Ionospheric range errors in modernized GPS and future Galileo systems

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Abstract

The availability of the third frequency from the modernized GPS and future Galileo systems provides an opportunity to eliminate the first and second order ionospheric terms in the refractive index formula. But higher order ionospheric errors caused by the differential bending of the signal and the third order ionospheric term are not fully removed. This paper examines the magnitude of these errors for a large number of ionosphere profiles reconstructed from GPS measurements carried out onboard LEO satellites. It has been found that triple-frequency residual range errors exceed centimeter level at low elevation angles and during times of high total electron content (TEC).

1. Introduction

For single-frequency GPS users, the signal delay due to refraction through the ionosphere is the largest (a few meters to tens of meters at the zenith [1]) and most variable source of positioning error. Taking advantage of the dispersive nature of the ionosphere, dual-frequency users can eliminate the first order ionospheric term by differencing the signal at two frequencies. The remaining second and third order errors are estimated typically to be ~0-2 cm and ~0-2 mm at zenith, respectively [2]. A third-frequency will be available from the modernized GPS and future Galileo systems. Therefore, triple-frequency users will have an opportunity to eliminate the first and the second order ionospheric errors by double differencing the code or phase measurements. However, with a triple-frequency combination, the ionospheric error will not disappear completely. Higher order ionospheric errors caused by the differential bending of the signal and the third order ionospheric term are not fully removed in this approach. Due to the dispersive nature of the ionosphere, GPS signals at L1, L2 and L5 frequency travel along different ray paths through the ionosphere. In each case the ray path is longer than the length of the free space path. The corresponding excess path in addition to the free space path or true range may achieve several centimeters at low elevations under high solar activity conditions. Moreover, the total electron content will be different along L1, L2 and L5 paths and causes additional error in range estimation. This paper examines the magnitude of the triple-frequency residual range errors due to total electron content difference, excess path length and third order ionospheric term. For this purpose, GPS signals are traced through the ionosphere for a large number of electron density profiles. The electron density profiles are reconstructed from the GPS radio occultation measurements onboard the CHAMP satellite and available at http://w3swaci.dlr.de/.

2. Triple-frequency residual errors

Assuming a right-hand circularly polarized signal, the GNSS triple-frequency code (Ψ) and phase (Φ) pseudo-range equations can be written in terms of true range ρ and higher order ionospheric errors as

$$\frac{1}{D}\left[A\left(\Phi_1 f_1^2-\Phi_2 f_2^2\right)-C\left(\Phi_1 f_1^2-\Phi_3 f_3^2\right)\right]=\rho \quad \underbrace{-\Delta s_{b(TEC)}-\Delta s_3+\Delta s_{b(length)}}_{RRE} \tag{1}$$

$$\frac{1}{D}\left[A\left(\Psi_1 f_1^2-\Psi_2 f_2^2\right)-C\left(\Psi_1 f_1^2-\Psi_3 f_3^2\right)\right]=\rho \quad \underbrace{+\Delta s_{b(TEC)}+3\Delta s_3+\Delta s_{b(length)}}_{(RRE)_{gr}} \tag{2}$$

In which

$$\Delta s_{b(TEC)} = \frac{40.3}{D}\left[C(\Delta TEC_3 - \Delta TEC_1) - A(\Delta TEC_2 - \Delta TEC_1)\right] \tag{3}$$

$$\Delta s_3 = \frac{t}{3D}\frac{(f_2 - f_3)}{f_2 f_3} \tag{4}$$

$$\Delta s_{b(length)} = \frac{1}{D}\left[C\left(f_3^2 d_3^{b(length)} - f_1^2 d_1^{b(length)}\right) - A\left(f_2^2 d_2^{b(length)} - f_1^2 d_1^{b(length)}\right)\right] \tag{5}$$

$$t = 2437\int n_e^2 ds + 4.74\times 10^{22}\int n_e B^2 (1+\cos^2\Theta) ds \tag{6}$$

$$A = \frac{f_1 f_2}{f_1 - f_2}$$
$$C = \frac{f_1 f_3}{f_1 - f_3} \tag{7}$$
$$D = f_1(f_2 - f_3)(f_1 + f_2 + f_3)$$

where $\Delta s_{b(TEC)}$, $\Delta s_{b(length)}$ and Δs_3 are the triple-frequency residual errors due to TEC difference at three GNSS frequencies f_1, f_2 and f_3 (e.g., L1, L2, L5 signal frequency), excess path length in addition to the free space path length and the third order ionospheric term, and *RRE* and $(RRE)_{gr}$ are the total residual range errors in phase and code combinations, respectively. In Eq. (6) n_e is the electron concentration, *ds* is the ray path element, Θ is the angle between the ray direction and the magnetic field vector $\boldsymbol{B}$ and B is the magnitude of $\boldsymbol{B}$. The quantity ΔTEC_i (i = 1, 2, 3) is the difference between TEC along the straight line of sight (LoS) and the ray path, and $d_i^{b(length)}$ is the excess path length of the signal in addition to the free space path length.

3. Simulation results

A two-dimensional ray tracing program is developed to trace electromagnetic signal through the ionosphere. The effects of the Earth's magnetic field on the signal propagation are taken into account considering the International Geomagnetic Reference Field (IGRF) model. As already mentioned, CHAMP vertical electron density profiles are employed for the ionosphere. More than two hundred thousands CHAMP profiles with global coverage are available. However, only the profiles derived during high and low solar activity years 2002 and 2006 are used for triple-frequency residual error estimations. These profiles are plotted in Fig. 1 together with TEC values obtained at different elevation angles.

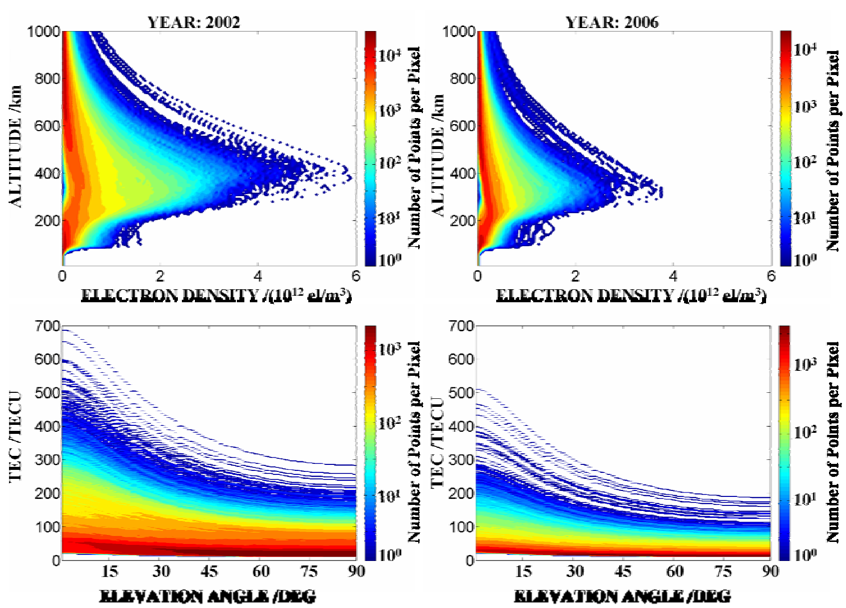

Fig. 1: CHAMP vertical electron density profiles during 2002 (about 31,000 profiles) and 2006 (about 23,500 profiles) (top plots) and corresponding TEC values at different elevation angles (bottom plots). Pixel size in XY scales: 0.05 unit × 10 unit (top plots) and 1 unit × 2 unit (bottom plots)

The L1, L2 and L5 signals are traced for different Earth-GPS paths by the ray tracing program. Then the triple-frequency residual errors $\Delta s_{b(TEC)}$, $\Delta s_{b(length)}$ and Δs_3 are estimated and plotted in Figs. 2 and 3.

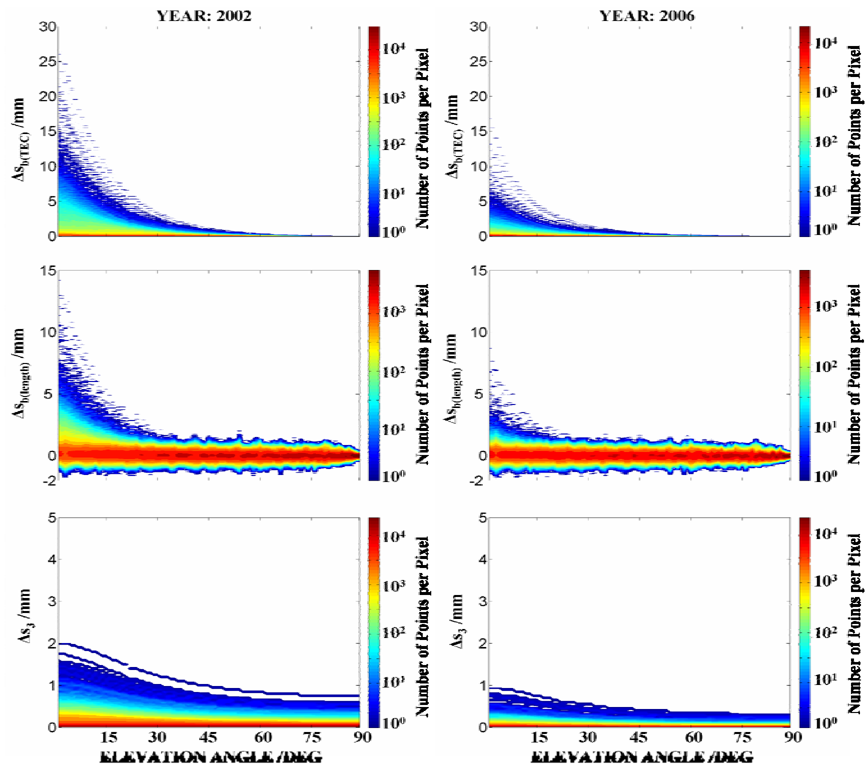

Fig. 2: Triple-frequency range errors due to signal bending (e.g., $\Delta s_{b(TEC)}$, $\Delta s_{b(length)}$) and third order ionospheric term (Δs_3). Pixel size in XY scales: 1 unit × 0.05 unit

Figure 2 (top plots) shows that $\Delta s_{b(TEC)}$ can be as big as about 2.5 mm at low elevation angles (e.g., 1°) and during times of high TEC (e.g. ~ 650 TEC). The maximum $\Delta s_{b(length)}$ and Δs_3 are estimated to be about 14 and 2 mm, respectively (see middle and bottom plots of Fig. 2). However, for most of the profiles $\Delta s_{b(TEC)}$, $\Delta s_{b(length)}$ and Δs_3 are found to be in sub-millimeter level. Comparing higher order errors at high and low solar activity years, we see that the errors are significantly higher during 2002 compared to those at 2006. The total residual range errors in phase and code measurements *RRE* and $(RRE)_{gr}$ are plotted in Fig. 3. Comparing absolute values, we see that the $(RRE)_{gr}$ is much higher than *RRE*. Fig. 3 shows that, during times of high TEC higher order ionospheric errors exceed 4 and 1 cm level in triple-frequency code and carrier-phase measurements, respectively. Therefore, these errors require to be corrected if millimeter level accuracy is needed in precise GNSS positioning.

4. Triple-frequency error corrections

The ray tracing calculations have been carried out to compute $\Delta s_{b(TEC)}$, $\Delta s_{b(length)}$ and Δs_3 for a large number of geometrical and ionospheric/plasmaspheric conditions. Functional dependencies have been studied separately for each parameter to develop empirical formulas for unknown components ΔTEC_i and $d_i^{b(length)}$ in Eqs. (3) and (5) as

$$\Delta TEC_i = \frac{457.91 \cdot \exp(-2.18\beta) \cdot (h_m + R_e) N_m TEC}{f_i^2 (h_m)^{0.35} \sqrt{(h_m + R_e)^2 - (R_h + R_e)^2 \cos^2 \beta}} \quad (8)$$

$$d_i^{b(length)} = \frac{3.1 \times 10^3 \exp(-2.13\beta)(h_m + R_e) N_m \cdot TEC}{f_i^4 (h_m)^{1/8} \sqrt{(h_m + R_e)^2 - (R_h + R_e)^2 \cos^2 \beta}} \quad (9)$$

where ΔTEC_i will be measured in el/m^2 and $d_i^{b(length)}$ in meters if the receiver height from the Earth's surface R_h, the Earth's mean radius R_e are in km, the maximum ionosphere ionization N_m in el/m^3 and its height h_m in km, the receiver to satellite elevation angle β in radians and f is in Hz. Now $\Delta s_{b(TEC)}$ and $\Delta s_{b(length)}$ can be estimated in meters by Eqs. (3) and (5) in conjunction with Eqs. (8) and (9), respectively. It has been found that, about 70% of these errors will be removed on an average for such approximations.

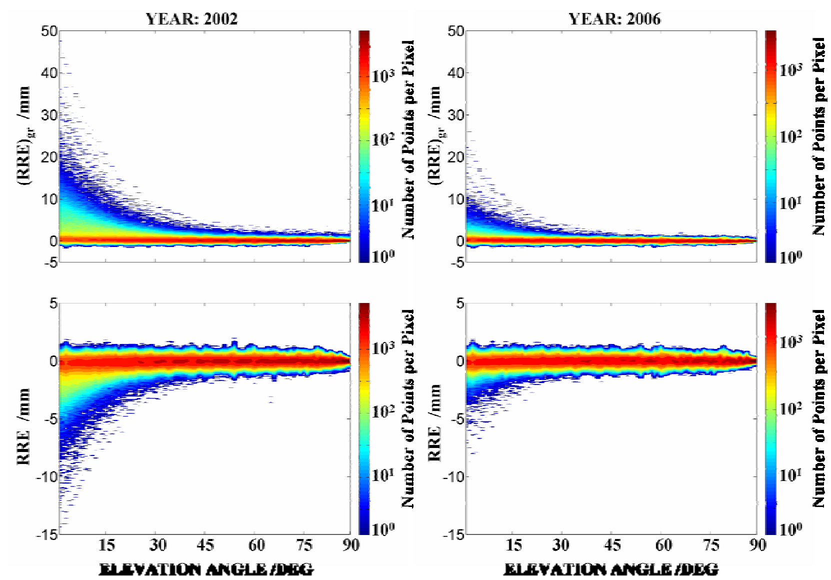

Fig. 3: Triple-frequency residual range errors RRE and $(RRE)_{gr}$. Pixel size in XY scales: 1 unit × 0.05 unit

5. Conclusion

A rigorous treatment of higher order ionospheric errors using a large number of ionosphere vertical profiles (about 54,500) derived from GPS measurements onboard CHAMP satellite shows that the triple-frequency residual range errors are significant at low elevation angles (< 15°) and cannot be neglected during times of high TEC. It has been found that in phase pseudo-range measurement triple-frequency *RRE* exceeds 1 cm level and in code pseudo-range measurement $(RRE)_{gr}$ exceeds 4 cm level at 1° elevation angle. The interoperability of GPS and Galileo will allow an additional frequency E5b (1207.14 MHz) to be used in code and phase pseudoranges. This will make possible to combine four frequencies (e.g., L1, L2, L5 and E5b) for higher order ionospheric corrections. Sample calculation shows that combining these four frequencies, the ionospheric effects can be cancelled out successfully from range estimations and no ionospheric correction is required.

6. References

1. J. A. Klobuchar, "Ionospheric Effects on GPS", in B. W. Parkinson, J. J. Spilker (eds.), *Global Positioning System: Theory and Applications*, *Vol I*, pp. 485-515, American Institute of Aeronautics & Astronautics, ISBN 156347106X, 1996
2. S. Bassiri and G. A. Hajj, "Higher-order ionospheric effects on the global positioning system observables and means of modeling them", *manuscripta geodaetica*, *18*(6), 280-289, 1993