On the Use of A Priori Data in Microwave Tomography

***Andreas Fhager*[1], *Chien-Chih Chen*[1], *Mikael Persson*[1]**

[1]Department of Signals and Systems, Chalmers University of Technology, Sweden,
andreas.fhager@chalmers.se, mikael.persson@chalmers.se

Abstract

In this paper an iterative electromagnetic time-domain inversion algorithm is described and modified to include a priori data of the shape and dielectric properties of an object being imaged. The algorithm is based on solving the regular and the adjoint Maxwell's equations in order to compute gradients, which are used to update the dielectric profile with the conjugate-gradient method. As an alternative a global optimization algorithm, the particle swarm optimization method, has also been investigated. The results show that by exploiting a priori dielectric data in this way a significant improvement in the resolving ability can be obtained.

1. Introduction

The inverse electromagnetic problem in the image reconstruction process is inherently ill-posed and nonlinear. An iterative reconstruction algorithm, which involves a cost function that is either maximized or minimized, is therefore necessary. However the nonlinearity of the problem often causes a local search algorithm to get trapped in a local minimum, leading to incorrect reconstructions, [10]. Possible ways to overcome this includes introducing a priori information about the object being imaged, [11]. It can also be necessary to use a frequency-hopping technique where low frequency content of the electromagnetic pulse is used to reduce the nonlinearities. This is then followed by using successively higher frequency content to improve the resolution, [5]. The problem of getting trapped in a local minimum can as an alternative often be helped with a clever initial guess or by initializing the optimization with an ideal model of the targets under reconstruction. This is for instance viable in biomedical imaging where the organs and tissues can be assumed known, [3], or when testing an object for a defect when the unperturbed object otherwise is known, [1]. To overcome the ill-posedness it is furthermore necessary to apply a regularization procedure that is introducing constraints on the reconstructed image. A good regularization scheme should make use of constraints corresponding to known physical properties of the object being imaged. The Tikonov method is an example of a method that is often used to impose a smoothness condition on the reconstruction, [13]. Other methods are for example edge preserving procedures, [4] and methods based on singular value decomposition, [2]. The regularization procedure can also be considered as a way to utilize a priori information about the target properties in the reconstruction since the regularization should favor known physical properties of the target.

The present paper is based on our earlier work on microwave tomography using an iterative non linear algorithm, [9], which is now modified to include a priori data. The particle swarm optimization algorithm has also been used and evaluated for its feasibility in exploiting a priori data. Its use in the microwave tomography application is described in this paper together with a numerical example.

2. The reconstruction algorithm

The iterative electromagnetic time-domain inversion algorithm, modified to incorporate a priori knowledge of the shape and dielectric properties, is similar to what is used in our microwave tomography experiment, [8], [9]. The algorithm assumes that measurements of transient pulses have been made for several transmitter/receiver combinations surrounding the target region. In the reconstruction process an initial dielectric distribution is assumed, and if no a priori information of the targets is included in the algorithm it is set equal to the uniform background properties. It is further assumed that the material is non magnetic. The objective of the reconstruction procedure is to minimize the cost functional, F, defined as

$$F(\varepsilon,\sigma)=\int_0^T\sum_{m=1}^{M}\sum_{n=1}^{N}\left|E_m(\varepsilon,\sigma,R_n,t)-E_m^{meas}(R_n,t)\right|^2dt, \tag{1}$$

where $E_m(\varepsilon, \sigma, R_n, t)$ is the calculated field from the computational model in successive iterations of the inversion algorithm. $E_m^{meas}(R_n, t)$ is the measured data. In this work simulations are however used to generate also the measured data. Furthermore M is the number of transmitters and N is the number of receivers. We have described a gradient based method including a priori data in the reference, [7] that is used in the numerical example in this paper. To modify the algorithm to take a priori data into account the gradients were used in a modified search procedure outlined in the following. The description is without loss of generality made for the case with $\varepsilon_{object} > \varepsilon_{background}$, $\sigma_{object} > \sigma_{background}$. First the gradients were normalized according to

$$G_{\varepsilon}^{normalised}(x) = \frac{G_{\varepsilon}(x)}{\max\{G_{\varepsilon}(x)\}},$$
$$G_{\sigma}^{normalised}(x) = \frac{G_{\sigma}(x)}{\max\{G_{\sigma}(x)\}}. \tag{2}$$

Based on these normalized gradients an optimization procedure was used to find a threshold level, g_{th} that was defined in the range $0 < g_{th} < 1$. All the grid cells in the reconstruction domain where

$$G_{\varepsilon}^{normalised}(x) > g_{th},$$
$$G_{\sigma}^{normalised}(x) > g_{th} \tag{3}$$

were assigned the a priori dielectric values of the object, all other grid cells were assigned the background values. In a similar way a mechanism for moving and changing the size of the object was deduced. An analog procedure can also be deduced for objects with properties $\varepsilon_{object} < \varepsilon_{background}$, $\sigma_{object} < \sigma_{background}$.

2.1 Numerical example

A numerical example showing the potential benefit of using an algorithm is presented next. The reconstructed object was a large object, with a relative permittivity 12.0, and two small included objects, with a relative permittivity 55.0. Using the a priori algorithm a very accurate reconstruction was made according to the image in figure 1 (a). In this case the frequency content of the electromagnetic pulse was 900 MHz centre frequency and Gaussian spectral contents with a full width half maximum bandwidth (FWHM) 900MHz. If instead reconstructing the same image with the original algorithm without utilizing the a priori data and using the same spectral content the reconstructed image can be seen in figure 1 (b). To achieve a similar resolution of the two smaller objects as is the case with the a priori algorithm it was necessary to use a spectral content with centre frequency 2.0 GHz and a bandwidth 2.0 GHz, figure 1 (c). This improvement in resolution when using a priori data in the reconstruction could be very beneficial in an experimental situation where the bandwidth of the experimental equipment is a critical design parameter. In conclusion by utilizing the a priori data it was possible to obtain a resolution which was significantly improved compared to the case when a priori information was not used.

(a) (b) (c)

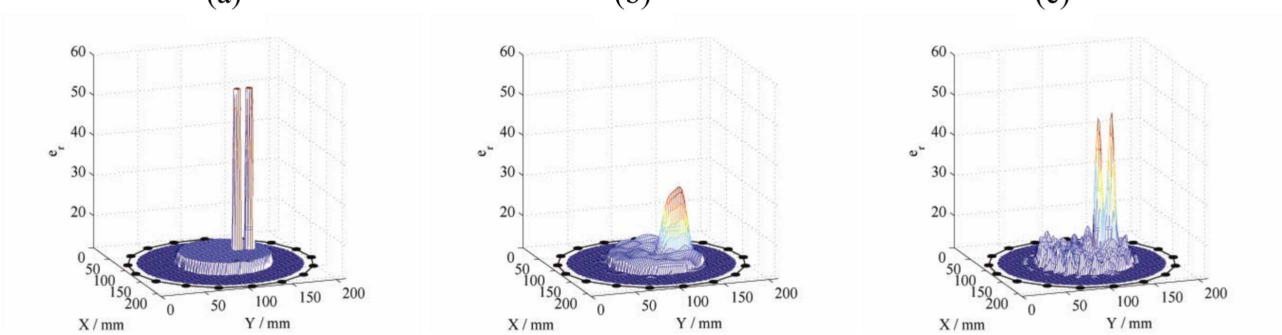

Figure 1. (a) A reconstruction of two small objects inside a large object utilising dielectric *a priori* data of the dielectric properties. This reconstruction constitutes a very accurate image of the original object. (b) Reconstruction of the same object as in (a), using no a priori data and the same spectral content of the electromagnetic pulse. The two small objects were not resolved. (c) Reconstruction of the same object as in (a), using no a priori data but a higher frequency content of the electromagnetic pulse. The centre frequency and the full width half maximum bandwidth was 2 GHz respectively.

3. Particle Swarm Optimization

The results in previous section indicate a significant benefit in the resolving ability in the reconstructed image when utilizing the a priori data. However the designated algorithm has turned out to be very sensitive to the changes in the settings. At the same time attempting to avoid the potential problem of a local search algorithm getting trapped in a local minimum a global optimization algorithm, i.e. the Particle Swarm Optimization (PSO) has been investigated. The concept of PSO is to mimic the social behavior of different species, such as bees finding flowers [12]. A basic PSO optimizer performs the following five tasks, [6]:

1. *Initialization:* Each particle is initialized with a random position and velocity.
2. *Fitness evaluation:* The "fitness", defined in terms of a penalty functional value, *F*, of each particle is evaluated.
3. *Personal best update*: The fitness value of each particle is compared with its own best fitness so far. The personal best (*pbest*) value is updated if needed.
4. *Global best update*: The fitness value of each particle is compared with its neighborhood particles. The global best value (*gbest*) is updated if needed.
5. *Velocity and position update:* The velocity vector of each particle is changed and the particle is moved to a new position according to the following two equations:

$$v(t) = w \cdot v(t-1) + c_1 rand() \cdot (x_{pbest} - x(t)) + c_2 rand() \cdot (x_{gbest} - x(t)) \tag{4}$$

$$x(t) = x(t-1) + v(t) \tag{5}$$

Here w denotes an inertia weight in the range 0 to 1, c_1 and c_2 are acceleration coefficients weighting the influence from the personal best and the global best fitness and *rand()* is a random number in the range 0 to 1. To use this optimization scheme for the microwave tomography reconstruction problem of finding the diameter, d_i position, (x_i, y_i), and dielectric properties, ε, σ, of circular objects in the same way as in the previous section the objective functional is reformulated into. In general several objects can be identified and thus i is used to label a number of objects. However in the following example we are only searching for a single object.

$$F(x_i, y_i, d_i, \varepsilon_i, \sigma_i) = \int_0^T \sum_{m=1}^{M} \sum_{n=1}^{N} \left| E_m(x_i, y_i, d_i, \varepsilon_i, \sigma_i, R_n, t) - E_m^{meas}(R_n, t) \right|^2 dt. \tag{6}$$

3.1 Numerical Example

As an illustration and a feasibility study of the particle swarm optimization method for finding an object with given dielectric properties the following test case was set up. The centre of a circular object was set at $(x, y) = (50, 50)$ grid cells. The diameter was 10 grid cells and the relative permittivity was 3.0 and the conductivity 0.1 in a background with ε_r=1.0 and σ= 0.0. In the PSO algorithm 5 particles were used together with equal acceleration coefficients $c_1 = c_2 = 1.49$ and a linearly decreasing inertial weight between 0.9-0.4 for the velocity update. The image reconstruction domain was 113 by 113 grids with grid resolution 2 mm. The FDTD numerical model consisted of eight antennas, and the Gaussian-amplitude sinusoidal transmitting signal had centre frequency 4.0 GHz and bandwidth 4.0 GHz. The reconstruction process is illustrated in figure 2, where the convergence of the different particles as a function of iteration number is shown. In (a) the convergence of the x-coordinate is shown, in (b) the y-coordinate and in (c) the diameter. With only three different search parameters the computational burden in this algorithm was comparable to the local search algorithm described in previous section. Without increasing the computational burden significantly it is also in a straight forward manner possible to extend the algorithm to also find the dielectric properties of the object within a preferred dielectric range. In conclusion this algorithm might consequently be a feasible alternative for using when imposing a priori knowledge of the objects under reconstruction.

4. Conclusion

Two different methods have been investigated for their applicability in exploiting a priori dielectric data in the microwave tomography problem. In general it has been found that by taking a priori dielectric knowledge of the imaging problem into account in the reconstruction process the resolving ability of the algorithm can significantly be improved.

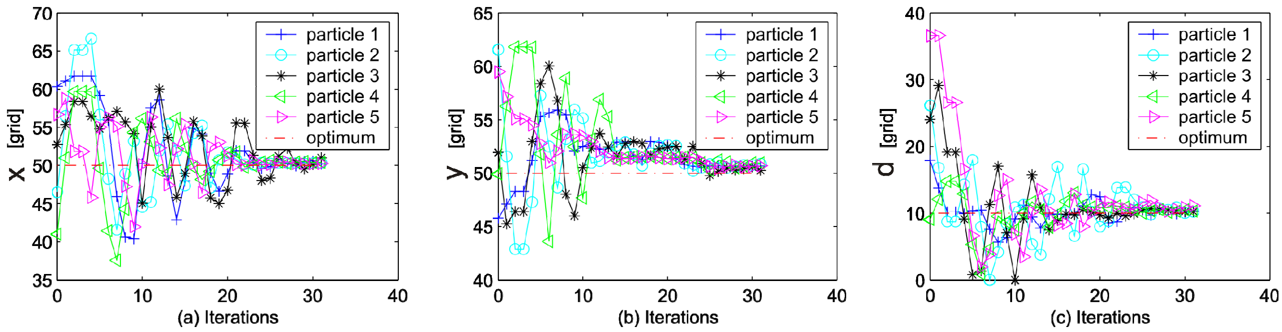

Figure 2. A PSO run showing the trajectories of five particles for reconstructions of the object centre (*x,y*), and diameter *d*. The trajectories show how the position and diameter of the object is converging in about 30 iterations.

A global optimization method, i.e. the particle swarm optimization method, has also been investigated for its potential use in the reconstruction problem. Global methods usually suffer from an inherently expensive computational burden, but in the case when reconstructing only a few unknown parameters the computational load can be on the same order of magnitude as a local search method. Since a priori data can easily be implemented into the particle swarm method by defining the search space we consider this as a viable method for reconstructing discrete, well defined objects. Another advantage with this method is that the risk of getting trapped in a local minimum is eliminated.

5. References

[1] M. Benedetti, M. Donelli, G. Franceschini, M. Pastorino and A. Massa, “Effective exploitation of the *a priori* information through a microwave imaging procedure based on the SMW for NDE/NDT applications,” *IEEE Trans. Geosc. Remote Sensing*, **vol. 43**, no 11, pp. 2584–2592, 2005.

[2] F. X. Canning, “Singular-value decomposition of integral equations of EM and applications to the cavity resonance problem,” *IEEE Trans. Antennas Propagat.,* **vol. 37**, no 9, pp. 1156–1163, 1989.

[3] S. Caorsi, G. L. Gragnani, M. Pastorino and M. Sartore, “Electromagnetic imaging of infinite dielectric cylinders using a modified Born approximation and including *a priori* information on the unknown cross sections,” *IEE Proc.-Microw. Antennas Propag.,* **vol. 141**, no 6, pp. 445–450, 1994.

[4] P. Charbonnier, L. Blanc-Féraud, G. Aubert and M. Barlaud, “Deterministic edge-preserving regularization in computed imaging,” *IEEE Trans. Image Processing*, **vol. 6**, no 2, pp. 298–311, 1997.

[5] W. C. Chew and J. H. Lin, “A frequency-hopping approach for microwave imaging of large inhomogeneous bodies,” *IEEE Microwave Guided Waves Letters*, **vol. 5**, no 12, pp. 439–441, 1995.

[6] A. P. Engelbrecht: *Particle Swarm Optimisation*, Computational Intelligence - Part IV Swarm Intelligence, John Wiley & Sons Ltd, 2002.

[7] A. Fhager, M. Persson, “Using a priori data to improve the reconstruction of small objects in microwave tomography” *IEEE Trans. Microwave Theory Techniques*, **vol. 55,** no. 11, pp. 2454-2462, 2007.

[8] A. Fhager, P. Hashemzadeh, L. Bååth and M. Persson, “Microwave imaging for mammography using an iterative time-domain reconstruction algorithm; Initial experiments,” in *Proceedings of the 16th International Zurich Symposium on Electromagnetic Compatibility; Topical Meetings*, pp. 65–70, 2005.

[9] P. Hashemzadeh, A. Fhager and M. Persson, “Experimental investigation of an optimisation approach to microwave tomography,” *Electromagnetic Biology and Medicine*, **vol. 25**, no 1, pp. 1–12, 2006.

[10] T. Isernia, V. Pascazio and R. Pierri, “On the local minima in a tomographic imaging technique,” *IEEE Trans. Geoscience and Remote Sensing,* **vol. 39**, no 7, pp. 1596–1607, 2001.

[11] N. Joachimowicz, C. Pichot and J. P. Hugonin, “Inverse Scattering: an iterative numerical method for electromagnetic imaging,” *IEEE Trans. Ant. Prop.*, **vol. 39**, no 12, pp. 1742–1753, 1991.

[12] J. Robinson and Y. Rahmat-Samii: “Particle Swarm Optimisationm in Electromagnetics,” *IEEE Trans. Antennas Propagat.,* **vol. 52**, no. 2, pp. 397-407, 2004.

[13] A. N. Tikonov and V. Arsenine, *Solutions of ill-posed problems,* Winston, New York, 1977.