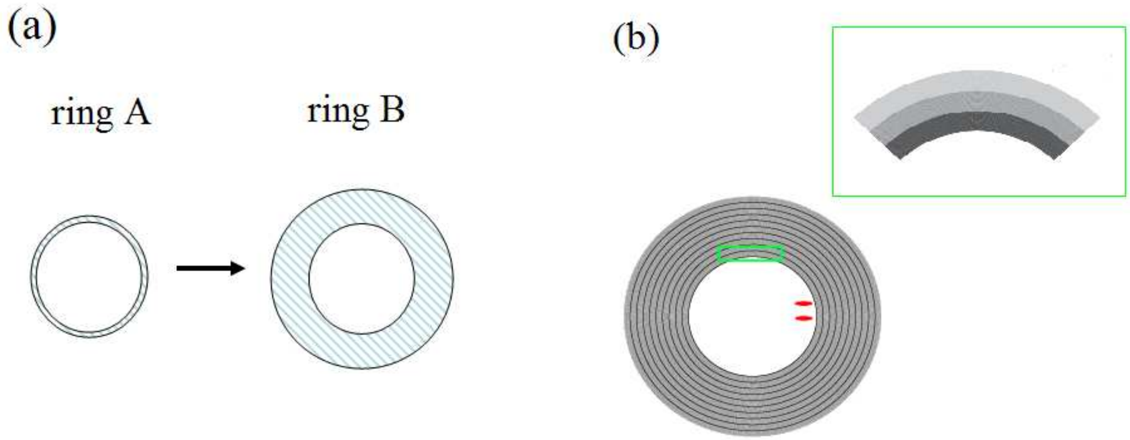


Fig. 1 (a) Hyperlens spatial transform that maps thin virtual ring A to a thick ring B, (b) Schematic of the layered realization of the dielectric permittivity distribution obtained from the map (a) Also shown are point sources separated by a distance below the diffraction limit kept close to the inner boundary. (inset) Single layer enclosed by the green box is a metalayer comprising of SiC/air/$SiO_2$ with individual layer thickness varying with $\rho$

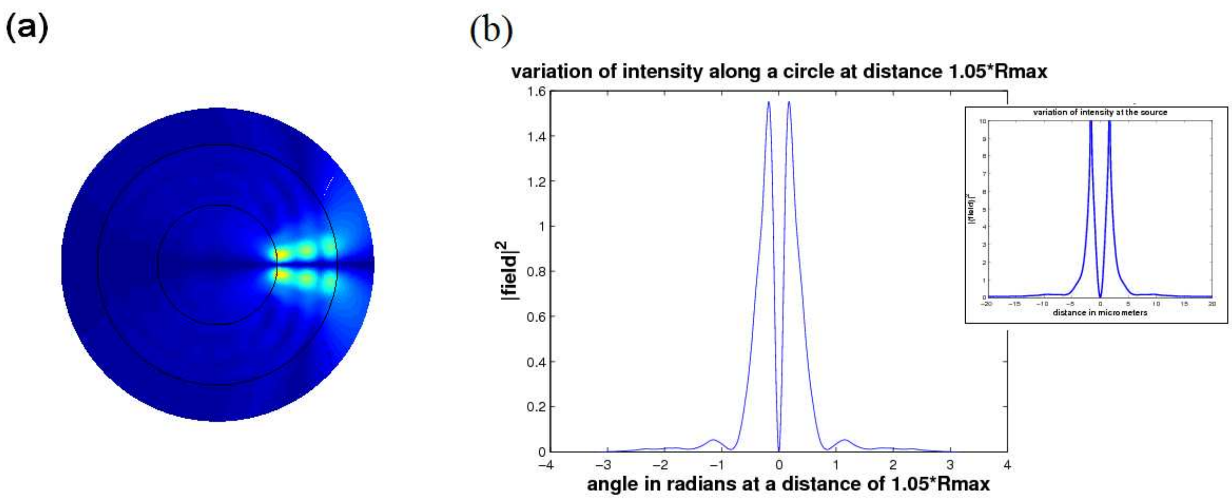


Fig. 2: (a) False color plot of intensity due to two point sources (phase difference of $\pi$) kept a distance $d=\lambda/3$ apart in a hyperlens with inner radius a ~ $\lambda$, outer radius $l$ ~ 2 $\lambda$, virtual radius ~ 1.02$\lambda$ , total 75 layers of alternating SiC/air/$SiO_2$ with layer thicknesses that vary with $\rho$ (b) Image of the two point sources outside the hyperlens and intensity variation at the source (inset); note the magnified image is beyond the diffraction limit accessible to conventional optics.

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