A Trust Region Method for EIT Application for Human Brain

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1. Introduction

In electrical impedance tomography (EIT) the internal electrical conductivity can be reconstructed from voltage measurements on the surface of an object under study. The EIT reconstruction problem is a nonlinear ill-posed inverse problem, and special approaches must be implemented to recover a stable solution. As EIT is an ill-posed problem, small perturbations in the measured boundary voltages can cause arbitrarily large errors in the estimated internal electrical conductivity. Hence, in this situation it is difficult to obtain a satisfactory solution from the nonlinear and ill-posed EIT problem. So it is necessary to use regularization techniques because of the ill-posed nature of the problem. In this paper, we introduce the regularization problem as a quadratic constrained least squares problem. In the area of optimization this problem is a special case of trust-region methods, which is known as the trust-region subproblem (TRS) (Björck,1996). A comprehensive study regarding different approaches for solving the regularization problem as a trust-region subproblem is given in (Rojas, 1998). We adopted the recently developed method for solving the trust-region subproblem in the regularization case (Grodzevich, 2004) for the EIT problem. In this method the regularized solution was obtained using a parameterized trust region approach to estimate the region of maximum curvature of the L-curve.

2. Trust Region Subproblem

We formulate the EIT regularization problem as the following quadratically constrained least squares problem

$$R_{\Delta} := \min \left\| Jx - \delta U \right\|_2 \quad \text{s.t.} \left\| x \right\|_2 \le \Delta \tag{1}$$

where $x \in \Re^n$ is the perturbation in conductivity distribution, σ, that maps to the differential measurements $\delta U \in \Re^m$, and $J \in \Re^{m \times n}$ is the Jacobian matrix. The solutions for (1) are the same as the following problem, which can be formulated by squaring both objective and constraint in (1),

$$\eta_{\Delta} := \min \quad Q(x) := x^T Hx - 2G^T x \quad \text{s.t.} \left\| x \right\|_2^2 \le \Delta^2 \tag{2}$$

where $H = J^T J$ and $G = J^T \delta U$. The optimization problem in (2) is called the trust region subproblem (TRS). The solution for the regularization problem (1) is found by solving the TRS (2) sequentially. The TRS can be used to form the L-curve,

$$L(J, \delta U) = \left\{ \left(\log(\Delta), \log\left\| Jx_{\Delta} - \delta U \right\| \right) : \Delta > 0 \right\} \tag{3}$$

The regularization parameter, Δ can be found through the point of maximum curvature, or the elbow, on the L-curve (Hansen, 1992). Therefore, the trust region radius, Δ needs to change iteratively to steer the algorithm to the elbow of the L-curve. A feasible vector $x^* = x_{\Delta}$ is a solution to (2) if and only if (Sorensen, 1997):

$$\begin{aligned} &(H - \lambda^* I) x_{\Delta} = G \\ &(H - \lambda^* I) \ge 0 \\ &\lambda^* (\left\| x_{\Delta} \right\|_2^2 - \Delta^2) = 0 \\ &\lambda^* \le 0 \end{aligned} \tag{4}$$

for a Lagrange multiplier $\lambda^* = \lambda_{\Delta}$. So the objective function value in (1) and Δ represent a unique point on the L-curve. More details regarding derivation of the above parameters are given in (Grodzevich 2004).

3. Methods

The experiments were conducted on a cylindrical phantom of 10cm height and 5cm radius containing 48 circular stainless steel electrodes in three rings with each ring composed of 16 electrodes connected peripherally around the cylinder. The phantom was filled with 0.2M saline and connected to an EIT system. The test objects were a metal rod 6 cm tall with diameter 1.5 cm and a plastic rod with 5.5 cm height and 1.3 cm diameter. Figure 1 depicts the objects and the overhead view of the electrode geometry.

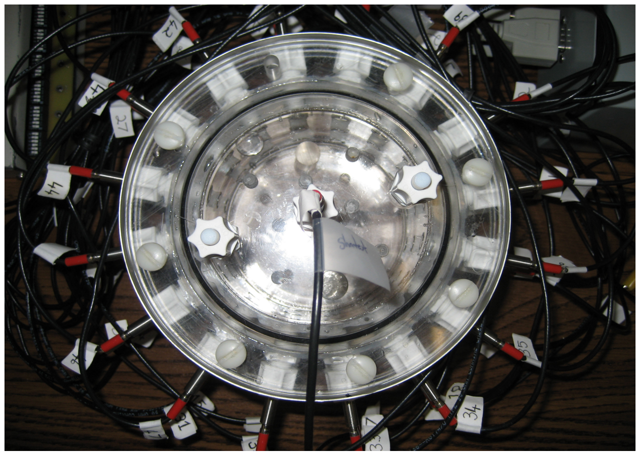

Fig.1 A cylindrical phantom with two test objects.

4. Results

Figure 2 shows the background-subtracted images for TRS method. The metal rod appeared as red while the plastic rod appears blue. Figure 3 shows the different points on L-curve. The best possible solution that was obtained using the Tikhonov method is also shown in the graph. Eventhough the curve is not strongly L-shaped, both horizontal and vertical parts are distinguishable. The TRS algorithm was able to locate the elbow and this was close to the best solution provided by the Tikhonov approach.

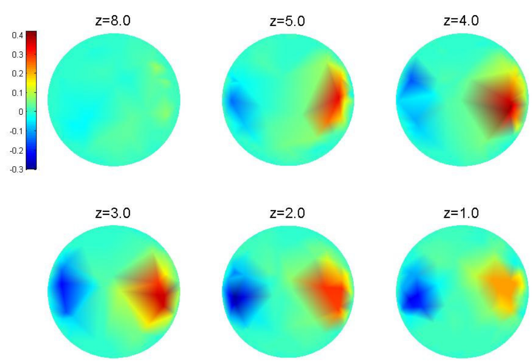

Fig.2 Reconstructed images with TRS at different heights of phantom.

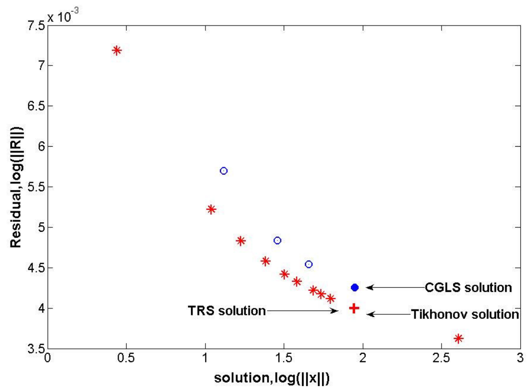

Fig.3 L-curve with TRS and best solution with Tikhonov.

5. Discussion

The two test objects were clearly distinguishable at correct position at all different heights as indicated in Figure 2. The TRS algorithm was able to follow points on the curvature and locate the elbow as shown in Figure 3. The TRS selected solution (marked with +) was close to the best possible Tikhonov solution. The advantage of the constrained least square approach in comparison to the Tikhonov method is that the physical properties of the problem could be used to estimate the norm of constraint Δ. The TRS was solved using a parameterized trust region to estimate the region of maximum curvature of the L-curve. Our simulations demonstrate that the TRS method is robust in obtaining high-quality reconstruction in EIT problems. It may be possible to use it in the future to image impedance changes related to brain study.

References

Björck, A., 1996. Numerical Methods for Least Squares Problems. SIAM, Philadelphia

Hansen, P.C., 1992. Analysis of discrete ill-posed problems by means of the L-curve. SIAM Rev., **34(4),**561-580

Grodzevich, O., 2004. Regularization using a parameterized Trust-Region subproblem. M.Sc. Thesis, Department of Combinatorics and Optimization, University of Waterloo, Canada

Rojas, M., 1998. A Large-Scale Trust-Region Approach to the Regularization of Discrete Ill-Posed Problems. Ph.D. Thesis, Technical Report TR98-19, Department of Computational and Applied Mathematics, Rice University, Houston

Sorensen, D.C., 1997. Minimization of a large-scale quadratic function subject to a spherical constraint. SIAM Journal on Optimization, **7(1)**, 141-161